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ABSTRACT

This curriculum bulletin is one of a planned series of bulletins designed to meet the needs of teachers and supervisors who are working to improve the achievement level of mathematics in the schools. The eighty sequential units of this bulletin are organized into 3 categories - (1) Sets, number and numeration, (2) Operations, and (3) Geometry and measurement. A "Note to Teacher" is included in several of the units to provide further clarification of mathematical concepts connected with the unit and/or to understand reasons for the developmental material. This bulletin is designed to provide articulation with Grade 4 mathematics and with the mathematics of Grades 6, 7, and 8. This publication is the last of a four year sequence in Intermediate School Mathematics based on their philosophy of what should and can be taught in Grades 5 through 8. (RP)

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Mathematics

Grade 5

SE 007 840

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Mathematics
Grade 5

Bureau of Curriculum Development
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FOREWORD

The MATHEMATICS, GRADE 5, CURRICULUM BULLETIN is one of a planned series of bulletins designed to meet the needs of teachers and supervisors who are working to improve the achievement level of mathematics in our schools. The material has been planned to help teachers meet the diverse mathematical needs of the children in fifth-grade classes in our schools. In addition to the emphasis that is always placed on arithmetic computational skills, this bulletin shows how to include other areas considered important, such as, concepts, skills, and ideas from Algebra and Geometry.

Use of the new bulletin will provide articulation with grade 4 mathematics and with the mathematics of grades 6, 7, and 8. The publication completes a four-year sequence in Intermediate School Mathematics based on the newer mathematical philosophy of what should and can be taught to children in grades 5 through 8. Revisions will be made in this publication as a result of use in schools during the introductory period.

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Dr. Joseph O. Loretan,* Deputy Superintendent of Schools and Mrs. Helene Lloyd, Assistant Superintendent supervised the project.

Assistant Superintendent Dr. William Bristow, Director of The Bureau of Curriculum Research and members of his Bureau cooperated in the initial discussions.

Mrs. Leona Critchley and Mrs. Alice Lombardi, Staff Coordinators of the Bureau of Mathematics, planned and prepared the sequence and organization, wrote the units, and edited the final draft for publication.

Mrs. Eva Pollack, Staff Coordinator of the Bureau of Mathematics, reviewed the units, made suggestions, and rewrote parts of some units.

Mr. Grossman, as Acting Director of Mathematics edited and suggested revisions and additions.

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Grateful acknowledgment is made to all those responsible for the production of Mathematics Cycles Grade 5 and Mathematics Cycles Grade 6.

******Deceased*

INTRODUCTION

This is the first part of a two part bulletin that has been prepared as a revision of the earlier Mathematics 5 Cycles. It includes all of the topics in Mathematics Cycles — Grade 5(1) enriched and expanded to include the newer emphasis of a modern mathematics curriculum as found in the 1965 edition of the Mathematics 6 Bulletin. All relevant material in these bulletins has been utilized.

This will mean that students completing this course will be in a better position to complete a more thorough course in Mathematics 6.

It *is* important for children to develop speed and accuracy in computation, but in addition to computational skills, it is important for children to develop understanding not only of arithmetic concepts but of concepts from algebra and geometry.

The 80 units of this bulletin are organized into 3 categories:

Sets; Number; Numeration
Operations
Geometry and Measurement

These categories are shown in the Scope and Sequence Chart on page xx. This chart may also be considered a Table of Contents.

The 80 units are sequentially planned. For example: After an introduction to sets, under the category "Sets," children are led to see the relation between Union of Sets and Addition of Whole Numbers under the category "Operations." This pattern is followed throughout.

The units also follow a *spiral* pattern, in that development of concepts and operations are repeated at increasing levels of understanding.

Concepts from Algebra, such as: open sentences, relations between numbers, graphing of solution sets, are included in the exercises of most of the units. Concepts from geometry are also included. A "Note to Teacher" is included in those units where it was felt the teacher might want further clarification of the mathematical concepts connected with the unit and/or to understand reasons for the developmental material.

Objectives for each unit are clearly stated immediately before the "Teaching Suggestions" designed for the implementation of those objectives.

Review of background necessary for the introduction of new topics is suggested where necessary. For example: Before adding and subtracting fractional numbers using the least common denominator method, suggestions are made for renaming fractions, regrouping fractions, etc.

Asterisks before a unit or before an item within that unit indicate that these developments may be used at the discretion of the teacher.

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(1) Mathematics Cycles, Grades 5 — Curriculum Bulletin 1961-62 Series, No. 6

SCOPE AND SEQUENCE

Pages vi to xi

Grade 5

SCOPE AND SEQUENCE

* — indicates optional units
page numbers below — indicate location of units

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Grade 5

SCOPE AND SEQUENCE

* — indicates optional units
page numbers below — indicate location of units

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33. Set of Fractional Numbers: Twelfths; Ninths; Sevenths p. 237		
	34. Addition of Fractional Numbers: Properties Applied; Horizontal and Vertical Format p. 245	
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Grade 5

SCOPE AND SEQUENCE

* — indicates optional units
page numbers below — indicate location of units

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SETS; NUMBER; NUMERATION

UNIT 1 - SETS

NOTE TO TEACHER

Why Sets?

A set may be defined as a collection or group of objects, ideas, or numbers.

The use of sets of concrete objects provides a visual and tactile experience for the development of the abstract concept of "number" and for operations on number.

Since geometric and algebraic, as well as arithmetic ideas, can be expressed in terms of sets, set concepts and terminology have a wide application.

Meaning of Set

A mathematical name for a collection of things is "Set".

A set may be composed of similar or dissimilar objects, things or ideas, which we decide to consider as a unit.

All the mathematics books that we use may be similar, but each is a different, discrete book. The collection of Mathematics Books can be considered as a set.

A collection of completely dissimilar things such as a pencil, a stapler, an orange, may also be considered as a set. These may be the set of objects on the Teacher's desk.

To convey the idea that a collection is a set, the collection must be clearly defined.

A collection of "Interesting Books" is not a clearly defined set because what is interesting to one may not be interesting to another. The set "Mathematics Books in our Classroom" is clearly defined.

Elements of a Set

Each object, thing or idea in a set is called a member or an element of that set. For example, the elements of a set of golf clubs are the individual clubs in that set.

Describing the set clearly helps to determine whether an object is or is not an element of the set.

For example, when we talk about, "The Set of The Five Boroughs of New York City", there is no doubt that Richmond is an element of that set.

We may also describe this set by tabulating its elements, for example, "The Set Whose Elements are: Manhattan, Bronx, Brooklyn, Queens, Richmond".

A set may consist of many elements. The set of all teachers in New York City is a set containing many elements.

A set may consist of only one member, for example, "The Set of the Teachers in this Room".

A set may contain no members, for example, "The Set of All Teachers Who Are 15 Feet Tall".

Notation for Sets

The elements of a set may be tabulated within braces. e.g., {Debbie, Gail, Kathy} can be read as: "The set whose elements are Debbie, Gail, Kathy".

It should be understood that when we list the elements of a set we never list the same element more than once.

$\{ \triangle, \square \}$ can be read as "The set whose elements are triangle, square".

Sets may also be identified by the use of a capital letter.

For example:

$A = \{\text{Debbie, Gail, Kathy}\}$ read as: "Set A is the set whose elements are Debbie, Gail, Kathy".

$B = \{\text{Bill}\}$ read as: "Set B is the set whose only member is Bill".

$C = \{\text{apple, banana}\}$ can be read as: "Set C is the set whose elements are apple, banana".

When the sets are referred to again they may be recorded as: A, B, and C.

The empty set can be designated by braces only, $\{ \}$, or by the symbol $\emptyset$. The empty set is sometimes called the null set.

The "belonging to" symbol for showing that an object is an element of a set is $\in$. The statement, "Kathy $\in$ A", is read: Kathy is an element of Set A".

The symbol for an element that is not in a set is $\notin$. The statement, "Ann $\notin$ A", is read: "Ann is not an element of Set A".

TEACHING SUGGESTIONS

Objective To help children understand the meaning of: Elements of a Set; Sets; Subset; Set Notation.

Procedure The previous exposure of the children to these concepts will help the teacher determine the extent to which she will use the following procedures.

1. Begin if necessary by presenting physical objects such as: a set of dishes; a set of checkers; etc. Elicit that a collection of things is called a set. Tell children that the mathematical name for a collection of objects is "Set".

2. Display and discuss sets of objects. Include some examples in which the elements of the set are similar, for example: two crayons; some in which the elements are dissimilar, for example; a book, a ball, and a piece of chalk.

   Children discuss dissimilar objects as a set.

3. Have children use available objects to create their own sets on their desks and then to describe their sets.

   Have them describe sets of similar things. Make sure they understand that each thing is a discrete object.

4. Children follow the same procedure using sets of dissimilar objects. They discuss why these are considered as sets.

5. Discuss sets that are not easily displayed.
   (The set of characters in a book; the set of states in the U.S.)

6. Tell children that each object in a set is called a member or an element of the set. Have children name elements in displayed sets; in sets of similar objects; in sets of dissimilar objects.

7. Tell children that a set may have only one element. Have them think of and discuss sets that have only one element.

   [the class aquarium; the principal of the school;
   the capital building of New York State.]

8. Ask the set of tigers in the room to stand. When no one stands ask:

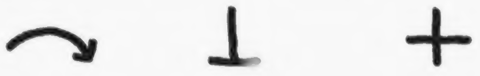
   Is there a set of tigers? [Yes]
   Is there a set of tigers in this room? [No, the set of tigers is the empty set].

9. Tell children that a set may have no elements and that this kind of set is called the empty set, or the null set. Have children suggest sets which contain no elements.

   [The set of women who have been presidents of the United States;
   The set of living dinossaurs; The set of astronauts in Grade 5.]

10. Discuss the use of symbols as a way of conveying ideas.

For example, Highway signs are a set of symbols that communicate ideas of curve, intersecting roads, etc.



11. Introduce set symbols.

A way of specifying a set is to list the names of the elements or symbols for the elements between braces { }.

{a piece of chalk, a globe, a milk container}

{□, Δ, * , 0 } is read as: the set whose elements are a square, triangle, star, circle.

Make sure that children understand that the same set may be recorded in many different ways, for example:

{ Δ, * , □ , 0 }, { 0, □, Δ , * }

or

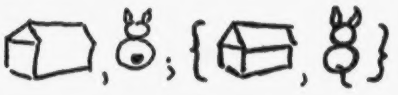
{The living presidents of the U.S.}

or

{L:B: Johnson, Harry Truman, Eisenhower}

Tell children that the order of listing the members of a set does not matter. Children should note that elements in a set are separated by commas. The same element appears only once in a listing.

12. Record on the board:

Mary, Jane; {Jane, Mary} 

Have children identify those that are sets.
[Only those within braces are sets.]

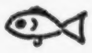
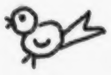
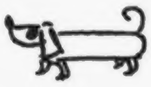
13. Tell children that a set of many objects can be thought of as a single idea and may be named by a capital letter, e.g., A, B, C.

Record: A = {Bob, Sam, Jack}

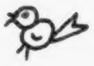
Read to children as: Set A is the set whose elements are Bob, Sam, Jack.

Present other sets in the same way. Have children write their own sets using capital letters.

14. Introduce the symbol for element of a set ($\in$).

Record: A = { , ,  }

Children read: "A is the set whose elements are a fish, a bird, a dog".

Record:  $\in$ A

Read to children as: "A bird is an element of set A".

PRACTICE and / or EVALUATION
SUGGESTED EXERCISES

1. Name as many words as you can that mean set.

   [Collection, group, team, etc.]

2. Write the set of odd numbers from 2 through 9.

   [$\{3, 5, 7, 9\}$]

3. Record next to the following sets, using set notation, those that have no elements:

   The set of dogs that fly.
   The set of children in your fifth grade who have green hair.
   The set of dishes in the closet.
   The set of letters of the alphabet.
   The set of even whole numbers less than 1.

4. List the elements of the following sets:

   The set of Great Lakes.
   The set of children who are officers of your class.
   The set of the capital city of New York State.

5. Record in set notation two examples of sets containing more than one element; two examples of sets containing only one element; two examples of sets containing no elements.

6. Write the following statement in set notation:

   Set A is the empty set.

   [A = { }]

7. Consider the following sets:

   A = {John, Mary, baby-Sue}

   F = {Monday, Tuesday, Wednesday, Thursday, Friday}

   S = { ▭ , ✱ , Δ }

   Write these sentences in set notation:

   John is an element of set A.
   Tuesday is an element of set F.
   A rectangle is not a member of set A.
   Baby-Sue belongs to set A.
   A triangle is an element of set S.

8. Use set notation to write:

   The set of letters in your name.
   The set of numbers between 20 and 30.
   The set of numbers that are less than 10.

SETS; NUMBER; NUMERATION

UNIT 2 - SETS: One-to-One Correspondence; Equivalent Sets; Number as a Property of a Set; The Set of Counting Numbers

NOTE TO TEACHER

One-to-One Correspondence

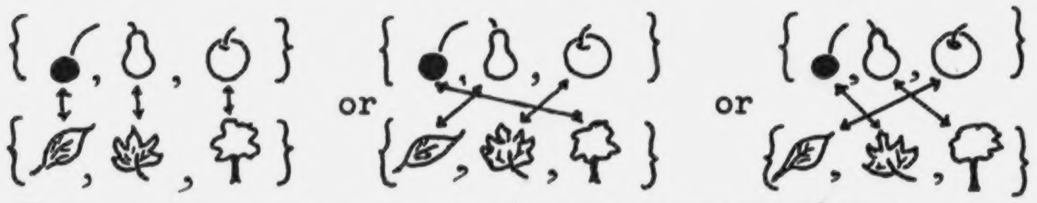
Matching objects in one set to objects in another set is not new. Children have always been encouraged to note when there is one book for each child, a chair for each child, a glove for each hand.

When the elements of two sets can be paired so that each element of one set is associated with one and only one element of the other set, and no element of either set is excluded in this pairing, the two sets are said to be in one-to-one correspondence.

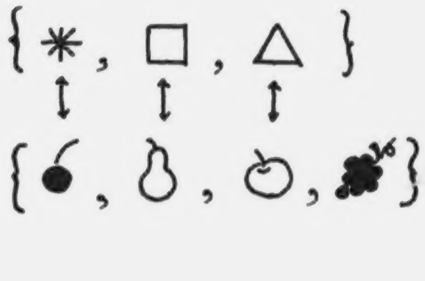
The elements of the two sets that are being matched may or may not be similar.

The sets below can be matched in a one-to-one correspondence.

The order in which they are matched is irrelevant.



These sets are not in one-to-one correspondence.



$\{a, b, c, d\}$

$\{x, y\}$

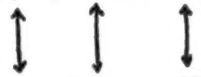
Equivalent Sets

Two sets of elements that can be placed in one-to-one correspondence are said to be equivalent sets.

Equivalent sets have the same number of elements.

The elements of the sets below have been put into one-to-one correspondence.

Set A is {✳, Δ, □}



Set B is {O, ▭, X}

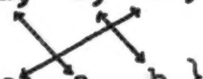
Each element of Set A is paired with one and only one element of Set B in each instance and each element of Set B is paired with one and only one element of Set A.

Set A is equivalent to Set B

Equal Sets

When two sets have the same elements and therefore have the same number of elements, they are said to be equal sets.

Set A is {a, b, c}



Set B is {c, a, b}

The elements of Set A and Set B are the same.

Every element of Set A is an element of Set B.

Every element of Set B is an element of Set A.

The number of elements in Set A and in Set B is the same.

Therefore:

Set A equals Set B

Set A and Set B are different names for the same set, (which is the usual meaning of "equal" in mathematical situations).

When we compare sets we find that one set may be *equivalent* to another set; one set may be *equal* to another set; one set may have *more* or *fewer* elements. In the last case it is neither equal to nor equivalent to the other set.

Number as a Property of a Set

Consider a set which contains 5 green objects. Its *elements* have the color property of "greeness".

The *set* has a number property of "five-ness".

When sets can be placed in a one-to-one correspondence, they have the same number property.

$\{ a, \; b, \; c, \; d \}$ has a number property of "4-ness".

$\{ x, \; y, \; r, \; s \}$ has a number property of "4-ness".

Both sets have the same number property.
The numbers are equal.

When two sets can not be placed in a one-to-one correspondence, they do not have the same number property.

One number is greater than or is less than the other.

Mathematical symbols for inequalities of number are:

"Is greater than", " > ", for example:

$$7 > 5 \; ; \qquad 5 - 2 > 6 - 4$$

"Is less than", " < ", for example:

$$2 < 3 \; ; \qquad 3 + 8 < 4 + 9$$

The Set of Counting Numbers

When children began to count, they began with one object. They associated the number 1 with a set containing one object; the number 2 with a set containing two objects and so on.

The set of numbers used in counting beginning with 1 is called the, "Set of Counting Numbers" and is shown in set notation as $\{1, 2, 3, 4, 5, \ldots\}$

Children should understand that no matter how far they count there is still another number. This is indicated by three dots as shown in the set above, standing for "and so forth".

TEACHING SUGGESTIONS

Objectives: To reinforce the concepts of one-to-one correspondence.
To develop number as a property of sets.

Procedure

1. Question children about the common property of familar things.

   What property do rain, milk and the ocean have in common?
   [liquid]

   What property do a cookie, a chocolate bar, a piece of pie have in common?
   [sweet, edible, fattening]

2. Compare sets of things in the classroom where a 1 - 1 matching is obvious.
   One pencil for one child
   One sheet of construction paper for each child
   One plastic spoon for each jar of paint

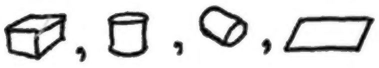
3. Discuss these matchings with the children.

4. Show two sets of dissimilar objects that are in one-to-one correspondence on a display table, for example:
   { a book, a pencil, a paint jar }
   { a ball, a bat, a stapler }

   Discuss the dissimilarity of the elements of the sets.

5. Ask children: What property do the sets on the display table have in common?
   [The number 3]

6. Compare other sets of objects that are in one-to-one correspondence to emphasize that number is a property of set. For example:

   a. { O, Δ, ✳ , x } and {  } [4]

   b. The number of children in Grade Five who are 3 years old.
      [{ }]

      and

      The number of children in Grade One who are 25 years old.
      [{ }]

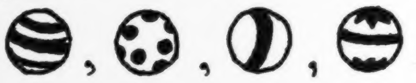
7. Help children to see that sets that can be placed in one-to-one correspondence have the same number.

8. Tell children:

   a. A way we indicate the number of elements in a set is by using the letter N (for number) before the set. For example:
      N {✳, Δ, O}

   b. When we write N { ✳ , Δ, O } = 3 we are saying that the number of elements in the set whose members are star, triangle, circle, is 3.

9. Children complete the following:

A = {  }

Number of elements = □ [4]

N (A) = □ [4]

G = {Sue, Betty, Jane}

Number of elements = □ [3]

N (G) = □ [3]

J = { □ }

Number of elements = □ [1]

N (J) = □ [1]

10. Discuss with children:

    a. How they arrived at the number of each set. [Counting]

    b. Counting as the assignment of names to successively larger quantities.

    c. The Set of counting numbers { 1, 2, 3, . . .}

    d. That each successive counting number is one more than the number before it.

11. Children write in set notation:

    First five counting numbers
    Even numbers between 2 - 10
    Even numbers between 25 - 43
    Odd numbers between 2 - 10
    Odd numbers between 25 - 43

12. Discuss the endless set of counting numbers.

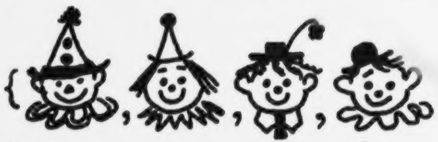
    Children note that no matter how far they count there are still more counting numbers.

    Tell children we use three dots (. . .) meaning "and so forth" to indicate such a continuation.

13. Review notation for the number of a set.

    [N (A)]

14. Present the following for children to complete:

A = {  } - - - - Number of elements in A = ☐

B = { 8, 3, 2, 1, 6, 4 } - - - - - - - - Number of elements in B = ☐

C = {Mary, Bob, John} - - - - - - - - - N (C) = ☐

D = { ✳ , Δ, O } - - - - - - - - - N (D) = ☐

E = { ✳ , O, Δ } - - - - - - - - - N (E) = ☐

They note that Sets E and D have the same elements and the same number. The numbers are equal; $3 = 3$ Why?

[The quantity represented is the same; the cardinal number is the same; the idea of "three" is the same; the elements can be matched with none left over; etc.
The Sets contain the same elements.]

15. To obtain the (counting) number of a set, such as

$$A = \{ \square, \ \triangle, \ 0, \ + \}$$

Compare set A with the set of counting numbers $\{1, 2, 3, 4, 5, \ldots\}$ and set up the following one-to-one correspondence:

$$\begin{array}{cccc} \{ \square, & \triangle, & 0, & + \} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{ 1, & 2, & 3, & 4 \} \end{array}$$

Have children notice our use of "counting" and that the last counting number needed is the number of the set.

PRACTICE AND / OR EVALUATION
SUGGESTED EXERCISES

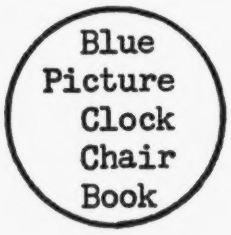
1. Write in set notation the set of counting numbers less than 15.

$$[\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 \}]$$

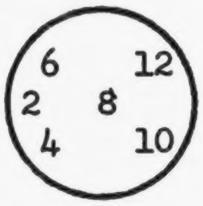
2. Show in set notation the set of all counting numbers greater than 30.

$$[\{ 31, 32, 33, 34 \ \ldots \}]$$

3. Here are two collections of things.



A



B

Show in set notation the number of elements in each set.

$$\begin{bmatrix} N (A) = 5 \\ N (B) = 6 \end{bmatrix}$$

4. Show that the counting number of a set you choose is 6.

   [N { 0, x, □, ✳ , +, ÷ } = 6]

   or

   [A = { 0, x, □, ✳ , +, ÷ }
   N (A) = 6]

5. Tell how you would find the number of elements in the set of girls in your class who are present today.

   [By counting]

6. Write two different ways in which the set of girls may be described.

   [By listing
   By description]

7. Show two sets using set notation.
   Compare these to show one-to-one correspondence.

SETS; NUMBER; NUMERATION

UNIT 3 - SET OF WHOLE NUMBERS: CONCEPT OF NUMBER AND NUMERAL; READING AND WRITING NUMERALS

NOTE TO TEACHER

Number and Numeral

To be able to distinguish between a System of Numbers and a System of Numeration, the difference in meaning between the terms number and numeral should be reviewed.

Number is the property of a set that denotes the idea of "How Many"; of quantity. This idea exists in the mind only.

A numeral is but the symbol or a name for a number. Three, Trois, Tres, 3, III are all symbols for the idea of the same number. Punched holes are symbols used with computers to convey the idea of number or to transmit information about number. A numeral is any symbol, agreed upon, for communicating the idea of number.

Children learn about the idea of quantity - the number of objects - before they read or write the symbols or numerals for representing the idea.

The symbol is not the number just as writing "blue" on the chalkboard is not the actual color blue. Symbols are simply ways of representing ideas of number.

Any number may be represented in many ways.

There are times when distinguishing between the words, "Number" and "Numeral" becomes cumbersome so that often we use the word

"Number" when we mean "Numeral". However, it is wise to try to use precise mathematical language whenever convenient.

The Set of Whole Numbers

The set of counting numbers, sometimes called the set of natural numbers is part of the set of whole numbers. The set of counting numbers can be symbolized in set notation as

(1, 2, 3, 4, 5 . . .)

Zero is not a counting number. The set of whole numbers includes zero which is the cardinal number of the empty set.

The set of whole numbers is recorded in set notation as

(0, 1, 2, 3 . . .)

Note that the set of whole numbers, unlike the set of counting numbers, has as its smallest number, zero. Like the set of counting numbers, the set of whole numbers has no greatest number.

Both sets are infinite (have an infinite number of elements).

TEACHING SUGGESTIONS

Objective: To help children understand:
- Meaning of number, numeral
- Zero as a whole number
- There is no greatest number

To reinforce:
- Counting
- Reading and writing numerals

Procedures

Concept of Number and Numeral

1. Children name other children in the classroom. Teacher lists the names,

then erases them. Discussion helps children realize that the children have not been erased.

Only the symbols or names for the children have been erased.

2. Use the same procedure for:
    Symbols on a musical scale and the notes they represent.
    Names for colors and the colors they represent, etc.

3. Display a set of six objects.
   Children identify the number of members of that set.
   Teacher records that number as: **six**, 6, VI, 𝍸 |
   Question the children:
    What is the same about these recordings?
    What is different about them?
    If we erase these symbols, have we erased the set of objects?

4. Tell children that the idea of "How Many" in the set is a number.
   The symbol is a numeral.

5. Teacher displays sets of various sizes.
   Children identify the number of each set.
   Children write the numeral for each number.

6. Discuss:
    Numbers are ideas which cannot be seen, written, erased.
    Numerals which stand for numbers can be seen, written, erased.

7. Write the following on the chalkboard and discuss the distinction between symbols and objects, number and numeral.

   A tiger - What do you see here?
   9, 5 - Which is larger?

The Set of Whole Numbers

1. Use a number line labelled from 0 to 20. Discuss:

    What is the next larger number after 5; after 3; after 15?
    What number comes just **before** 5; before 3; before 21?
    What is the next whole number after 1?
    Is there any number that comes just before 1?

2. Teacher writes numerals on the board: 16, 10, 8, 13, 0
   Ask children:
    What is the greatest number shown?
    What is the least number shown?

3. Tell children that the numbers under discussion are called whole numbers; that the set of whole numbers includes zero.

4. Teacher writes numerals on the board: 17, 38, 45, 0, 4
Direct children to rewrite them in order from least to greatest.
Ask children to continue to write numbers in order after the greatest number shown.

   Ask them what they think the largest number would be, were they to continue.

   Elicit from them that there is no largest number. Why?

5. Encourage children to tell what the set of whole numbers is.

   [The set of whole numbers is the set of numbers whose least number is zero, and which has no greatest number.
   Children will express this idea in their own words.]

6. Show children the following, {0, 1, 2, 3, 4 . . .}
Discuss the use of the three dots.

PRACTICE and / or EVALUATION
SUGGESTED EXERCISES

1. List the set of the first 10 whole numbers.

   [{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}]

2. List the set of whole numbers greater than 51.

   [{52, 53, 54 . . .}]

3. What is the greatest whole number in the set of whole numbers?

   [There is no greatest whole number]

4. List the elements of the following:

   The set of whole numbers between 100 and 101.

   [{ }]

   The set of whole numbers less than 5 and greater than 3.

   [{ 4 }]

5. List a set containing five elements each representing the number 9. Answers may vary.

$[\ \{\ \text{IX},\ 8 + 1,\ 4\frac{1}{2} + 4\frac{1}{2},\ 3 \times 3,\ 27 \div 3\ \}\ ]$

6. Children write the numerals for the following:

Five hundred seven
Twelve hundred twenty-five
Three thousand forty

7. Children write the following in words.

706 1342 3029 2500

8. Dictate numbers such as the following:

3286 41 1870 1028
605 2300 3 4009

Note: Number symbols such as 5280 may be read as 52 hundred eighty as well as 5 thousand 2 hundred eighty.

9. Children supply the missing numerals, counting by tens:
For example - 134, 144, 154

268, ____, ____ 1380, ____, ____
990, ____, ____ 3507, ____, ____

10. Children write the missing numerals in each sequence:

1970, 1980, 1990, ____, ____, 2020
1530, 1520, 1510, ____, ____, 1480
____, ____, 1000, 1001, 1002, ____
3500, 3600, 3700, ____, 3900, ____
____, ____, 2000, 2100, 2200, ____

11. Children count forward in each of the following series, limit determined by teacher.

Interval: Numbers through 9

47, 51, 55 . . .
69, 77, 85 . . .
178, 184, 190 . . .
305, 314, 323 . . .
577, 586, 595 . . .

Interval: 9 - 99

30, 60, 90 . . .
150, 200, 250 . . .
60, 120, 180 . . .
5049, 5059, 5069 . . .
3028, 3048, 3068 . . .

21, 42, 63 . . .
17, 34, 68 . . .
28, 56, 84 . . .
62, 124, 186 . . .
215, 230, 245 . . .

Interval: 99 - 999

300, 600, 900 . . .
600, 1200, 1800 . . .
700, 1400, 2100 . . .
2368, 2468, 2568 . . .
6072, 6272, 6472 . . .

250, 500, 750 . . .
130, 260, 390 . . .
402, 804, 1206 . . .
325, 650, 975 . . .
215, 430, 645 . . .

Interval: 1000, 2000, 4000, etc.

1000, 3000, 5000 . . .
1300, 2300, 3300 . . .
2529, 3529, 4529 . . .
1480, 3480, 5480 . . .
1461, 3461, 5461 . . .

12. Children count backward in each of the following series:

Interval: Numbers through 9

67, 63, 59 . . .
219, 211, 203 . . .
648, 643, 638 . . .
351, 342, 333 . . .
429, 422, 415 . . .

Interval: 30, 40, 50, etc.

180, 150, 120 . . .
350, 300, 250 . . .
300, 240, 180 . . .
5089, 5079, 5069 . . .
2098, 2078, 2058 . . .

Interval: 100, 300, 700, etc.

1800, 1500, 1200 . . .
2428, 2328, 2228 . . .
3500, 2800, 2100 . . .
5472, 5272, 5072 . . .
3925, 3725, 3525 . . .

Interval: 1000, 2000, etc.

9000, 8000, 7000 . . .
5300, 4300, 3300 . . .
7680, 6680, 5680 . . .
8320, 6320, 4320 . . .
6529, 5529, 4529 . . .

GEOMETRY AND MEASUREMENT

UNIT 4 - GEOMETRY: SETS OF POINTS; CURVES; NUMBER LINE

NOTE TO TEACHER

Points

We continue our exploration of Sets with their extension to Sets of Points.

A point is an idea.
A point cannot be seen or felt. It cannot be measured.

A point may be represented as a dot, as the end of a sharply pointed object, as the location where two walls and a ceiling meet, etc.

A point can be represented as a fixed location which does not move.

If the dot on the paper is erased, or the paper moved, the point still exists, and would have to be described in some other way, perhaps by a set of directions.

Space can be defined as the "set of all points".

Curves

Think of the idea of a path between two points in space.

All paths are sets of points in space and are called curves whether the paths are "straight" or not. The mathematical meaning of curve is different from the common meaning of curve.

A string stretched between two points, the representation on a map of the road between two cities are both representations of curves.

Curves include straight lines.

Line Segment

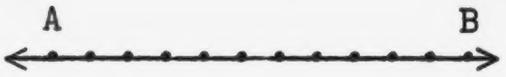
A line segment is an idea. It is a set of points that may be represented by a special curve drawn on paper connecting two dots. When we say "line segment" we mean "straight line segment".

The line segment is the shortest curve between two points. When represented it includes the end points of the line segment.

The symbol for a line segment is $\overline{AB}$. Points A and B are the end points of the line segment $\overline{AB}$,

Lines

A line can be thought of as the extension of a line segment in both directions.

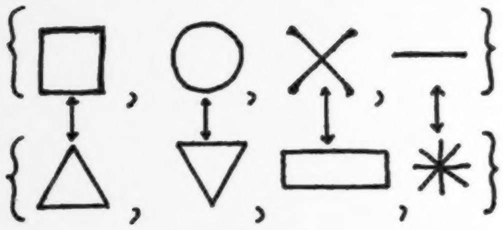


The symbol for the line above is: $\overleftrightarrow{AB}$
Notice how the two arrows indicate extension in both directions.

Number Lines: Sets of Points on a Line

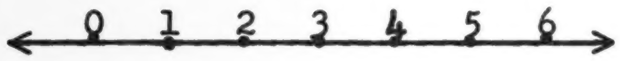
Because every point on a line has a position and because there is a distance between every two points on a line which may be compared with the distance between any two other points on that line, the number line is an invaluable device for helping children as they deal with operations on numbers and to see the one-to-one correspondence between some points on the line and the numbers under consideration.

We have previously discussed the association of a number with a set of things. For example: we associated "4" with any set whose members can be put into a one-to-one correspondence with any other set containing 4 elements, e.g.,



Both of these sets have a common number property. "4" is the counting number that tells how many elements in each of these sets.

"4" may also be associated with a specific point on the number line, thus:



There is on this number line, one and only one point corresponding to any one whole number.

The numbers are ordered. That is, they can be arranged in a sequence. Any number to the right of a number on the number line is greater than any number to the left of that number.

TEACHING SUGGESTIONS

Objective: To introduce geometric concepts of points and lines.
To introduce properties of lines.
To reinforce the number line.

Procedure:

1. Ask each child to touch a spot on his desk.
   Children locate the point.

   [6 inches from the right edge, 3 inches from the bottom edge.]

2. Teacher mentions a location that is not precise. For example: a spot to the right of the door and above the floor.

   Teacher asks children:

   Do you know exactly the spot meant? [No] Why not?
   How can you find out exactly the spot to which I am referring?

   [Answers will vary]

Teacher directs children to suggest ways of describing the spot so that it is exactly located.

3. Tell children that any precise location in space is called a point.

4. Teacher holds a sheet of paper against the chalkboard. Put a finger lightly on a point on the paper. Slide the paper away still keeping the finger at the same location.

Question children:

What did I do to the paper?	[Took it away]
Where is my finger?	[In the same place]
Did the point go away?	[No]

5. Children mark a dot on paper.
They erase the dot.
Elicit from children that the point is still in the same location even though the dot was erased.

6. Tell children that points, like numbers, are mathematical ideas. The dot is just a representation, symbol or model of the point, just as the numeral is a symbol for a number.

7. Discuss: How small is a mathematical point? ; a dot? ; etc.

8. Ask one child to hold the end of a string. Bring about the understanding that the end of the string is a point. Ask another child to hold the other end of the string. Establish this end as another endpoint.

Elicit from the children that the string makes a path from one point to another point.

Have the children stand so that the string is not taut.

Discuss idea of a "curved line".

They stretch the string taut.
Compare curved path with straight path.

[straight path yields the shortest distance between 2 points]

Children drop the string. Establish, through discussion, that the path the string traversed is still there, even though the string is not.

9. Children mark a dot on the paper. They mark a series of dots

in line with the first dot.

•   •   •   •   •   •

They place dots in between the dots already on the paper.

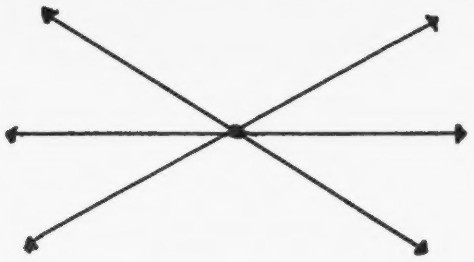
• • • • • • • • • • •

They continue to place dots between the existing dots until they are dense.

They draw a line through the dots.
They note a line is a set of points.

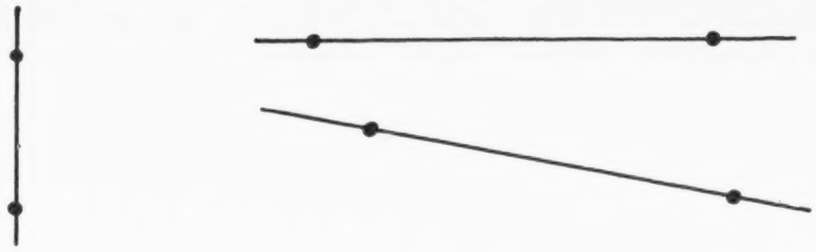
10. Children mark a dot on paper.
They draw a straight line through the dot.
They continue to draw lines through that point.

Children note that any number of lines can be drawn through one point.



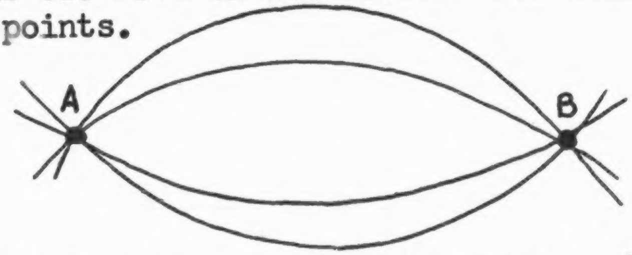
11. Direct children to mark a dot on a paper. They mark a second dot a distance away in any direction.

They draw straight lines passing through the points.

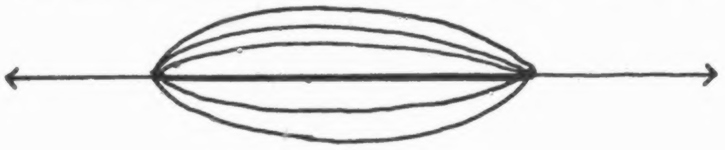


They see that one and only one straight line can be drawn through two points in one plane.

12. Teacher marks two dots on the board. She draws many curved lines between the points.

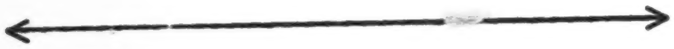


Using strings first, then by drawing lines, children experiment to find the shortest curve between points A and B.



Discuss these curves until children see that all lines are curves but that the curve that is the shortest distance between two points is a straight line segment.

13. Draw a line.



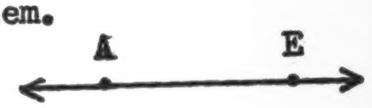
Ask children:

Why are the arrow heads there?
What are the end points of the line shown?

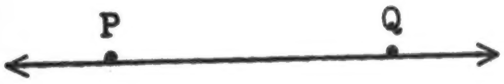
[There are no end points on a line.]

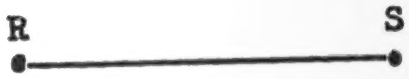
Tell children that a line is named by any two points on the line with the symbol " $\leftrightarrow$ " above them.

$\overleftrightarrow{AE}$ names the line



14. Present the following:





Discuss the line and line segment until children understand that a line segment is part of a line; that a line segment has specified end points.

In the diagram above "R" and "S" are end points of that line segment.

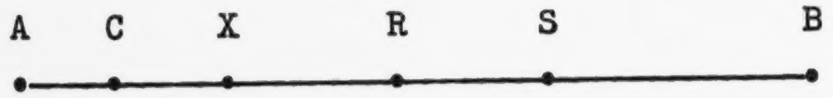
Tell children that the symbol for a line segment is " — " .

The line segment above is symbolized as "$\overline{RS}$"

15. Children draw, identify and symbolize lines and line segments.

16. Direct children to place a dot on a paper. They label it "A" . They place another dot and label it "B" . Children draw a line segment from A to B.

Ask children to mark off a series of points along the line segment. They label each point.



Discuss points on a line.

Can you always insert another point?
How many points do you think are on a line segment?
[An undetermined number.]

17. Children are familiar with the number line.

Emphasize through discussion that:

We are assigning whole numbers to certain points on the line.

Each number can be considered as a name for that point.

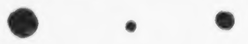
Zero is the least whole number.

Numbers to the right of any number are larger than that number.

Numbers to the left of any number are smaller than that number.

PRACTICE AND / OR EVALUATION
SUGGESTED EXERCISES

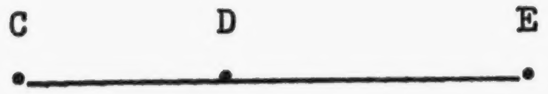
1. Which of the following is the best model of a point? Why?



2. Which of the following are the best answers? Why?

A point is a dot.
A point is an exact location in space.
A point is the end of a nail.
A point is an idea.

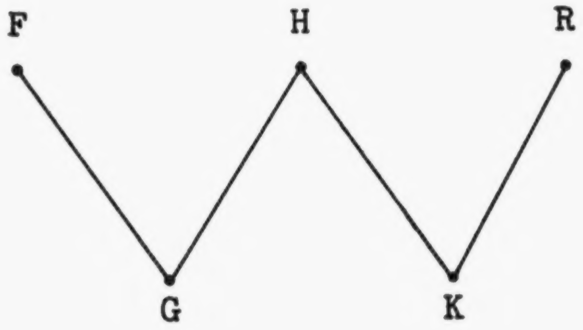
3. Name all the line segments represented in each figure below.



[$\overline{CD}$

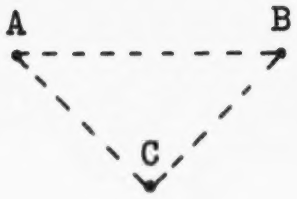
$\overline{DE}$

$\overline{CE}$]



[$\overline{FG}$ $\overline{GH}$ $\overline{HK}$ $\overline{KR}$]

4. Draw all possible line segments to connect the given points.
Name all the line segments.



[$\overline{AB}$ $\overline{BC}$ $\overline{CA}$]

5. Note points "P" and "R" below.
How many line segments can be drawn with end points "P" and "R" ?

* 6. Exploratory Exercises

Draw sketches to show:

How many points can two straight lines always have in common?

Will 2 straight lines always have at least one point in common?

[Remember the definition of a line.]

GEOMETRY AND MEASUREMENT

UNIT 5 - MEASUREMENT: TEMPERATURE

NOTE TO TEACHER

The derivation of the word Geometry comes from "geo", the earth and "metry," measurement. Formerly Geometry was concerned mainly with measurement of the earth.

Measurement is the process of assigning numbers to physical objects or physical quantities or to their mathematical abstraction, line segments etc.

All measurement involves a comparison with a unit of measure. The number of units in the object being measured is called the measure of this object.

All physical measurement is approximate.

When counting objects, the answer to the question, "How many are there?" can be exact.

However, in the measurement of distance, capacity, weight, time, etc., absolute accuracy is impossible. While we are saying, "It is 6 o'clock" the second hand of the clock has already moved on, and it is after 6 o'clock. When measuring with a standard ruler the width of the markings on the ruler and the degree to which the lead of the pencil is sharpened, contribute to variations.

Measurement concepts will be developed by using instruments and units of measure in experience situations. They include: schedules for railroad, bus and airplane travel; time tables for radio and television programs; records of changes in tides and temperature; time of sunrise and sunsets, etc.

Before using standard units of measure, children should be given many opportunities to use non-standard units such as hand, span, foot, unmarked containers.

Children should have many opportunities to estimate length, weight, height, etc. They should check their estimates with measurements made.

Standard instruments (scales, rulers, calendars) for measuring weight, length, time, etc. should be available.

Give children many opportunities to determine the appropriate instrument that can be used in a given situation. For example: in testing vision in the classroom, the distance from the eye chart is 20 feet. What instrument should we use to measure the distance? Would you use a 6 inch ruler or a yard-stick? Why? etc.

Provide ample practice for expressing estimation of quantity in the most suitable unit of measure. The length of a room can be estimated as 18 feet; a piece of paper as 12 inches long.

Indirect Measurement

Some objects cannot be measured directly; their measurements are arrived at by indirect methods. We may call these "indirect measurements". For example:

> The height of a very tall building, a tree or an inaccessible object such as a mountain, is arrived at indirectly. We measure directly certain lengths and angles, and then perform computations on the (approximate) measures obtained.

Temperature

It is not possible to measure temperature directly. To "measure" temperature, we make use of a property of heat. When a substance gets warmer, it usually expands. The instrument used to measure temperature is the thermometer in which a column of liquid expands or contracts in relation to the amount of heat to which it is exposed. We measure the length of the column of liquid to estimate the temperature. The unit of measure on the thermometer is the degree.

The thermometer can be considered a number line. Two scales, the Fahrenheit and the Centegrade, can be introduced. Tabular and graphical comparisons can be made. An important use of these scales is the opportunity to work with positive and negative numbers.

TEACHING SUGGESTIONS

Objective: Continued development of concepts of temperature.

Procedures

1. Have several thermometers available for the children. Children hold their hands around the bulb of the thermometer to discover that:

    Mercury is a liquid that easily expands and contracts.
    The measure of the length of the column of mercury indicates temperature.

2. Discuss the:

    Scale on the thermometer as part of a number line.
    Markings on the thermometer as end points of the segments.
    Variations of length of intervals on different thermometers (Scale).
    Measurement and measurement of temperature as an approximate value and as an indirect measure.

3. Discuss the:

    Unit of measure for temperature as a degree.
    Symbol for the unit, " ° "
    Meaning of zero degrees on the thermometer
    Boiling point and freezing point on the Fahrenheit Scale.
    Variations of boiling point and freezing points of various substances.
    Include milk, alcohol, mercury, water, etc.

    Boiling Point of Water = 212° F
    Freezing Point of Water = 32° F etc.

4. Children read and interpret the readings on various types of thermometers. They read temperatures below zero.

5. Use experience charts to compare outdoor and indoor temperature, daily temperature, etc.

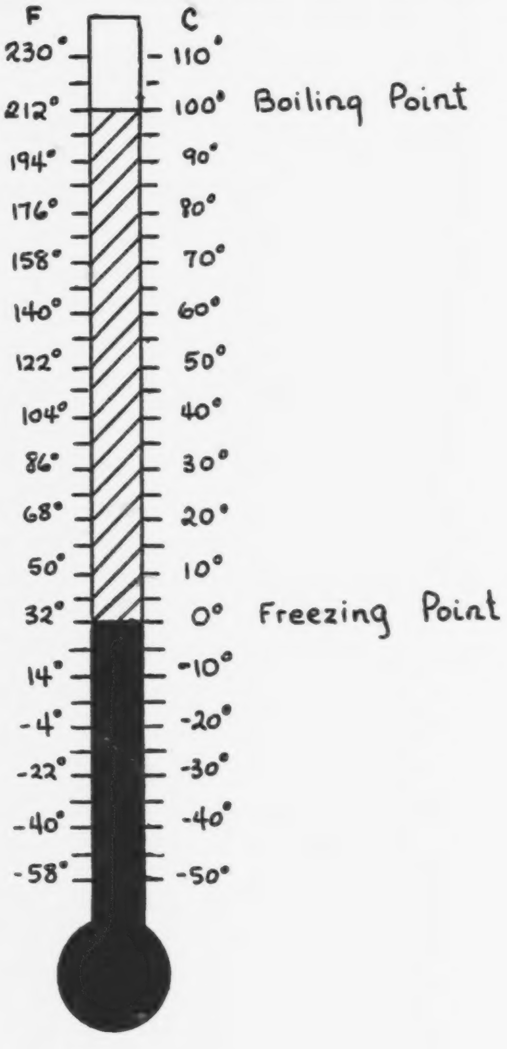
	Temp. at 10 A.M.	Temp. at 2 P.M.
Mon.	60°	68°
Tues.	48°	57°
Wed.		
Thurs.		
Fri.		

	Outdoor Temp.	Indoor Temp.
Mon.	60°	72°
Tues.	48°	72°
Wed.		
Thurs.		
Fri.		

✱ 6. Discuss idea of Centigrade Scale **(Optional)**

$$0^\circ\ C = 32^\circ\ F$$
$$100^\circ\ C = 212^\circ\ F$$

Centigrade is used in European countries and in Science.


F
C
230°
110°
212°
100°
Boiling Point
194°
90°
176°
80°
158°
70°
140°
60°
122°
50°
104°
40°
86°
30°
68°
20°
50°
10°
32°
0°
Freezing Point
14°
-10°
-4°
-20°
-22°
-30°
-40°
-40°
-58°
-50°

EVALUATION and / or Practice

SUGGESTED EXERCISES

1. Which is warmer, - 10° or - 20° ? [-10°]

2. If the temperature at 8 o'clock is 2° below zero and at noon it is 16° above zero, how much warmer has it gotten?

   [18 degrees warmer]

3. The temperature this morning was 10°. It has gotten 15° colder. What is the temperature now? [-5°]

4. Use variously scaled thermometers to answer the following:

   How many spaces are there on the scale from the 60° to the 70° mark, etc.

   How many degrees does each space represent?

   [Answers will vary depending upon the thermometer used]

5. Match the following temperatures with the appropriate item in the column at the right.

350°	Ice-skating Weather
32°	Boiling Water
10°	Swimming Weather
212°	Temperature at which water freezes
95°	Oven Temperature

6. The temperature changed from 5° to 2° below zero last night. This afternoon it went from 8° to 14° . Which was the greater change?

   [5° to 2° below zero]

7. Relate to science experiment.

Materials needed: Colored water, alcohol, mercury, long glass tubes.

Pour a small amount of a liquid into a tube.
Children cover the base of the tube with their hands and note how the liquid expands and rises in the tube as it gets warmer.
They note which liquid reacts most quickly.

Children construct a scale for each tube to note variations in temperature.

8. Additional exercises may be found in textbooks.

OPERATIONS

UNIT 6 – SET OF WHOLE NUMBERS: UNION OF SETS; ADDITION

NOTE TO TEACHER

Two sets may be combined or joined to form a new set.

The new set is called the union of the two sets. The term "union" is applied both to the operation and to the resulting set.

Understanding of the union of two sets is basic to the understanding of addition of whole numbers.

Sets having no elements in common are called Disjoint Sets. For example,

$A = \{$ Susan, Mary, Judy $\}$

$B = \{$ Ken, Frank, Paul $\}$

are disjoint sets.

The union of Set A and Set B is a new Set, C, where

$C = \{$Susan, Mary, Judy, Ken, Frank, Paul$\}$

The symbol for union is " $\cup$ ".

Here $C = A \cup B$

If X is the set of boys in the classroom and if Y is the set of girls in the classroom, we form a union of Sets X and Y to arrive at the Set of children in the room, Set Z.

We record this as: $Z = X \cup Y$

Union of Sets and Addition of Numbers

The term "Union" applies to sets and the term "Addition" applies to numbers.

For example,

Let $A = \{ a, b, c \}$
$B = \{ h, t, b \}$

Then $A \cup B = C$ where
$C = \{ a, b, c, h, t, b \}$

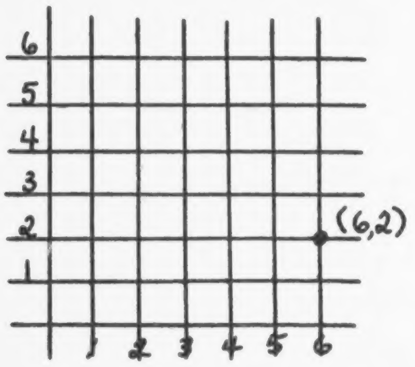
and $N(A) + N(B) = N(C)$
$3 + 3 = 6$

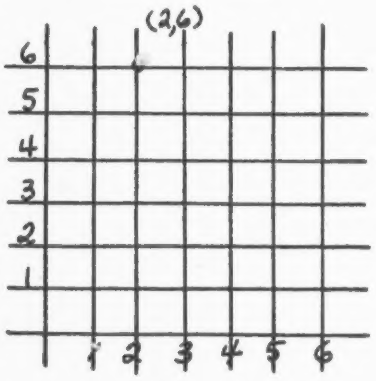
Addition is a binary operation in which an ordered pair of numbers is operated on to yield a third number called the sum. Each number of the ordered pair is called an addend.

An ordered pair of numbers involves two considerations:

The numbers
The order in which they are considered.

For example: The ordered pair (6, 2) is not the same as the ordered pair (2, 6) although both involve the same pair of numbers and both yield the same sum. This can be shown on the quadrants below.





Definition of the Operation of Addition for Whole Numbers

If A and B are disjoint sets as above, we have seen that:

$$N(A) + N(B) = N(A \cup B) \quad \text{since} \quad C = A \cup B$$

This is the way we can define the operation of the addition of two whole numbers which we will call a and b:

If $N(A) = a$
and $N(B) = b$
Where A and B are disjoint sets
then $a + b$ is equal to $N(A \cup B)$

By applying this definition we can verify each of the following properties.

Properties of Addition: Set of Whole Numbers

Operations on numbers are subject to certain rules or properties.

Some properties of operation for addition are the:

Associative Property

When three or more numbers are to be added, the order in which they are grouped does not affect the sum.

For example:

4, 3, 2 may be grouped as
$(4 + 3) + 2$ or as
$4 + (3 + 2)$

and $(4 + 3) + 2 = 4 + (3 + 2)$

Commutative Property

The order in which two numbers are added does not affect the sum.

For example:
$3 + 4 = 4 + 3$

Identity Element for Addition is Zero

$6 + 0 = 6$ and $0 + 6 = 6$

Closure

The set of whole numbers is closed with respect to addition.

When whole numbers are added, the sum is always a whole number.

Concepts from Algebra

A mathematical sentence is any statement of equality or inequality involving numbers.

A mathematical sentence may be either open, true, false .

An open sentence contains one or more place holders.

$4 + \square = 9$ is an example of an open sentence

$4 + 5 = 9$ is an example of a true sentence

$\{ 5 \}$ is the truth set for the open sentence $4 + \square = 9$

A given set or a set from which it would be reasonable to choose the number for the truth set is called the replacement set. For $4 + \square = 9$ the set of whole numbers would be considered the replacement set.

The symbols $\square$ or n or Δ, etc. are called placeholders or variables.

$8 > (n \times 2)$ is also an open sentence.

If we establish the replacement set as the set of whole numbers, then the truth set for $8 > (n \times 2)$ is $\{0, 1, 2, 3\}$

$4 + 5 = 8$ is an example of a false statement.

Basic Addition Facts

An addition fact is a statement about the two addends, each from 0 through 9 and their sum - a number from 0 through 18. Thus:

$8 + 4 = 12$ is an addition fact but
$12 + 4 = 16$ is not called an addition fact.

Teachers must aim for automatic response to basic facts by their children.

Automatic response is achieved through periodic drill.

Drill

Drill should be organized according to specific patterns which emphasize one type of relationship at a time.

A few minutes of each lesson should be devoted to drill.

Drill should be related to the major topic under consideration, wherever possible.

TEACHING SUGGESTIONS

Objectives:

To develop understanding of the union of sets.
To relate addition of numbers to union of sets.
To test and / or reinforce automatic response to addition facts.
To reinforce extensions of addition facts to higher decades.
To introduce concepts from algebra.

Procedures

Union of Sets

1. Display two sets of objects, for example, a ruler and a scissors; a crayon and a pencil

   Children note that the objects are different.

2. Children show these in set notation, as:

   Let $A = \{ \text{scissors, ruler} \}$

   Let $B = \{ \text{pencil, crayon} \}$

3. Ask children how to form a new set using the elements of A and B.

   [Put them together, join them, etc.]

4. Ask children:

   What are the elements of this new set which we will call Set C?

   Teacher records as child responds

   C = {scissors, ruler, crayon, pencil}

   Does Set C include all of the elements of Set A? of Set B?

5. Tell children that the symbol for joining sets is " ∪ " and that it is read as: "Union of".

   Record in set notation the action of joining the sets.

   {scissors, ruler} ∪ {crayon, pencil} = {scissors, ruler, pencil, crayon}
   A ∪ B = C

   Emphasize that the term "union" applies to sets.

6. Reinforce set notation:

   Braces; symbol for empty set; symbol for union; symbol for the number property of a set.

7. Present the problem:

   Alice, Joan and Mary went skating. At the rink they met Harry, Tom and Bob. The two groups skated together.

   Children show this in set notation .

   { Alice, Joan, Mary } ∪ { Harry, Tom, Bob }

   Children show the union of the two sets.

   Let A = {Alice, Joan, Mary}
   B = {Harry, Tom, Bob}
   C = {Alice, Joan, Mary, Harry, Tom, Bob}

   Then , A ∪ B = C

8. Use the problem above to reinforce number as a property of sets.

Children show, in set notation, the number of elements in Set A, Set B and Set C as:

N(A) = □ [3]
N(B) = □ [3]
N(C) = □ [6]

Relate Union of Sets to Addition of Numbers

Ask children (referring to the sets above)

What are the elements of Set A?

[Alice, Joan, Mary]

What is the number of Set A?

[N(A) = 3]

What are the elements of Set B?

[Harry, Tom, Bob]

What is the number of Set B?

[N(B) = 3]

What are the elements of Set C?

[Alice, Joan, Mary, Harry, Tom, Bob]

What is the number of Set C?

[N(C) = 6]

Show in set notation that the number of Set A added to the number of Set B , will give the number of Set C where C = A U B.

$$\begin{bmatrix} N(A) + N(B) = N(C) \\ N(A) + N(B) = N(A \cup B) \end{bmatrix}$$

Show the above using only numerals.

[3 + 3 = 6]

Emphasize that:

The binary operation "Union" applies to sets. We combine sets.

The binary operation "Addition" applies to numbers. We add numbers.

Facts and Extensions

1. Test for automatic response to basic addition facts.

   Drill or reteach as indicated by results.

2. Test children's responses to the extension of addition facts to higher decades.

A

Sums in the Same Decade

17 + 2 = □
24 + 2 = □
46 + 2 = □

15 + 4 = □
26 + 4 = □
64 + 4 = □

B

Sums reaching the Next Decade

41 + 9 = □
14 + 6 = □
38 + 2 = □

17 + 3 = □
55 + 5 = □
66 + 4 = □

C

Sums in the Next Decade

57 + 5 = □
46 + 5 = □
14 + 8 = □

89 + 7 = □
18 + 3 = □
59 + 8 = □

D

Sums in the Hundreds

324 + 3 = □
436 + 4 = □
518 + 5 = □

Sums in the Thousands

- Children make the open sentences true.

1324 + 3 = □
2436 + 4 = □
1617 + 6 = □
1598 + 5 = □

Read as hundreds rather than as thousands to facilitate arriving at sums. For example:

1324 + 3 is read as: 13 hundred 24 plus 3

3. Drill or teach as indicated by results, emphasizing the following mathematical relationships.

Applying Commutative Property

$2 + 9$ thought of as $9 + 2$
$5 + 23$ thought of as $23 + 5$
$6 + 184$ thought of as $184 + 6$

Deriving Near-Doubles from Doubles

From $8 + 8$ we derive, $8+7$, $8+9$, $7+8$, $9+8$
From $25 + 25$ we derive, $25 + 24$, $25 + 26$, $24 + 25$, $26 + 25$

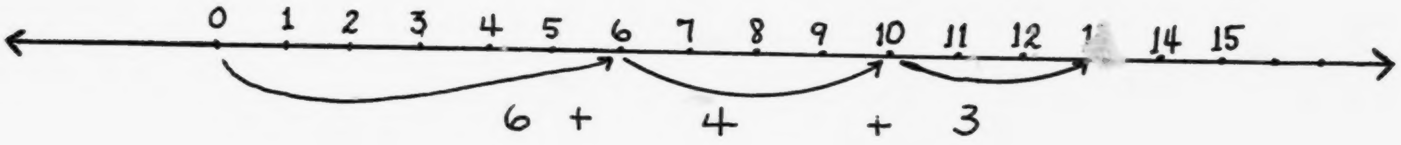
Applying Associative Property

$9 + 6$ thought through as
$9 + (1 + 5) = (9 + 1) + 5 = 10 + 5 = 15$

$38 + 5$ thought through as
$38 + (2 + 3) = (38 + 2) + 3 = 40 + 3 = 43$

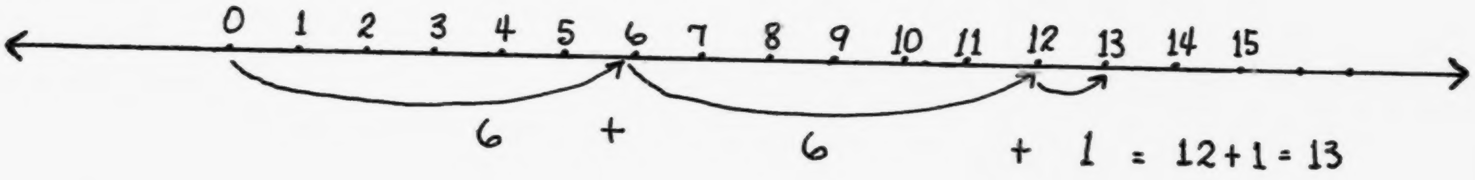
$243 + 9$ thought through as
$243 + (7 + 2) = (243 + 7) + 2 = 250 + 2 = 252$

4. Use the number line to reinforce addition facts and extension of facts to higher decades.



$$6 + 7 = 6 + (4 + 3) = (6 + 4) + 3 = 10 + 3 = 13$$

or



Extend number line for higher decades.

Concepts from Algebra

1. Tell children that:

   Symbols □, Δ, n, <>, etc. in a mathematical sentence are called placeholders or variables.

   Mathematical statements containing variables are called open sentences.

2. Direct children:

Replace the variable to make a true statement

$$8 + \square = 17$$
$$15 + \square = 21$$
$$9 + \square = 14$$

Which of the statements below are true? Which are false? Which are open?

$$3 \times 7 = 21$$
$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$
$$\square - 7 = 10$$
$$48 + 22 = 60$$
$$\frac{1}{2} - \frac{1}{4} = 1\frac{1}{4}$$

Why cannot $\square - 7 = 10$ be called a true statement ;**A false statement?**

Write an open sentence; A true sentence; **A false statement?**

3. Make this open sentence true: $236 + \square = 243$ [7]

Tell children that $\{7\}$ is called the truth set for the open sentence $236 + \square = 243$. Ask, "Why?"

Let children find the truth set for the following open sentences. In each case ask how they obtained the answer.

$6 + \square = 27$; $3 + (2 \times \square) = 11$; $2 + (3 \times \square) = 2$

4. Have children make the following open sentences true.

Commutative Property

$$3 + 36 = 36 + n$$
$$7 + 148 = n + 7$$
$$9 + 2145 = 2145 + n$$

etc.

Associative Property

$$105 + 7 = 110 + n$$
$$1526 + 4 + \square = 1532$$
$$1397 + 8 = 1400 + \square$$
$$225 + 5 + n = 233$$

Use of the terms "Commutative" and "Associative" would depend upon the maturity of the children.

5. Replacement Set

Given the set $\{7\frac{1}{2}, \frac{1}{2}, 3, 5, 6\}$ children choose the values for n to make the following statement true.

$$8 > (n + 2) \qquad [n = \tfrac{1}{2}, \text{ or } 3, \text{ or } 5]$$

Tell children that the set of numbers from which one or more numbers can be chosen to replace a variable is called a replacement set.

Why is the name, "Replacement Set" given to this set?

Children note that more than one choice is possible for the truth set or solution set.

6. Children find the truth set for the following:

Open Sentence	Replacement Set
a. $4 < \square < 9$	The set of whole numbers
b. $\square + \Delta = 7$	$\{1, 2, 3 \ldots 10\}$
c. $37 + 5 > n$	$\{43, 41, 29, 50\}$

PRACTICE and / or EVALUATION
SUGGESTED EXERCISES

1. Using any or all of the following: 4, 6, 8

    Write two statements of inequality.

2. Tell what whole number n is, so that each mathematical sentence below, is true.

$n + 50 = 50 + n$ $\quad$ [$\{0, 1, 2, 3 \ldots\}$ or any whole number]

$13 + \square = 24$

$9 + 8 < n$ $\quad$ [$\{18, 19, 20 \ldots\}$]

$26 + 13 > n$ $\quad$ [$\{0, 1, 2, 3 \ldots 38\}$]

3. Additional exercises may be found in textbooks.

SETS; NUMBER; NUMERATION

UNIT 7 - SUBSETS

NOTE TO TEACHER

A set contained in another set is called a subset of the original set.

Each element of the subset is by definition an element of the given set.

For example: All of the children in a classroom may be thought of as a set of children.

The set of girls is a subset of the set of children, The set of boys is another subset of the class of children.

The set of even numbers is a subset of the set of whole numbers.

The set of odd numbers is a subset of the set of whole numbers.

The symbol for "is a subset of" is "$\subset$" .

If Set $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and
Set $B = \{2, 4, 6, 8\}$

Then Set B is a subset of Set A because each of the elements of Set B is also an element of Set A.

This is symbolized as: $B \subset A$ and is read as: Set B is a subset of Set A.

Understanding of subsets is important in arithmetic. For example: We can explain subtraction of two whole numbers by removing a subset from a given set and examining the number of the three sets involved.

TEACHING SUGGESTIONS

Objective: To help children understand the meaning of subset.

Procedures

1. The problem of rearranging the bookshelves is presented to the class. Three children, Mary, Alice and John are chosen.
2. Identify these children as elements of a set.
3. Children record this given set using set notation.

   [{Mary, Alice, John}]

4. Guide children to see that there are sets within a given set.

   Ask children:

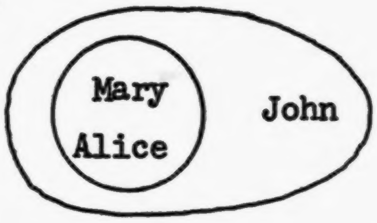
   Which children within this set can we use to help rearrange the books on the back shelves?

   [Answers may vary]

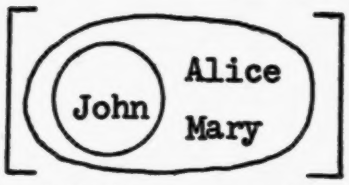
   How would you describe Mary and Alice in relation to the set?

   [Elements of the set]

   Use a diagram to show this.



   Show, using diagrams, other children from the given set who might be used to help.



   Record different sets of helpers, using set notation.

   [{Mary, John}] etc.

   Ask children:

   What can you tell about the relationship of the sets you have recorded to the given set.

[Children helping are in the set of children you chose;
Mary and Alice are part of the names you put on the board;
etc.]

5. Tell children that sets that are part of a given set are called subsets of that set.

6. Ask children whether we might use Alice, Mary and John to help.

[Yes]

Tell children that any set may also be considered a subset of itself.

7. Ask children what notation they would use to show that we do not wish to use any of the children of the given set.

[{ }]

Ask children whether there is any member in the empty set that is not an element of the given set.

Tell children that because there is no element in the empty set that is not in the given set the empty set is a subset of every set.

8. Tell children that the symbol for subset is " $\subset$ " .

For example: $\{ \text{Alice} \} \subset \{ \text{Alice, John, Mary} \}$ is read as: The set that contains Alice is a subset of the set whose elements are Alice, John, Mary.

We may also record this in another way.

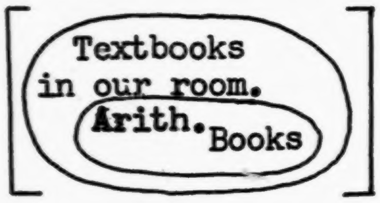
$D = \{ \text{Alice, John, Mary} \}$

$F = \{ \text{Alice} \}$

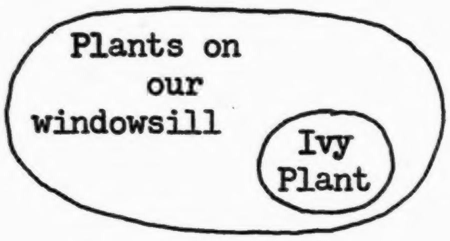
Then we may record the same idea as: $F \subset D$ read as: "Set F is a subset of Set D".

PRACTICE EXERCISES

1. Use a diagram to show that the set of Arithmetic books is a subset of the set of all the textbooks in our room.

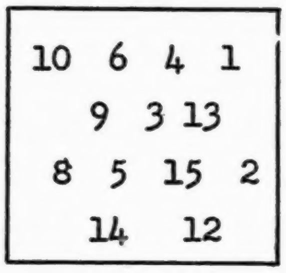


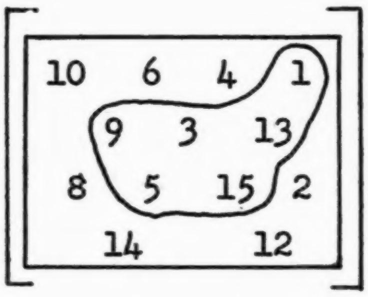
2. Use set notation to describe the diagram below.


Plants on our windowsill
Ivy Plant

[{Ivy plant} ⊂ {Plants on our windowsill}]

3. Consider the diagram below. Encircle the subset of odd numbers.


10 6 4 1
9 3 13
8 5 15 2
14 12


10 6 4 1
9 3 13
8 5 15 2
14 12

4. Use data on the chart to give information required about {John, Tom, Don, Alan, Bill}

Name	Color of Tie	Color of Shirt
John	Blue	White
Don	Red	Grey
Alan	Grey	Blue
Bill	Blue	White
Tom	Grey	Grey

a. Show in set notation that the boys whose ties are blue is a subset of the given set.

[{John, Bill} ⊂ {John, Don, Alan, Bill, Tom}]

b. Show in set notation that the boy wearing a red tie is a subset of the given set.

[{Don} ⊂ {John, Don, Alan, Bill, Tom}]

c. If Frank also is wearing a red tie is the following statement true? Explain.

{Don, Frank} ⊂ {John, Don, Alan, Bill, Tom}

5. List in set notation all the subsets of the set of rivers around New York City.

{Hudson, Harlem, East}
{Hudson} ; {Hudson, East} ; {Hudson, Harlem}
{Harlem} ; {Harlem, East}
{ East } ; { }

*6. How many subsets are there in a set that contains 3 elements? **(Optional)**

*7. Distinguish between: **(Optional)**

$$\text{Alice} \in \{\text{Alice, John}\}$$

and

$$\{\text{Alice}\} \subset \{\text{Alice, John}\}$$

*8. How many subsets are there of a set that contains 4 elements? **(Optional)**

OPERATIONS

UNIT 8 - SET OF WHOLE NUMBERS: SUBTRACTION - RELATED TO COMPLEMENT OF A SET; MEANINGS; PROPERTIES; FACTS; EXTENSIONS TO HIGHER DECADES

NOTE TO TEACHER

Although children have dealt with subtraction in earlier grades a recapitulation of subtraction in connection with sets may help them to solve verbal problems.

An interpretation of subtraction in the set of whole numbers depends upon:

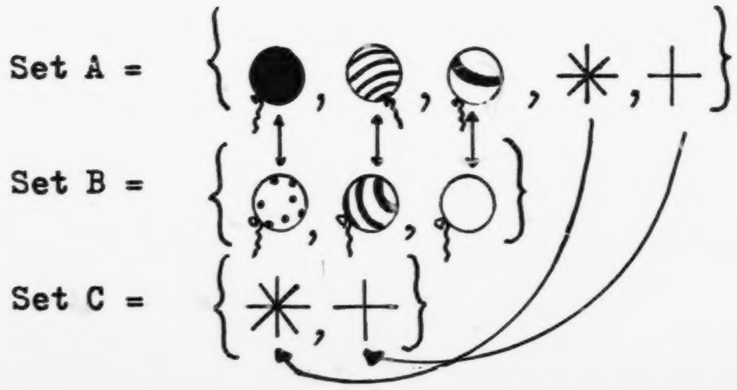
1. The meaning of addition in the set of whole numbers in terms of the union of disjoint sets.
2. The understanding that subtracting a number is the inverse operation of adding that number.

Interpretation of Subtraction in Terms of One-to-One Correspondence; Related to Finding the Difference in Subtraction of Numbers

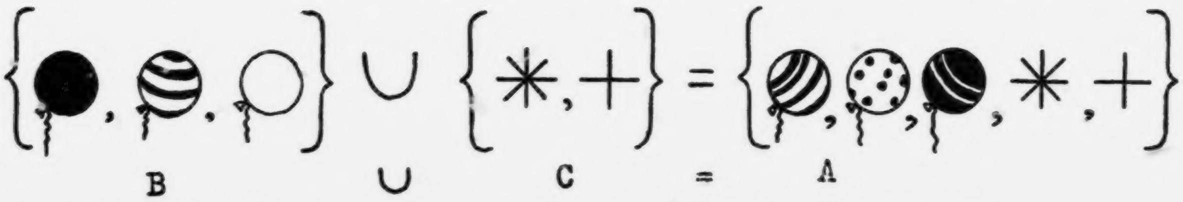
Illustrative Example

Choose a set with 5 elements, Set A and a set with 3 different elements, Set B.

Find another Set C such that when the union of Set B and Set C is formed, the elements of this union can be placed in one-to-one correspondence with the elements of Set A.

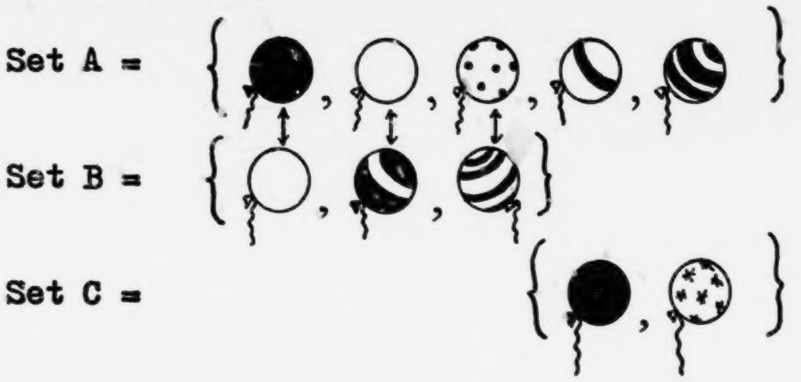


Therefore:



The elements of the union of sets B and C may now be placed in one-to-one correspondence with the elements of Set A.

When the elements of Set A are matched with the elements of Set B the elements that are not matched form the new Set C.



Relating Matching of Sets to Subtraction of Numbers

Referring to the conditions stated above, we find that if we know the number of elements in Set A (5) and the number of elements in Set B (3), we can find the number of elements in Set C (2).

We may think in terms of addition:
What number added to 3 will result in 5?
This may be stated as: $3 + \square = 5$
We must find a second addend which when added to 3 will give the sum 5.

Addition is an operation on two numbers, to find a third number called the sum. The two numbers are called addends.

Addend + Addend = Sum
3 + 2 = □

Subtraction then is the operation of finding a missing addend when the sum and the other addend are known.

Addend + ▭ = Sum
3 + □ = 5

In terms of subtraction this is finding the difference.

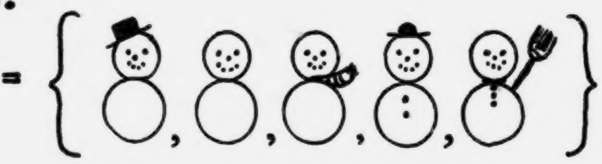
Interpretation In Terms of Subsets (Complement of a Set); Related to Finding "How Many Are Left" In Subtraction of Numbers

Illustrative Example

Choose a Set A with 5 elements from which we wish to remove a subset Set B of 3 elements. The subset that is left can be called the remainder set, which we will call Set C.

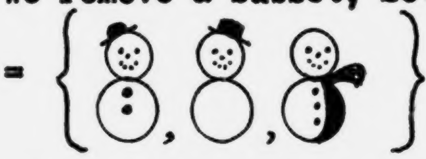
Set C is said to be a complement of Set B relative to Set A.
For example:

Set A =



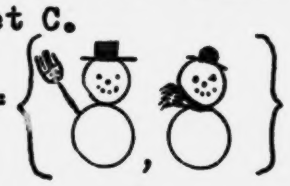
From Set A we remove a subset, Set B

Set B =



We are left with the elements of another subset which we will call Set C.

Set C =

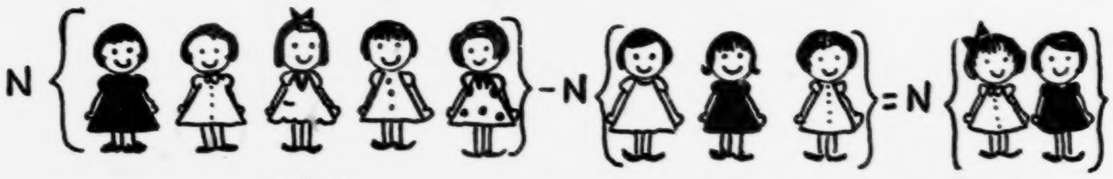


Set C is the remainder set.
Set C is the complement of Set B relative to Set A.

Should we wish to remove Subset C from Set A, then Subset B would be the remainder set or the complement of Set C relative to Set A.

Relating Removing of a Subset to Subtraction of Numbers

When we wish to remove a number of elements from a set the operation of subtraction is involved:



N (A) − N (B) = N (C)

N (A) = 5

N (B) = 3

5 − 3 = 2

Subtraction is a binary operation on an ordered pair of numbers.

A binary operation means that only two numbers may be operated on at any one time to produce a third number.

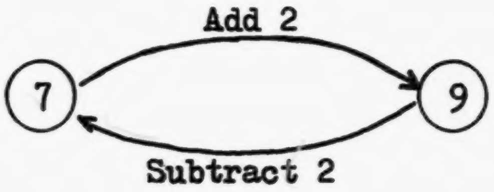
Finding "The Difference" and finding "How Many Are Left" situations are both solved by the operation of subtraction.

Some Properties of Subtraction

Inverse Operation

An action often has associated with it an inverse action. We open a door, we close a door; we put on shoes, we take off shoes.

Mathematical operations also have inverse operations. For example: After we add 2 to 7 to arrive at 9, to get back to 7 we subtract 2 from 9.



This may be thought of as a "doing" and an "undoing."

"Subtracting a number" is the inverse operation of "adding that number."

Addition. . . . . $7 + \underline{2} = 9$

Related Subtraction . . . $9 - \underline{2} = 7$

Commutative Property

Subtraction is not commutative

$4 - 3 = 1$ but
$3 - 4 \neq 1$

Associative Property

Subtraction is not Associative

$(6 - 4) - 2 = 0$ but
$6 - (4 - 2) \neq 0$

Property of Closure

The set of whole numbers is not closed with respect to subtraction. When whole numbers are subtracted the result is not always a whole number. For example: 6 - 8 is not within the set of whole numbers.

Role of Zero in Subtraction

Zero subtracted from any number results in that same number.

$$6 - 0 = 6$$

However, a whole number subtracted from 0 does not result in a whole number or in that same number.

$$0 - 6 \neq 0$$
$$0 - 6 \neq 6$$

(Compare with addition where $6 + 0 = 6$ and $0 + 6 = 6$).

Basic Subtraction Facts

A basic subtraction fact involves three numbers, at least two of which are selected from 0 through 9. For example:

$15 - 7 = 8$ is a subtraction fact

$25 - 7 = 18$ is not considered a basic subtraction fact. It is an extension of a basic subtraction fact.

Drill and Testing

Subtraction facts should be presented concurrently with their related addition facts.

$$8 + 7 = 15; \quad 15 - \square = 8$$

TEACHING SUGGESTIONS

Objectives: To relate the operation of subtraction to relative complements of sets. (The "take away" aspect of subtraction).

To relate the operation of subtraction to ideas of Union of Sets and One-to-One Correspondence. (Comparison Difference aspect of subtraction).

To apply properties of Subtraction to drill and/or reteaching of facts and extensions of facts to higher decades.

Procedures:

Finding a Remainder Set - Relative Complement of a Set

1. Direct 5 children to come to the front of the room. Mary, Jane, Eva, Al, Tony. This is the set of children who will arrange books. Al and Tony are called to the office.

   Ask children to:

   Record set of children who were to help arrange books.
   Call this Set A .
   $A = \{\text{Mary, Jane, Eva, Al, Tony}\}$

   Record subset of children called out of room.
   Call this Set B.
   $B = \{\text{Al, Tony}\}$

   Record the set left to help with books.
   Call this Set C.
   $C = \{\text{Mary, Jane, Eva}\}$
   Why can we not call it Set A?

   Tell children Set C is called the remainder set.

2. Show a set of objects on desk.
   Ask children to come and remove a subset.
   Ask children to describe the remainder set.

3. Display a set of objects, e.g. a disc, a book, a pencil, a plant, a jar of paint. Call it Set A.

   Ask children to record this set showing the number of elements. [$N(A) = 5$]

   Ask children to remove a subset whose number is 2 from Set A.
   Call the subset, Set B.
   Ask children to record this subset showing the number of elements. [$N(B) = 2$]

   Call the remainder set, Set C.
   Ask children to record the number of elements in the remainder set. [$N(C) = 3$]

4. Ask children to write a true mathematical sentence using numerals only to show the action just performed.
   [$5 - 2 = 3$]

5. Refer to the remainder set displayed on the table.
Ask children what they would do to show the original set.

[Put back the set of objects we took away.
Put back the pencil and the plant.
Put back Set B, etc.]

6. Ask children:

What do we call this operation on sets? [Union]

To record the action using set notation. [$C \cup B = A$]

To record "$C \cup B = A$" showing that a number property is involved. [$N(C) + N(B) = N(A)$]

To remove set B from set A again.
To record the removal of subset B showing the number property of the sets. [$N(A) - N(B) = N(C)$]

7. Children show these actions using equations

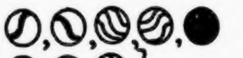
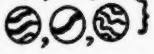
$3 + 2 = 5$
$5 - 2 = 3$

Relate Finding the Difference to One-to-One Correspondence For Comparison-Difference Subtraction

1. Present a problem:

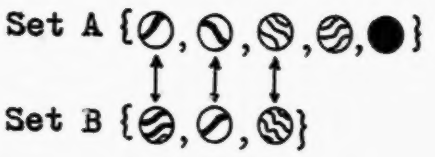
Frank has a set of marbles which we will call Set A.
Jerry has a set of marbles which we will call Set B.
We want to compare the two sets to find how many more or less one set has than the other.

Record these in set notation.

Set A = {}
Set B = {}

2. Tell children we will compare the elements of Set A with the elements of Set B to find the number of elements that match.

Record:



Ask children: How many elements in Set A?

Record as children respond: [N(A) = 5]

Ask children: How many elements in Set B?

Record as children respond: [N(B) = 3]

Ask children: How many elements are matched?

Record as children respond:
[3 elements of Set A are matched with 3 elements of Set B]

How many elements are not matched? Which set has more elements? Which set has fewer? What is the difference between the number of elements in Set A and the number of elements in Set B?

Direct children to show this action:

Using Set Notation [N(A) - N(B) = N(C)]

Using Numerals only [5 - 3 = 2]

3. Direct children:

Find a new set which when joined to Set B will form a set all of whose elements will match all of the elements in Set A. Call this new set, Set C.

How many elements are in Set C?

Write a mathematical sentence that shows the action of finding how - many - more are needed. [3 + ☐ = 5]

Write a mathematical sentence to show the thinking that will solve this. [5 - 3 = 2]

Subtracting Numbers

1. Test for automatic response to basic subtraction facts.

2. Drill or re-test as indicated by results.

Present each subtraction fact with its related addition fact. (Inverse Operation)

7 + 2 = 9
9 - 2 = 7

2 + 7 = 9
9 - 7 = 2

9 + 7 = 16
16 - 7 = 9

7 + 9 = 16
16 - 9 = 7 etc.

Reaching ten by regrouping the number to be subtracted.

15 - 8 thought through as:
15 - 5 - 3, then as 10 - 3 = 7

63 - 7 thought through as:
63 - 3 - 4, then as 60 - 4 = 56

452 - 5 thought through as:
452 - 2 - 3, then as 450 - 3 = 447

3. Test children's responses to extension of facts to higher decades.

A

Remainders in the Same Decade

18 - 3 = ☐
27 - 4 = ☐
39 - 7 = ☐
29 - 5 = ☐
96 - 3 = ☐
24 - 2 = ☐

B

Subtracting from Whole Decades

60 - 7 = ☐
40 - 8 = ☐
20 - 3 = ☐
50 - 6 = ☐
70 - 2 = ☐
90 - 4 = ☐

C

Remainders in the Preceding Decade

74 - 6 = ☐
75 - 6 = ☐
25 - 9 = ☐
51 - 9 = ☐
24 - 5 = ☐
73 - 6 = ☐

D

Extensions to Hundreds

374 - 3 = ☐
475 - 4 = ☐
525 - 5 = ☐

E

Extensions to Thousands

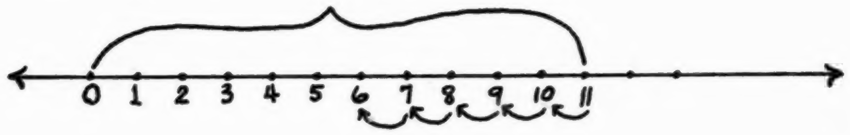
1327 - 3 = ☐
2440 - 4 = ☐
1623 - 6 = ☐
1603 - 5 = ☐ etc.

4. **Provide additional practice using varied forms.**

$$\begin{aligned} 235 - 8 &= 230 - \square \\ 2134 - 7 &= 2130 - n \\ 174 - n &= 170 - 3 \\ 1405 - 8 &= 1400 - \square \end{aligned}$$

5. **The number line is an effective way of providing practice. For example:**

a. $11 - \square = 6$

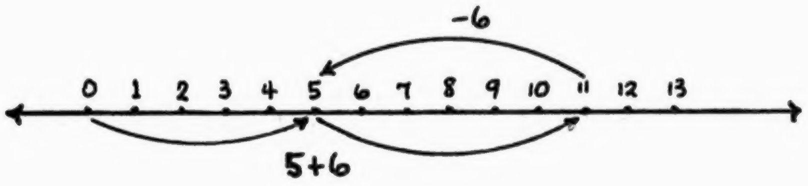

0
1
2
3
4
5
6
7
8
9
10
11

(Optional) * b. Use the same number line. Ask children to solve:

$6 - 11 = \square$

c. $5 + 6 = \square$

$11 - 6 = \square$

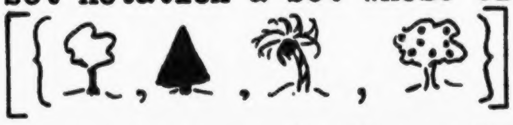

-6
0
1
2
3
4
5
6
7
8
9
10
11
12
13
5+6

6. Present true and false mathematical sentences.
Children indicate which are true and which are false, and why?
Ask them to change false sentences to true sentences.

$8 - 3 = 5$ because $7 + 2 = 5$	[F. $8 - 3 = 5$ because $5 + 3 = 8$]
$16 - 9 = 7 - 0$	[T. because $16 - 9 = 7$; $7 - 0 = 7$; $7 = 7$]
$48 + 9 = 57 - 9$	[F. because it is not an equality]
$48 + 9 = 57$ is the inverse operation of $57 - 9 = 48$	[T.]
$317 - 9 = 317 - 7 - 3$	[F. because 308 does not equal 307. To make it true, the sentence must read $317 - 9 = 317 - 7 - 2$]

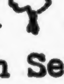
EVALUATION AND/OR PRACTICE

1. Record in set notation a set whose elements are 4 trees.



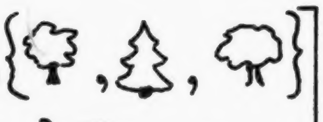
Call it Set A and associate the number four with this set. [N(A) = 4]

Record any subset of this set. Call this subset Set B.

[Set B = {  } (Answers may vary)]

Associate the number with Set B. [N(B) = 1 or . . .]

Find the remainder set. Call it Set C. Record the number of the remainder set.

[Set C = 
N(C) = 3 or . . .]

2. Complete the following chart:

The number of the Original Set	The number of the Subset removed	The number of the Remainder Set
19	0	_____
49	5	_____
89	7	_____
19	_____	_____
39	_____	_____

3. Complete the following:

   In Set F there are 14 chairs.
   G is a subset of F and there are 8 armchairs in set G.
   Remove the elements of subset G.
   Show, using set notation, the number of elements in the remainder set. [N(H) = 6]

4. Additional problems may be found in textbooks.

SETS; NUMBERS; NUMERATION

UNIT 9 - SYSTEMS OF NUMERATION: BASE 10; EXPANDED NOTATION

NOTE TO TEACHER

A system of numeration must be distinguished from a number system.

A number system involves:

Ideas of quantity
Operations on the numbers
The properties that apply to those operations.

Addition and Multiplication are the major operations.

On the other hand, a system of numeration involves:

A set of symbols
Rules for using the symbols to represent and name numbers.

Common Characteristics of Systems of Numeration

1. Only a finite number of different symbols are used.
2. Any of these symbols or combinations of symbols may be repeated, and may in different positions represent different numbers.

The decimal or base ten system of numeration is only one of many systems although it is the one most commonly used.

The Hindu-Arabic or Decimal System of Numeration

Symbols

The decimal system uses ten symbols - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

It is a place value system. The number, ten, represented by a 1 and 0 in fixed places plays a special role. Ten is called the base of this system.

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits.

Place Value:, Grouping By Ten; Multiplicative Aspect.

Place value is the property of a system of numeration which assigns a value to a digit according to its ordered place in the numeral.

In the base-ten numeral 11, the place value of the 1 on the right is 1 one; the place value of the 1 on the left is 1 ten.

In the base-ten numeral 32, the 2 indicates a grouping of 2 ones; the 3 indicates a grouping of 3 tens.

The place of each digit represents a value ten times as great as the value of the place of the digit immediately to its right.

The place of each digit represents a value one tenth as great as the place of the digit immediately to its left.

For the numeral 1111 the value of the digits can be shown as:

10 x 10 x 10 Thousand	10 x 10 Hundred	Ten	One
100 x 10 x 1	10 x 10 x 1	10 x 1	1 x 1

For the numeral 4,736 the value of the digits can be shown as:

4 x 10 x 10 x 10	7 x 10 x 10	3 x 10	6 x 1

Expanded Notation: Additive Aspect

Expanded notation is a way of showing clearly the value of a number by adding the values represented by each digit. For example: 132 may be shown in expanded notation as:

132 = 100 + 30 + 2

132 = (10 x 10) + (3 x 10) + 2

132 = (1 x 10 x 10) + (3 x 10) + (2 x 1)

Role of Zero As a Digit

Zero is one of the digits, just as 1, 2, 3...9 are. As such, it can be considered a placeholder to show that there are no ones or no tens, etc. in a representation of a number.

For example, in the numeral 203, zero indicates the absence of tens.

TEACHING SUGGESTIONS

Objective: To emphasize the relationship of tens to ones, hundreds to tens, thousands to hundreds, etc.

Procedure

1. Test children's understanding of place value.

Which numeral has the digit 6 in the hundred's place?

186 603 560 3256

What number does the 8, in each of the following, represent?

871 80 1208

Write the numeral for the number that is ten more than 960.

What is the largest number which can be represented by using each of these digits $\{3, 5, 8, 4\}$ only once?

In writing the numeral for two thousand nine hundred three, in which place do we write the zero?

In which of these numerals does 5 represent half of one hundred?

750 45 500 50000

If the 3 in each of the following were changed to 7, which of the numbers would be increased the greatest?

2038 6318 3021 7324

Replace the placeholder to make a true sentence.

$300 + n + 7 = 597$ $3405 = 3000 + n + 5$

$268 = n + 8$ $n + 600 + 50 + \square = 4651$

Mark the numeral on each line that is closest in value to the one in column A.

A			
600:	200	400	900
300:	325	270	288
450:	430	500	410

2. Develop Place Value Through 10,000

Use Squared Material when necessary.
To represent 1000, prepare a strip of 10 "Hundred-Squares" taped together. Several of these strips may be used to represent larger numbers. Relate materials to the written symbols.

Start with 1000 and gradually increase to larger numbers, e.g., 1000 - 2000; 2000 - 3000; etc.

Caution: Use only teachers' demonstration materials (1") or only children's materials (1/2"). Do not combine teachers' and children's Squared Materials when representing a number.

Stress the value of the digits, e.g., 3475 as:

3000 + 400 + 70 + 5

or as

Thousands	Hundreds	Tens	Ones
3	4	7	5

Emphasize those concepts with which children have difficulty e.g.,

1001, 1010, 1101, 2001, 2010, 2101, etc.

3. Give children practice in using expanded notation.

373 = 300 + n + 3
136 = n + 6
639 = n + 30 + 9

n + 600 + 50 + 1 = 4651
2435 = 2400 + n + 5
3000 + 600 + n + 5 = 3605

SETS; NUMBERS; NUMERATION

UNIT 10 - SET OF FRACTIONAL NUMBERS: CONCEPTS AND COMPARISONS; EQUIVALENTS; COUNTING

NOTE TO TEACHER

All previous topics in this bulletin dealt with the set of whole numbers.

Number lines have been marked at selected points which correspond to whole numbers.

Now we wish to name points between those represented by whole numbers.

The number system must be extended to provide for more accurate measurement and to help deal with division which is not closed within the set of whole numbers.

The extended set of numbers is called the Set of Fractional Numbers.

A fraction can be considered either as a number or as a numeral.

Any whole number may be expressed in fraction form. For example: $\frac{2}{2}$, $\frac{0}{1}$, $\frac{9}{3}$, etc.

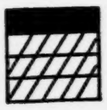
A fraction is the indicated division of an ordered pair of numbers where the second member of the pair is not zero. The second number of the ordered pair is called the denominator. The denominator indicates the number of parts of the same size into which a unit has been divided.

The first number of the ordered pair is called the numerator. The numerator indicates the number of parts of the same size being considered.

A fraction may be interpreted in several ways.

1. As one or more parts of the same size into which a unit has been divided. For example:

   $\frac{1}{4}$, $\frac{3}{4}$

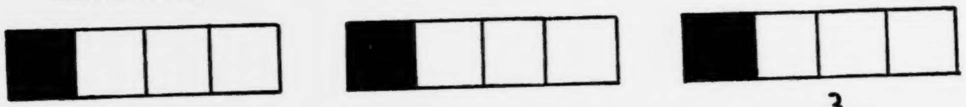


2. As a comparison or ratio. For example:

   3:4 or 3 out of 4

3. As one of each of the parts of the same size into which more than one unit has been divided. For example:

   $\frac{3}{4}$ meaning $\frac{1}{4}$ of 3 or one of each of the parts of the same size into which 3 units have been divided.



4. As an indicated division. For example: $\frac{3}{4}$ meaning $3 \div 4$.

Fractions can be represented in the following forms:

The (common) fractional form, $\frac{7}{10}$

The decimal form 0.7

The decimal form is used for convenience when the denominator is ten or a power of ten.

Mixed form is the name given to the symbol for a number involving a whole number and fraction. For example:

$3\frac{3}{4}$, 3.75

Children have used physical models and representative materials to develop fraction concepts. They have worked with units such as line segments, circular and rectangular regions. They have worked with collections of things. They have developed concepts of equivalency by comparing regions of the same size and shape.

The study of fractional parts in Grade 5 extends the application of comparisons and equivalencies.

Equivalent fractions are thought of as different symbols or names for the same fractional number.

Physical models and the number line are used to introduce new concepts and where necessary, to reinforce previous learnings.

Children who need representative materials should prepare a kit (paper discs or commercial fractional parts) of the following: 2 wholes, 4 halves, 8 fourths, 16 eighths, 6 thirds.

TEACHING SUGGESTIONS

Objective: To reinforce and extend concepts of thirds, halves, fourths, eighths

Procedures

1. Reinforce meaning of symbols
   Begin with $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$

   Ask children, referring, for example, to the above fractions

   What does the denominator 2, 4, 8, 3 tell us?
   What does the numerator tell us?
   Which is larger, $\frac{1}{3}$ or $\frac{1}{2}$; $\frac{1}{3}$ or $\frac{1}{4}$? Why?

   For $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, etc. ask children:

   What does the denominator 4, 8, 3, etc. tell us?
   What does the numerator 2, 3, 4, etc, tell us?

Which is larger in value, $\frac{3}{8}$ or $\frac{5}{8}$? Why?

To read the following: $\frac{2}{4}$, $\frac{3}{8}$, $\frac{2}{2}$, $\frac{7}{8}$, etc.

To write, using numerals: two fourths, eight eighths, two thirds, etc.

Continue with mixed form. Ask children:

To read: $2\frac{3}{4}$, $3\frac{1}{8}$, $1\frac{7}{8}$, etc.

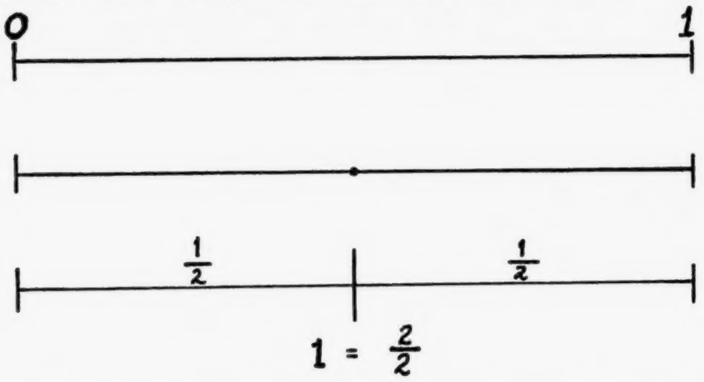
To write numerals for

one and one-half	two + three fourths
two plus three eighths	four and one fourth, etc.

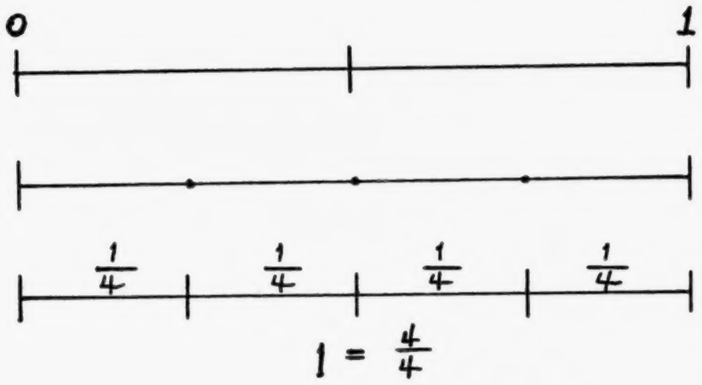
Emphasize that $2\frac{3}{4}$ means 2 plus $\frac{3}{4}$

2. Use line segment diagrams for reinforcement of concepts involving one unit.

Children draw a line segment to represent 1.
Ask children: How many halves may be shown on this line segment?



After line diagrams for halves have been developed children suggest ways to indicate fourths on a line segment.

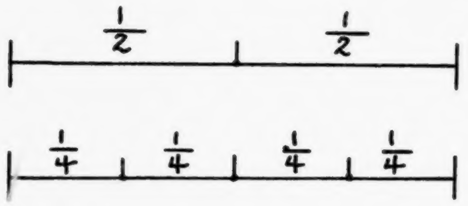


Discuss dividing the line into halves first, then each half into two equal parts. This reinforces the concept that one fourth is one half of one half.

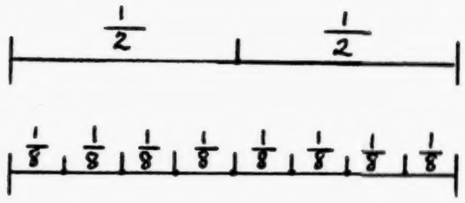
Extend to finding eighths on a line segment.
Divide line segment into halves, then into fourths, finally into eighths. Thus, one eighth is one half of one fourth, or one fourth of one half.

Children use line diagrams and rulers graduated in 8ths to deepen concepts of:

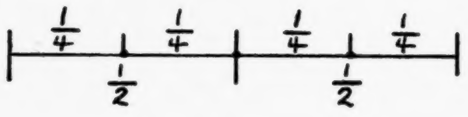
Halves and Fourths



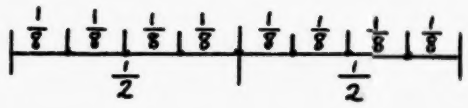
Halves and Eighths



Fourths and Halves



Eighths and Halves



Children mark the following true or false. They make false statements true.

$\frac{3}{8} = \frac{1}{4} + \frac{1}{6}$

$\frac{3}{8} = \frac{1}{2}$ of $\frac{6}{8}$

$\frac{3}{8} = \frac{4}{8}$ minus $\frac{1}{6}$

$\frac{6}{8} =$ 2 times $\frac{3}{8}$

$\frac{6}{8} = \frac{1}{2} + \frac{2}{8}$

Children complete the following operations.

$\frac{1}{2} + \frac{1}{8} = \frac{\square}{8} + \frac{1}{4}$

$\frac{3}{8} + \frac{1}{8} = \frac{\square}{8} + \frac{\square}{8}$

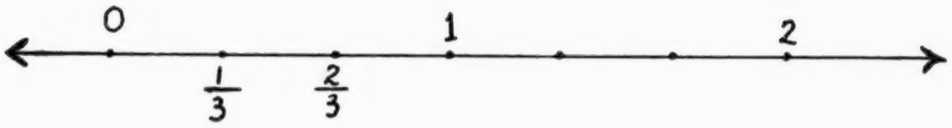
$\frac{3}{8} + \frac{1}{8} = \frac{\square}{4} + \frac{\square}{4}$

$\frac{3}{8} + \frac{1}{8} = \frac{\square}{4} + \frac{1}{8} + \frac{1}{8}$

Emphasize the fact that equivalent fractions are different names for the same number.

Wholes and thirds

Children rename 1 and 2 on the number line below as thirds.



Halves and Fourths

Children indicate that:

1 half has the same value as 2 fourths

$\frac{1}{2}$ = ? times $\frac{1}{4}$

$\frac{1}{4}$ has the same value as $\frac{1}{2}$ of $\frac{1}{2}$

$\frac{1}{4}$ is what part of $\frac{1}{2}$?

Halves and Eighths

Children replace the "n" with the correct numeral.

$\frac{2}{2} = \frac{n}{8}$, $\frac{4}{2} = \frac{n}{8}$, $\frac{8}{2} = \frac{n}{8}$

$\frac{1}{2} = \frac{\square}{8}$, $\frac{3}{2} = \frac{n}{8}$, $\frac{6}{2} = \frac{n}{8}$

$\frac{1}{2}$ = n times $\frac{1}{8}$; $\frac{1}{8}$ = n part of $\frac{1}{2}$; etc.

Fourths and Eighths

Children find solutions to the following:

$\frac{4}{4} = \frac{\square}{8}$, $\frac{5}{4} = \frac{n}{8}$, $\frac{6}{4} = \frac{n}{8}$

$\frac{2}{4} = \frac{\square}{4}$, $\frac{4}{4} = \frac{n}{8}$, $\frac{8}{4} = \frac{n}{8}$

$\frac{1}{4} = ?$ times $\frac{1}{8}$; $\frac{1}{8}$ is what part of $\frac{1}{4}$?

$\frac{3}{4} = ?$ times $\frac{3}{8}$; $\frac{3}{8}$ is what part of $\frac{3}{4}$?

Halves, Fourths and Eighths

Children note that:

$\frac{4}{8}$ is equivalent to 1 half or 2 fourths

$\frac{6}{4}$ equals:
1 whole and 2 fourths, or
1 whole and 1 half, or
1 and 1 half, or $1\frac{1}{2}$

Children substitute numerals for n and $\square$ to make the following equations true. (n and $\square$ represent different numerals)

$\frac{3}{4} = \frac{1}{n} + \frac{1}{\square}$ $\frac{5}{8} = \frac{1}{2} + \frac{\square}{8}$

$\frac{3}{4} = \frac{n}{2} + \frac{\square}{8}$ $\frac{5}{8} = \frac{3}{8} + \frac{\square}{4}$

$\frac{3}{4} = \frac{2}{\square} + \frac{1}{n}$ etc.

$\frac{3}{4} = \frac{n}{8} + \frac{1}{\square}$

Children make these statements true.

$1 = \frac{\square}{2} = \frac{\square}{3} = \frac{\square}{4} = \frac{\square}{8}$

$\frac{1}{2} = \frac{\square}{4} = \frac{\square}{8}$; $\frac{\square}{8} = 1$; $\frac{\square}{4} = 1$; $\frac{6}{8} = \frac{\square}{4}$; $\frac{2}{8} = \frac{\square}{4}$

$\frac{8}{8} = \frac{\square}{4}$; $\frac{3}{8} + \frac{1}{4} = \frac{n}{8}$; $\frac{5}{8} = \frac{1}{2} + \frac{n}{8}$; $\frac{5}{8} = \frac{1}{8} + \frac{1}{n}$

4. Comparisons - thirds, halves, fourths, eighths

Children compare: 1 whole divided into fourths, with wholes of same size divided into thirds, eighths, etc.

Children discover that:

The more equal parts into which the whole has been divided, the smaller the parts will be.

The fewer equal parts into which the whole has been divided, the larger each part will be.

For each of the following ask children:
Which is larger? Which is smaller? Why?

$\frac{3}{8}$ or $\frac{3}{4}$; $\frac{1}{3}$ or $\frac{7}{8}$; $\frac{2}{3}$ or $\frac{1}{2}$; $\frac{3}{4}$ or $\frac{3}{8}$;

$\frac{7}{8}$ or $\frac{1}{3}$; $\frac{1}{2}$ or $\frac{2}{3}$; etc.

Children use symbols (>, <) to show relationships between the pairs of fractions shown above.

Children write > or < between each group of fractional numerals below to make a true statement.

$\frac{1}{3}$ $\frac{1}{4}$; $\frac{1}{3}$ $\frac{1}{2}$; $\frac{5}{8}$ $\frac{3}{4}$; $\frac{7}{8}$ $\frac{3}{4}$;

$\frac{3}{8}$ $\frac{1}{2}$; $\frac{5}{8}$ $\frac{1}{2}$; $\frac{1}{2}$ $\frac{2}{3}$; $\frac{1}{3}$ $\frac{3}{4}$;

Children solve for n in the statements below. In some cases more than one solution is possible.

$\frac{1}{2} > \frac{n}{3}$ $n = ?$

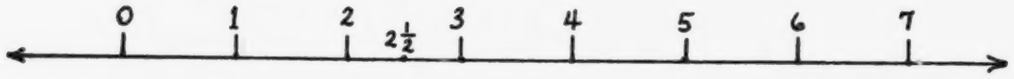
$\frac{1}{4} > \frac{n}{8}$ $n = ?$

$\frac{3}{4} > \frac{n}{8}$ $n = ?$

5. Use line diagrams and rulers graduated in eighths to reinforce or

to introduce fractional parts involving more than one unit.

Locating points on a line.



Ask children to find $1\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $2\frac{1}{2}$, $6\frac{1}{2}$, etc.
Have them place a dot and label.

Emphasize:

One point on the line represents $2\frac{1}{2}$.

$2\frac{1}{2}$ represents the distance or length from 0 to that point.

$2\frac{1}{2}$ is 2 and $\frac{1}{2}$ more.

$2\frac{1}{2}$ means $2 + \frac{1}{2}$.

Children draw a similar line segment on which to indicate fourths. They locate such points as:

$2\frac{1}{4}$, $4\frac{1}{4}$, $6\frac{1}{4}$, etc.

$\frac{2}{4}$, $3\frac{2}{4}$, $1\frac{2}{4}$, $5\frac{2}{4}$, etc.

$1\frac{3}{4}$, $2\frac{3}{4}$, $4\frac{3}{4}$, etc.

Discuss distance from 0 and distance between points, e.g.,

between $1\frac{3}{4}$ and $3\frac{3}{4}$

between $2\frac{1}{4}$ and $3\frac{3}{4}$

Proceed as above for eighths; thirds; halves; fourths and eighths.

Discuss:

a. Where 1 half, 1 fourth, 1 eighth are located on a number line.

b. Find on the number line:

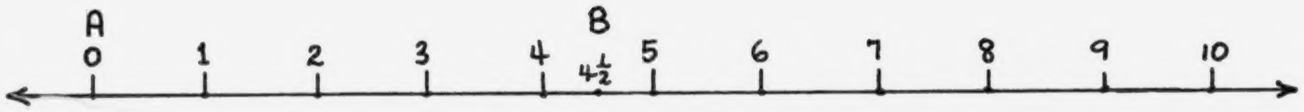
$1\frac{1}{2}$, $4\frac{1}{4}$, $5\frac{1}{8}$, $6\frac{2}{8}$, $7\frac{3}{4}$, $3\frac{5}{8}$

c. Find $1\frac{1}{2}$ on the number line. How many halves in $1\frac{1}{2}$?

Find $1\frac{1}{2}$ on the number line. How many fourths in $1\frac{1}{2}$?

6. Enrichment Activities

Present a number line.



Discuss:

Distance from A to B = distance from B to C. Find Point C.

A	B	C
0	$4\frac{1}{2}$	?
4	$13\frac{1}{2}$	?
$2\frac{1}{4}$	4	? etc.

Compare the number line with a ruler graduated in eighths.

Children find equivalents for more than 1 whole using line diagrams if needed.

A	B
$\frac{3}{2} = 1 + ?$	$2\frac{1}{4} = \frac{?}{4}$
$\frac{5}{2} = ? + \frac{1}{2}$ etc.	$4\frac{3}{4} = \frac{n}{4}$ etc.

C	D	E
$3\frac{3}{8} = \frac{n}{8}$	$1\frac{5}{8} = \frac{n}{8}$	$2\frac{1}{4} = \frac{n}{2} + \frac{\square}{4}$
$1\frac{5}{8} = \frac{n}{8}$	$1\frac{3}{8} = n + \frac{\square}{8}$	$3\frac{1}{2} = \frac{n}{4}$ etc.

Children relate to ruler:

Halves in 1 inch, 2 inches, 4 inches, etc.
Halves in $1\frac{1}{2}$ inches, $2\frac{1}{2}$ inches, etc.

Fourths in 1 inch, 2 inches, 3 inches, etc.
Fourths in $1\frac{1}{2}$ inches, $1\frac{1}{4}$ inches, etc.

Eighths in 1 inch, 2 inches, 3 inches, etc.
Eighths in $\frac{1}{2}$ inch, $\frac{1}{4}$ inch, $\frac{3}{4}$ inch, $1\frac{1}{4}$ inch, etc.

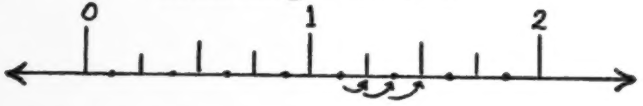
Eighths in $5\frac{1}{4}$ inches

Eighths in $3\frac{3}{4}$ inches

7. Counting

Children use fractional parts and / or number lines to count forward and backward by halves, fourths, eighths.

Counting Forward



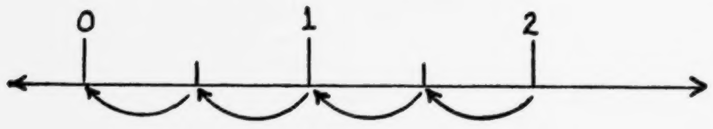
$1\frac{1}{8}$, $1\frac{2}{8}$, $1\frac{3}{8}$, $1\frac{4}{8}$, —, —, —, — Later: $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{3}{8}$, $1\frac{1}{2}$, —, —, —, —

$2\frac{1}{4}$, $2\frac{2}{4}$, $2\frac{3}{4}$, $2\frac{4}{4}$, —, —, —, — Later: $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, 3,
$3\frac{1}{4}$, $3\frac{1}{2}$, $3\frac{3}{4}$.

Children record series of fractions which they counted.

Note: Some children need to count forward and backward with simpler intervals, e.g., counting forward by fourths beginning with $\frac{1}{4}$.

Counting Backward



$\frac{4}{2}$, $\frac{3}{2}$, $\frac{2}{2}$, $\frac{1}{2}$, —, —, —, — Later: 2, $1\frac{1}{2}$, 1, $\frac{1}{2}$, etc.

$\frac{8}{4}$, $\frac{7}{4}$, $\frac{6}{4}$, $\frac{5}{4}$, —, —, —, — Later: 2, $1\frac{3}{4}$, $1\frac{1}{2}$, $1\frac{1}{4}$, etc.

$\frac{8}{8}$, $\frac{7}{8}$, $\frac{6}{8}$, $\frac{5}{8}$, —, —, —, — Later: 1, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, etc.

Children record series of fractions which they counted.

Children count forward or backward by $\frac{1}{2}$ beginning with fractions other than $\frac{1}{2}$. They use number line.

$\frac{1}{4}$, $\frac{3}{4}$, $1\frac{1}{4}$, —, —, —, —, —
$3\frac{3}{4}$, $4\frac{1}{4}$, $4\frac{3}{4}$, —, —, —, —
$4\frac{1}{4}$, $3\frac{3}{4}$, $3\frac{1}{4}$, —, —, —, —, —, etc.

Children count forward by 2 eighths beginning with any point on the line.

$\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{9}{8}$, etc. Later: $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $1\frac{1}{8}$, —, —

They count forward by 3 fourths beginning with any point on the line.

$\frac{1}{8}$, $\frac{7}{8}$, $\frac{13}{8}$, etc. Later: $\frac{1}{8}$, $\frac{7}{8}$, $1\frac{5}{8}$, etc.

They fill in the missing numerals:

$3\frac{1}{8}$, $3\frac{3}{8}$, —, —, $4\frac{1}{8}$, —, —, $4\frac{7}{8}$

GEOMETRY AND MEASUREMENT

UNIT 11 - MEASUREMENT: LENGTH

TEACHING SUGGESTIONS

Objective: To reinforce and extend concepts of length; Relationships among standard units of length.

Procedures

1. Continue to develop finding $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$ of a foot. Express in inches.

   Materials used: Various types of rulers; yardstick; tape measure.

   Discuss relationships:

   1 foot equals 12 inches (is the same length as)

   $\frac{1}{2}$ foot equals 6 inches

   $\frac{1}{4}$ foot equals 3 inches

   $\frac{1}{3}$ foot equals 4 inches

   $\frac{1}{3}$ yard equals 12 inches

   $\frac{1}{3}$ yard equals 1 foot

2. Extend development to include finding $\frac{1}{8}$ of a yard. Relate to $\frac{1}{4}$ of a yard. Express in inches.

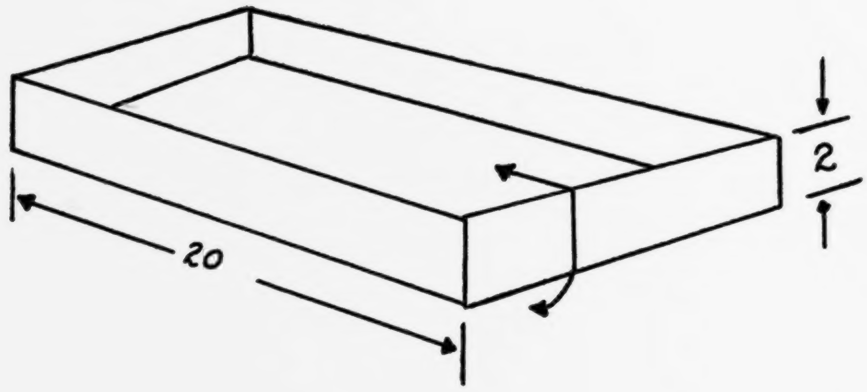
   $\frac{1}{4}$ yard = 9 inches

   $\frac{1}{8}$ yard = $\frac{1}{2}$ of $\frac{1}{4}$ yard

   $\frac{1}{8}$ yard = $\frac{1}{2}$ of 9 inches = $4\frac{1}{2}$ inches

3. Measures relating to mailing parcel post packages.

Have packages of different shapes for pupils to measure. They decide which packages can be sent by parcel post. Apply regulations: "Limit 70 pounds, measuring not more than 100 in. in length and girth combined." Girth is the distance around the package at its widest part.



e.g., Length = 20"
Girth = 44"
Combined length and girth = 64"

4. Reinforce Equivalents

Teacher and children organize a table of the fractional parts of the foot and the yard in inches and stress relationships in various ways.

Relationships among:

Yards, Feet, Inches	Feet and Inches	Yards and inches
1 ft. = 12 in.	1 ft. = 12 in.	1 yd. = 36 in.
1 yd. = 36 in.	1/2 ft. = 6 in.	1/2 yd. = 18 in.
1 yd. = 3 ft.	1/4 ft. = 3 in.	1/4 yd. = 9 in.
1/3 yd. = 1 ft.	1/3 ft. = 4 in.	3/4 yd. = 27 in.
1/2 yd. = 18 in.	2/3 ft. = 8 in.	1/8 yd. = 4 1/2 in.
		1/3 yd. = 12 in.
		2/3 yd. = 24 in.

There are numerous ways of reinforcing these concepts and relationships by organizing different tables in tabular form.

Encourage children to prepare their own tables showing equivalents.

Discuss alternate use of fractional parts of measurements.

We purchase $1\frac{1}{2}$ yd. of ribbon rather than 54 inches.
A carpenter may refer to a length of board as $3\frac{1}{4}$ feet instead of 39 inches.

5. Provide practice in changing to different units.

18 in. $= 1$ ft. 6 in. or $1\frac{1}{2}$ ft. or $\frac{1}{2}$ yd.

$1\frac{1}{2}$ yd. $= 3$ ft. 18 in. or 54 in. or $4\frac{1}{2}$ ft., etc.

$\frac{3}{4}$ yd. $= \square$ in.
Since $\frac{1}{4}$ yd. $= 9$ in., then $\frac{3}{4}$ yd. $= 9$ in. $+ 9$ in. $+ 9$ in. $= 27$ in.

6. Number of feet in one mile.

Suggested activities to develop the concept of the length of a mile.

Estimate and then pace off the length of the classroom.
Estimate and pace off distances in the playground.
Pace off a block.
Locate a familiar landmark about one mile from school.

Pupils visualize the distance of a mile by:

Observing the odometer while riding on a bicycle or in a car.
Relating distance and time.
Using pedometers on walks or hikes.

Note: Children should be aware of the fact that the odometer in a car or on a bicycle registers distance in tenths of a mile.

Discuss distances

Approximate number of blocks in a mile.
Number of blocks from home to school and from school to near-by places of interest.
Bus trips from school to places of interest.

Vacation trips by car .
Heights of mountains.
Heights at which planes travel.
Lengths of rivers and lakes.
Distances run in races.
Lengths of track in athletic stadiums.

7. Have children construct charts to show distance from school to:

Post Office	11 blocks	about $\frac{1}{2}$ mile
Museum	23 blocks	more than a mile
Park	8 blocks	less than $\frac{1}{2}$ mile
Library	5 blocks	about $\frac{1}{4}$ mile

* 8. Discuss units of measure of the Past: **(Optional)**

cubit
fathom
digit
furlong
league
span
hand
pace

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Tell which is more and why

8 inches or $\frac{1}{2}$ yard

$1\frac{1}{4}$ feet or $11\frac{1}{2}$ inches

2. Arrange the following units of length from the shortest to the longest unit.

yard inch foot mile

3. Draw a line segment $\frac{3}{4}$ of an inch in length.

   How many $\frac{1}{8}$ inches in this segment?

   How many $\frac{1}{4}$ inches?

4. Children answer the following:

   Ann had one yard of felt. She cut off 12 inches to make a pencil case. What part of the yard did she use?

   18 inches $\frac{1}{2}$ yard $\frac{1}{4}$ yard $\frac{1}{3}$ yard

   A piece of cloth is 38 inches long. Jane cut off 1 yard to make a towel. How much of the cloth was not used?

   The teacher cut a yard of leather into 8 equal pieces to make bookmarks. Each piece was

   $\frac{1}{2}$ yard 8 inches 9 inches $4\frac{1}{2}$ inches

5. How many feet are there in a mile; how many yards; how many inches?

*Enrichment Activities (Optional)

1. Extend concepts of the mile

   Children report on:

   History of the development of the distance of a mile.

   [Originally the distance of a mile was established as 1000 double paces. One pace is about 3 feet.]

   Relationship of length of football field to the mile.

   [A football field is 100 yards or 300 feet in length. Twenty such fields would be about one mile.]

   Terms, such as: standard mile, statute mile, nautical mile, knots

2. Measurements mentioned in literature, such as:

   "Charge of the Light Brigade" - Half a league onward.

   "Bible" - Noah's Ark; Solomon's Temple

3. Have children explore possible ways to obtain the approximate height of a very tall object, such as the school flagpole.

   Hint: Compare with height of school building where each story is approximately 10 feet high

   and / or

   Child stands next to flagpole. Another child measures the length of his shadow and the length of the shadow of the flagpole. What is the relationship of the child's height and the length of his shadow.

GEOMETRY AND MEASUREMENT

UNIT 12 - MEASUREMENT: TIME

NOTE TO TEACHER

In grade 5, problems involving denominate numbers are solved by computing mentally. Problem solving should include the use of equivalents. Problems may be made by teachers using data from charts. Additional problems may be obtained from various textbooks.

TEACHING SUGGESTIONS

Objectives: To introduce finding fractional parts of time.
To help children interpret and record time.
To help children solve problems.

Procedure

The Calendar

1. Reinforce

Number of months in one year
Names of the months of the year
Names of the seasons
Number and names of the months in each season

2. Introduce

Finding $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$ of a year in months
Relating $\frac{1}{4}$ of a year to the seasons

3. Typical Problems

Today is Sept. 25. My dental appointment is two weeks from today. What date will that be?

Halloween is on Oct. 31. Invitations for a party should be mailed ten days in advance. On what day and date shall I mail them?

(Children solve problems in a variety of ways.)

Suggested Experiences

Noting seasonal changes - weather, outdoor phenomena
Computing variations in length of daylight, length of night.
Marking data on calendar - classroom parties, assembly programs, special events, etc. - Computing length of time until event.
Days, weeks, school days in a month, etc.

Visits to dentist - twice a year, every six months, - semi-annually.
Estimate calendar time since last visit.

Ways in which to record dates - 10/3/66, 10 - 3 - 66

The Clock

1. Continue to develop:

Minutes in one hour, half hour, quarter hour, three-quarter hour
Telling time by the hour, half hour, quarter hour, 5 minute intervals, etc.

A.M. and P.M. - before noon and afternoon. Extend understanding to ante and post meridian.

A day as extending from midnight to midnight

2. Interpret and record time

Assembly Program	9:00 A.M.	
Recess	10:30 A.M.	(read as half past ten or ten thirty)
Supper	6:15 P.M.	(15 minutes after 6 or a quarter past 6 or 6 fifteen)
Sunrise	5:18 A.M.	(18 minutes after 5, or 5 eighteen)
Sunset	6:50 P.M.	(6 fifty, 10 minutes to 7)
Lunch	12:00	(12 noon)

3. Solve "verbal" problems in a variety of ways.

How long was it from supper to sunset?
How much more daylight did we have today than we did on Monday?

Our Trip To The Museum

Left school	9:30
Arrived at Museum	10:15
Lecture - Far North	10:25
Eskimo Exhibit	11:00
Film - Eskimo Life	12:05
Lunch	12:45

How long did it take us to get to the museum?

To solve, children think:
From 9:30 to 10:00 A.M. ($\frac{1}{2}$ hour), from 10:00 to 10:15 ($\frac{1}{4}$ hour).
The trip took $\frac{3}{4}$ of an hour.

Some children might compute in minutes:
From 9:30 to 10:00 (30 min.), from 10:00 to 10:15 is another 15 minutes. The trip to the museum took 45 minutes.

We came to school at 8:40. How much time did we have before we left for the museum?

How long was it from the time we arrived at the museum until lunch time?

We were due back at school at 3:00 o'clock. If the trip back takes 50 minutes, at what time must we leave the museum?

Some children may need to refer to a clock or a clock face.

Experiences, Activities and / or Homework

Children:
Compare variations in time of daylight, night.
Plan for time of preparation and duration of party.
Estimate amount of time for preparation of assembly programs, culmination of unit, etc. Use both calendar time and clock time.

Compute time for out-door activities
Plan schedule for the school day
Note and record time of sunset and sunrise (refer to newspaper)
Note changes in length of days
Construct and interpret charts based on information found in newspapers, e.g., time of sunrise and sunset
Note time of radio and television programs
Note time spent in various activities outside of school
Discuss best use of time at home
Use schedules and time tables (discussing travel time)

* Enrichment Activities **(Optional)**

Using Timetables and Bus Schedules

Select two trains, two buses, or a train and a bus. Determine which has the greater speed by comparing travel time between the same places.

Write a problem involving which train or bus to take, in order to keep an appointment in another city, at a specific time.

Children look up and explain

Other calendars, e.g. Gregorian

Ancient devices for telling time: Sundial; water-clock; etc.

UNIT 13 - SYSTEMS OF NUMERATION: ROMAN SYSTEM

NOTE TO TEACHER

The Roman System of Numeration is taught for two reasons.

1. Understanding of various systems of numeration helps to deepen children's understanding of the decimal system.

2. Roman numerals are still used on clock faces, for chapter headings in books, on cornerstones of buildings, etc.

We teach the Roman system as another way of recording numbers, another system of numeration.

We should stress the relationships within the system and comparisons with the decimal (Hindu-Arabic) system of numeration.

The Roman System uses seven symbols:

I, V, X, L, C, D, M.

The following principles are used in combining these symbols to represent numbers:

1. The Rule of Repetition: When the Roman Numeral, I, X, C, M is repeated its value is added. For example: XXX represents 10 + 10 + 10.

2. The Rule of Addition: The values of the Roman Symbols are added when the symbol representing the larger number is placed at the left in the numeral. For example:

   VII = V + I + I; DC = D + C

3. The Rule of Subtraction: When a numeral of lesser value is written to the left of a numeral of greater value the lesser value is to be subtracted from the greater. See Item 2 under "Teaching Suggestions". (This is a late adaptation of the original Roman System).

   The subtractive property is used specifically to represent numbers such as: four, nine, ninety, four hundred, nine hundred.

   IV = four XL = forty CD = four hundred

   IX = nine XC = ninety CM = nine hundred

Some comparisons that can be made with the Decimal system are:

1. There is no symbol for "zero" in the Roman System of Numeration.
2. Place Value is not used in the Roman System except when the subtraction rule is involved.
3. The Rule of Repetition is not used in the Decimal System.
4. The Rule of Subtraction is not used in the Decimal System.

TEACHING SUGGESTIONS

Objectives: To introduce new symbols and their values.
To compare the Roman System of Numeration with the Decimal System of Numeration.
To provide practice in reading and writing Roman Numerals.

Procedure

1. Evaluate children's understanding of the Roman System.

   Write the following in the decimal system of numeration:

   VII
   XIV
   XXIX
   XLVI
   XXXV
   XLIX
   CCII
   CXIV

Write the following using Roman numerals:

8	41	24	94
17	39	50	140

Write a Roman numeral of two symbols which shows that the value of one symbol is to be added to the value of the other symbol to obtain the value of the entire written symbol.

Write a Roman numeral of two symbols which shows that the value of one symbol is to be subtracted from the value of the other symbol to obtain the value of the entire written symbol.

Choose the Roman numeral on each line which is the sum for the additions at the left:

IV + VI	VVII	IVVI	X
IX + XI	XX	IXXI	XXII
XXI +XIX	XXXI	XXIXIX	XL
XXV +XXV	XXXX	L	XXXVV

2. Reinforce understanding of the Roman System

Discuss the value of the basic symbols (I, V, X, L, C) used in the Roman system.

Have children compare the Hindu-Arabic system of notation, which is a positional system, with the Roman system.

Be sure children understand that:
When a Roman numeral is repeated, its value is added.

II represents	1 + 1 or 2
III represents	1 + 1 + 1 or 3
XX represents	10 + 10 or 20
XXX represents	10 + 10 + 10 or 30
CC represents	100 + 100 or 200

Roman symbols "V" and "L" are not repeated, since the value of two V's is represented by X and the value of two L's is represented by C.

When a Roman numeral of lesser value is written before a numeral of greater value, it indicates subtraction of the lesser value from the greater value. The symbols V, L are never written to the left of numerals of greater value

I to the left of V	(IV) indicates	1 less than 5
I to the left of X	(IX) indicates	1 less than 10
X to the left of L	(XL) indicates	10 less than 50
X to the left of C	(XC) indicates	10 less than 100

The symbol I may be used subtractively with V and X only.

The symbol X may be used subtractively with L and C only.

3. Continued Development

Test children's ability to translate Roman numerals into Arabic numerals and Arabic numerals, through 399.

Introduce D as the Roman numeral representing 500. Have children write the Roman numerals for 390 **through 500.**

Provide practice in reading and writing Roman numerals **which** represent numbers through 500 and beyond.

EVALUATION and / or PRACTICE

SUGGESTED EXERCISES

1. Children read the following:

V	VI	IV	VIII	Etc.		
X	IX	XI	XXI	XV	XXX	Etc.
L	XL	LX	LXIX	XLV	LVII	Etc.
C	CCX	XC	CIX	CXI	CL	Etc.

2. **Children** write the following series using Roman numerals.

10 - 19	89 - 95	310 - 319
40 - 49	95 - 106	500 - 508
70 - 80	281 - 292	590 - 600

*3. Point out the advantage of the Decimal System. **(Optional)**

*4. Explore one or two computations involving addition, and / or multiplication using Roman numerals. **(Optional)**

*5. Explain why the Romans did not include a symbol for zero in their numerals. **(Optional)**

OPERATIONS

UNIT 14 - SET OF WHOLE NUMBERS: ADDITION AND SUBTRACTION; PROPERTIES APPLIED; HORIZONTAL FORMAT

NOTE TO TEACHER

Adding and subtracting by groups (mental computation) involves computing from left to right rather than from a visualized vertical algorithm.

This generally involves the application of the Associative Property for Addition. Children should recognize the need for quick mental computation in real life situations.

Encourage children to compute "mentally" with the numbers that they can deal with easily.

Encourage them to discover many ways to arrive at sums and remainders. There is no one correct or best way of arriving at a solution. For example:

To add:

58 + 26: A child might begin with the first addend and rename 26 as 20 + 2 + 4. He could then apply the associative property and think 58 + 20 = 78, 78 + 2 = 80, 80 + 4 = 84.

or

A child might rename 58 as 50 + 8 and rename 26 as 20 + 6. He could then apply the associative property and think 50 + 20 = 70, 8 + 6 = 14, 70 + 14 = 84

or

A child might think 60 + 26 = 86, 86 - 2 = 84, etc.

To subtract 192-29:

A child might think: 192-20 = 172, 172-9 = 163 or

A child might think: 192-30 = 162, but since I subtracted 1 too many, I must add 1; 162 + 1 = 163.

To compute using a horizontal format children need to have automatic response to basic addition and subtraction facts, and have ability in adding and subtracting numbers involving extensions beyond the facts. They need facility in regrouping numbers.

In presenting addition or subtraction exercises:

Record the entire exercise

or

Write one of the numerals and present the other orally

or

State the entire exercise orally.

Children record only the sum or the remainders.

Teacher may wish to write some of the partial sums or remainders as the child states the thinking.

TEACHING SUGGESTIONS

Objective: To develop facility in adding and subtracting "mentally".

Procedures

1. Test children's understanding of relationships.

   Children replace "n" with numerals, to make the equation a true statement.

Use of Associative Property	Indirect Application of Associative Property
45 + 23 = 65 + n	68 - 23 = 48 - n
62 + 19 = 72 + 8 + n	92 - 57 = 42 - n
58 + 36 = n + 6	81 - 45 = n - 5
324 + 150 = 424 + n	237 - 12 = 227 - n

Doubles and Near-Doubles

49 + 50 = 100 - n
250 + n = 500
125 + 126 = 250 + n
86 + 86 = 160 + n
57 + 57 = n + 14

Use of the Commutative Property

32 + 265 = 265 + n
n + 458 = 458 + 24
21 + 518 = n + 21
57 + n = 112 + 57

2. Provide practice in adding and subtracting using the horizontal format. From time to time present addition and subtraction exercises as shown below to emphasize that each addition fact has a related subtraction fact.

Have children explain this relationship. (Idea of inverse)

66 + 7 = □
236 + 7 = □
196 + 7 = □
2256 + 7 = □
1396 + 7 = □

73 - 7 = □
243 - 7 = □
203 - 7 = □
2263 - 7 = □
1403 - 7 = □

The following suggested exercises are graded as to difficulty. Children should find sums and remainders. The teacher should ask some children to explain how they arrived at a solution. They show how the Commutative and Associative Properties are being used to justify steps in the computation.

A

27 + 27 = □
65 + 65 = □
45 + 18 + 26 = □
12 + 36 + 27 + 23 = □
67 + 61 = □
56 + 57 = □
34 + 85 + 26 = □

80 - 39 = □
99 - 14 = □

B

243 + 34 = □
243 + 38 = □
507 + 68 = □
40 + 319 = □

270 - 30 = □
567 - 60 = □
184 - 32 = □
364 - 45 = □

C

140 + 140 = □
122 + 122 = □
125 + 125 = □
115 + 115 = □
135 + 135 = □

220 - 160 = □
290 - 145 = □
350 - 175 = □

3. Extend development to numbers through 9999.
Present the following graded series discussing the problems and ways of solving them. Most children will need to work with smaller numbers first.

Present many more examples of each type within a series. (Read 4-place numerals as hundreds to facilitate arriving at sums and remainders).

A

2300 + 3000 = □
1160 + 1000 = □
3226 + 2000 = □
5448 + 4000 = □

5300 - 3000 = □
2160 - 1000 = □
9448 - 4000 = □
7008 - 2000 = □

B

2416 + 60 = □
2006 + 60 = □
3652 + 20 = □
5048 + 30 = □

2476 - 60 = □
4870 - 40 = □
1396 - 50 = □
5078 - 30 = □

C

1538 + 200 = □
3215 + 300 = □
4351 + 100 = □
3642 + 200 = □

5117 + 700 = □

D

2760 - 200 = □
1938 - 400 = □
4651 - 300 = □
3842 - 500 = □

5617 - 700 = □

E

1163 + 15 = □ [May be thought through as (1163, 1173, 1178)]
2434 + 23 = □
62 + 4526 = □ [May be thought through as 4526 + 62]
3058 + 41 = □
5555 + 32 = □

1178 - 15 = □ [May be thought through as (1178, 1168, 1163)]
2457 - 23 = □
3149 - 37 = □
4189 - 22 = □
5276 - 43 = □

F

2536 + 24 = n [Thought through as (2536, 2556, 2560)]
1769 + 25 = n
1441 + 39 = n
2657 + 18 = n
3223 + 49 = n

3244 - 26 = n [Thought through as (3244, 3224, 3220, 3218)]
2560 - 34 = n
1480 - 39 = n
2675 - 18 = n
3272 - 47 = n

OPERATIONS

UNIT 15 - MULTIPLICATION OF WHOLE NUMBERS: FACTS AND EXTENSIONS BEYOND THE FACTS

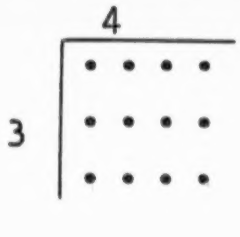
NOTE TO TEACHER

Interpretation of Multiplication

In Addition, an ordered pair of numbers is operated on to yield a third number called their sum.

In Multiplication, a unique third number, called the product, is similarly assigned to an ordered pair of numbers.

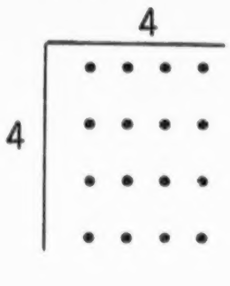
There are at least two interpretations involving the operation of multiplication on the set of whole numbers. In previous grades children have interpreted multiplication as repeated addition. This interpretation is shown below by the use of rectangular arrays. An array is an orderly arrangement of sets of things, numerals or other symbols.
Consider these arrays:



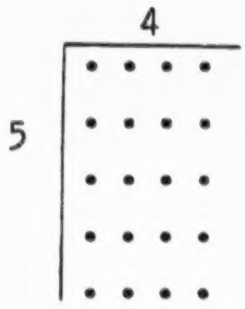
$3 \times 4 = 12$

or

$4 \times 3 = 12$



$4 \times 4 = 16$



$5 \times 4 = 20$

or

$4 \times 5 = 20$

In each of the arrays above the product may be computed by successive additions.
Thus, the product for 3 x 4 may be interpreted by considering 4 as an addend 3 times; $4 + 4 + 4 = 12$ or 3 as an addend 4 times as $3 + 3 + 3 + 3 = 12$.

Concept of Product Set

Another interpretation of multiplication of whole numbers involves a situation such as the following:

> Joan has a set of 3 new blouses;
> a red one, a yellow and a white one.
> She has a set of 2 new skirts;
> a grey one and a blue one.
> She wants to know how many different
> outfits she can have.

Possible matchings:

Grey skirt, red blouse
Grey skirt, yellow blouse
Grey skirt, white blouse

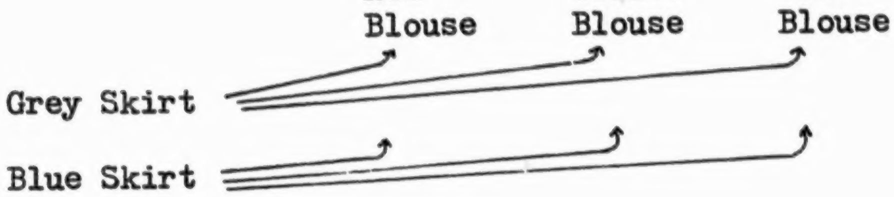
Blue skirt, red blouse
Blue skirt, yellow blouse
Blue skirt, white blouse

The Product Set (also called Cartesian Product) of two sets A and B is denoted by A x B and is defined as the set of all ordered pairs (a,b) where a is an element of Set A and b is an element of Set B.

Here if $A = \{\text{red, yellow, white}\}$ and
$B = \{\text{grey, blue}\}$
then the six ordered pairs above list the elements of A x B.

Notice that $N(A \times B) = 6$
$N(A) = 3$
$N(B) = 2$

and that $N(A \times B) = N(A) \times N(B)$ which is the reason why A x B is called the product set.
Thus A x B in the above illustration can be represented on a chart as:



and represented as an array:

			Blouses	
		Red	Yellow	White
Skirts	Grey	•	•	•
	Blue	•	•	•

There are then, six matchings possible and we see that by this interpretation of multiplication of the ordered pair (2, 3) we arrive at the same array with 6 elements as in the first interpretation.
The numbers operated on, 3 and 2, are called factors.
The result of the operation, 6, is called the product.

$\boxed{\text{Factor x Factor = Product}}$

Properties of Multiplication

Some of the properties that apply to multiplication and which will be used in this bulletin are:

Commutative Property for Multiplication which states that reversing the order of the factors does not affect the product.
For example: $2 \times 3 = 3 \times 2$

Associative Property of Multiplication which states that the order of associating a product involving more than two factors does not affect the product.
For example:
$6 \times 5 = (3 \times 2) \times 5 = 3 \times (2 \times 5) = 3 \times 10$

Distributive Property of Multiplication With Respect to Addition combines multiplication and addition.
For example:
For 6×23, we can write
$6 \times (20 + 3) = (6 \times 20) + (6 \times 3)$

For 23×6, we can write
$(20 + 3) \times 6 = (20 \times 6) + (3 \times 6)$
and we obtain the same product in both cases, because of commutativity.

Identity Element for Multiplication

The number "One" has a special property with respect to multiplication. The product of one and any number is that same number.
For example: 1 x 8 = 8; 8 x 1 = 8

Multiplicative Property of Zero

The product of zero and any number is zero.
For example: 0 x 8 = 0; 8 x 0 = 0; 0 x 0 = 0

Closure

The set of whole numbers is closed with respect to multiplication; that is, when multiplying any two whole numbers, the product is always a whole number.
For example: 8 x 4 = 32

Drill on Multiplication Facts and Extensions Beyond the Facts

Multiplication and division facts were developed in Grade 4. However, drill for automatic response is still essential in Grade 5.

After an interval of meaningful drill, and/or, development, the teacher should test for automatic response.

Keep drill periods short (5 - 10) minutes.
Children's responses may be written or oral.
Written responses are preferable.
Children record products or quotients only.

Present sets of multiplication facts according to specific patterns or relationships. One or more of these patterns may be presented during the drill period. Guide cards showing these patterns may be used by the teacher for easy reference.

Equations stressing properties of addition and multiplication can be used as another form of drill. These, too, should be organized according to specific patterns.

Relate the drill to the major topic to be developed wherever possible.

Extension beyond the basic facts are needed for mental computation in multiplication and as background for division.
Sentences should be presented in horizontal form.
The teacher may record facts using words and numerals or numerals only, e.g. 2 sixes or 2 x 6.
Children may read 3 x 6 as 3 sixes or 6 threes (3 - six times).
Note that, because of commutativity of multiplication we no longer give just one interpretation to 3 x 6.

TEACHING SUGGESTIONS

Objectives: To help children extend their concepts of multiplication through the use of arrays.

To teach the concept of factors, multiples.

To reinforce multiplication facts and extensions.

Arrays

1. Present a problem such as:

> Joan has a set of 3 new blouses; a red one, a yellow and a white one. She has a set of 2 new skirts; a grey one and a blue one. She wants to know how many different outfits she can wear.

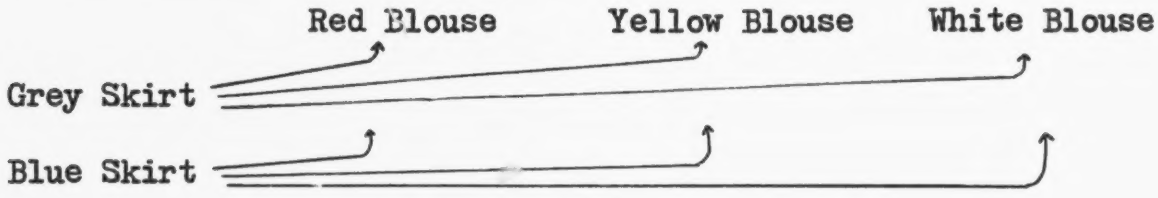
Ask children:

> In how many different ways can Joan match a blouse and skirt?

Record the different matchings as children discuss them.

Grey skirt, red blouse
Grey skirt, yellow blouse, etc.

2. Teacher and children should prepare a chart on the chalkboard showing the different matchings.



3. Tell children that what we have shown is called an array. The array is also shown in another way in the Note to Teacher, page

   Ask children:

   How many different blouses can Joan wear with her grey skirt? [3; red, yellow, white]

   How many different blouses can she wear with her blue skirt? [3; red, yellow, white]

   How many different skirts can Joan wear with her red blouse? [2; grey, blue]
   with her yellow blouse? [2]
   with her white blouse? [2]

   How many different outfits can Joan make matching 2 skirts and 3 blouses? [6]

   How can we determine the number of different outfits Joan can have without preparing an array? [2 x 3 or 3 x 2]

   How many outfits would Joan have if she had 4 blouses and 2 skirts?

   Tell children to examine the array and ask them whether they can tell what an array is. Use a multiplication chart to show an array of numerals. Elicit from them that an array is an orderly arrangement of objects or numerals.
   Point out that here we use rectangular arrays.

4. Present another problem:

   Three boys (John, Ted and Bill) and 3 girls (Mary, Sue and Alice) came to the school party. Draw an array to show how many different sets of partners there can be for dancing.

Direct children:

To make an array to show the matchings in the problem.

To tell how they can find the solution without counting the dots in the array.
[Multiply the number of rows by the number of columns]

To tell how they would find the solution without drawing an array. [Multiplication]

5. Ask children to draw an array to show 4 x 5.
Discuss the multiplication facts chart as an array.

x	0	1	2	3	etc
0	0				
1					
2					
3					
etc					

*6. Ask children to draw an array to show 1 x 5; to show 0 x 5 (Optional)

Factors

1. Ask children:

To give different numerals for a number such as 12 or 36.
Teacher records responses.

12 = 8 + 4
12 = 4 x 3
12 = 24 ÷ 2
12 = 20 - 8 etc.

36 = 30 + 6
36 = 18 + 18
36 = 9 x 4
36 = 72 ÷ 2

To select those names which involve only the operation of multiplication. [12 = 4 x 3; 36 = 9 x 4]

They may rename 12 and 36 as shown below:

12 = 2 x 6
12 = 6 x 2
12 = 12 x 1
12 = 3 x 4 etc.

36 = 4 x 9
36 = 6 x 6
36 = 9 x 4
36 = 18 x 2 etc.

To record many sentences involving multiplication using the numbers below as products:

24, 21, 81, 63, 45

e.g.[24 = 3 x 8; 24 = 4 x 6; 24 = 2 x 12; 24 = 6 x 4; 24 = 24 x 1]

2. Tell children that the name given to numbers which are multiplied to obtain a product is "factors."
Record: factor x factor = product
Teacher should write the sentence: 3 x 6 = 18.
Children identify the factors. [3, 6]
Present other multiplication sentences. Ask children to identify factors.
Elicit from the children that multiplication involves finding a product when two factors are given.

3. Patterns as shown in the multiplication table.
Use rexographed outlines or graph paper to present the multiplication "table" as shown below.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6							27
4	0	4								36
5										45
6										54
7										63
8										72
9										81

Include: The operational symbol "X"; 0 to 9 in the left hand column; 0 to 9 along the top row.

Have children complete one row at a time as shown above. Help them to observe patterns. Ask:

What are the intervals or the differences between two consecutive numbers in each vertical column?

What is the relationship between the intervals and the heading?

Have children examine the products in any one column.

How can you show that the product of any two factors is not affected by the order of the factors.

Note the horizontal and vertical rows of Zero.

Why are there complete lines of Zero?

What property is involved?

They note the numerals in the second row, and compare these with the heading.

Why are they the same? (Property of "1" in Multiplication)

Children note the numerals in the second column and compare these with the numerals at the left.

Why are these the same?

They should note that 12 appears four times. Why? 30 appears only twice. Why?

They find the numerals that appear only once. Why?

Multiples

1. Teacher should record as children count in sequence. (3,6,9,12,15,etc.)
   Children should be asked to count by threes through 30.
   They can list, in sequence, multiplication sentences using 3 as the constant factor. (3 = 1 x 3; 6 = 2 x 3; 9 = 3 x 3; 12 = 4 x 3; etc.)
   Each number in the sequence above is the product of two factors.
   What factor appears in each multiplication? [3]

2. Tell children that all products that have 3 as a factor are called multiples of 3. Instead of saying "3 is a factor of 12", it is often convenient to say, "12 is a multiple of 3".

3. Children should:

   List ten multiples of 3, beginning with 3. (3,6,9,12,15,etc.)

   Explain why 6 is a multiple of 3; 9 is a multiple of 3; 12 is a multiple of 3, etc.

   Name a multiple of 3.

Discuss zero as a multiple of 3, of 2, of every counting number.

Name the smallest multiple of 3; (0) the greatest (there is no greatest)

List the first 10 multiples of 2; of 3; of 4; of 6.

4. Teacher and/or children should record these sets of multiples.

A. Multiples of 2: {0, 2, 4, 6, 8, 10, 12, ... }
B. Multiples of 3: {0, 3, 6, 9, 12, 15, 18, ... }
C. Multiples of 4: {0, 4, 8, 12, 16, 20, 24, ...}
D. Multiples of 6: {0, 6, 12, 18, 24, 30, 36, ... }

Ask the children to examine the sets above.

What can you call 12 in Set A? [A multiple of 2]
What can you call 12 in Set B? [A multiple of 3]
What can you call 12 in Set C? [A multiple of 4]

12 is a common multiple of which numbers? (a common multiple of 2, 3, 4)

Find other multiples which appear in two or more of the sets. Explain.

5. Children should be encouraged to state the meaning of a multiple in their own words.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. Name any multiple of 5.

2. List four multiples of 7; of 8; of 9.

3. Is 48 a multiple of 6? of 8? of 12? of 24? of 36? Justify your answers.

4. Which of the numbers below is not a multiple of 9? Why?

    83, 72, 27, 35, 62, 54, 45, 108, 225

5. Write the first twelve multiples of 10; of 100; of 1000.

6. Compare the multiples of 2 and 4.

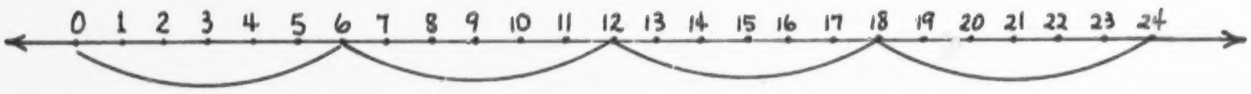
    Is every multiple of 2 also a multiple of 4? Why?

    Is every multiple of 4 also a multiple of 2? Why?

Number Line

Another approach to interpreting multiplication of whole numbers is through the use of the number line.

Present a number line:



Suggested children's activities:

Use the number line to count by threes, fours, fives, sixes, etc.

Begin with zero and state the multiples of 4, of 5, of 6, etc.

Which multiple of 6 would follow 24; 30; etc.

Facts and Extensions Beyond the Facts

1. Introduce the use of parentheses to avoid ambiguity in evaluating numerical phrases. (Order of Operations)
   Record: $4 \times 6 + 2 = ?$ Discuss possible interpretations.
   $(4 \times 6) + 2 = ?$ $4 \times (6 \times 2) = ?$
   What should $2 + 3 \times 4$ mean? Does it mean $(2 + 3) \times 4$ or $2 + (3 \times 4)$?

   Tell children that a convention used in mathematics is that in an expression such as the one above, without parentheses, multiplication should be done before addition; thus here $2 + 3 \times 4 = 14$.

   Provide drill and emphasize the role of parentheses.

   $(3 \times 6) + 6 = n$; $n =$

   $(5 \times 6) + 18 = n$; $n =$

   $12 + (3 \times 6) = n$; $n =$ etc.

   Children place parentheses to make the following statements true:

   $4 \times 6 + 12 = 36$; $8 \times 6 + 6 = 54$; $6 + 7 \times 6 = 48$; etc.

2. Reinforce additions to help children find products. Ability to add is essential for the application of the Distributive Property.

   Present the following additions. Follow with the multiplications that apply. For example:

   $18 + 6 = n$ For $4 \times 6 = (3 \times 6) + (1 \times 6)$

   $= 18 + 6$

   $= 24$

   $\downarrow$ $\downarrow$

   Adding 6 to multiples of 6

   Using addition to derive product. Application of Distributive Property.

   Discuss the following with children to help them see how unknown products may be derived from known products:

   $48 + 6 = n$ For $9 \times 6 = (8 \times 6) + (n \times 6) = \square$

   $300 + 6 = n$ For $51 \times 6 = (50 \times 6) + (1 \times n) = \square$

3. After drill and/or development of multiplication facts test children for automatic response to those facts. Use test results to organize children into groups for further drill.

   Suggested drills follow:

   Adding 6 to multiples of 6

   18 + 6, 48 + 6

   66 + 6, 72 + 6, 120 + 6, 300 + 6

   Adding multiples of 6 to multiples of 6

   18 + 12, 36 + 12, 24 + 18, 30 + 24

   60 + 12, 240 + 18, 180 + 36, 420 + 54, etc.

   Doubles of multiples of 6

   12 + 12, 18 + 18, 24 + 24

   60 + 60, 120 + 120, 240 + 240, 180 + 180, 360 + 360 etc.

Adding tens of sixes to tens of sixes

120 + 60, 180 + 60, 420 + 120, 360 + 180, etc.

4. For Multiplications

Emphasize relationships (doubling, adding groups, etc.) in deriving each product from the preceding product.

Doubling of sixes

2 x 6, 4 x 6, 8 x 6; 3 x 6, 6 x 6; etc.
8 x 6, 16 x 6, 32 x 6, etc.
6 x 6, 12 x 6, 24 x 6, etc.
30 x 6, 60 x 6, 120 x 6, etc.

Adding sixes

3 x 6, 4 x 6; 6 x 6, 7 x 6; 8 x 6, 9 x 6;
10 x 6 11 x 6; 20 x 6, 21 x 6; 30 x 6, 31 x 6; etc.
5 x 6, 7 x 6; 5 x 6, 9 x 6; 7 x 6, 9 x 6;
10 x 6, 14 x 6; 20 x 6, 27 x 6; 50 x 6, 53 x 6; etc.

Doubling and adding sixes

4 x 6, 8 x 62, 9 x 6; 3 x 6, 6 x 6, 7 x 6;
10 x 6, 20 x 6, 21 x 6; 20 x 6, 40 x 6, 43 x 6; etc.

Subtracting sixes

5 x 6, 4 x 6; 10 x 6, 9 x 6;
20 x 6, 19 x 6; 100 x 6, 90 x 6; 100 x 6, 99 x 6; etc.

Halving with sixes

100 x 6, 50 x 6; 50 x 6, 25 x 6; 25 x 6, $12\frac{1}{2}$ x 6;
300 x 6,150 x 6; 150 x 6, 75 x 6; etc.

Using Equations

Use patterns to arrive at solutions. Ask children to explain their thinking. When children record equations, the solutions for "n" should be recorded separately, for example:

$$4 \times 6 = 24 \qquad 13 \times 6 = 60 + n$$
$$2 \times 6 = n \qquad n = 18$$
$$n = 12$$

	From $4 \times 6 = 24$, derive:	From $5 \times 6 = 30$, derive:
(by doubling)	$8 \times 6 = n$; $n =$	$10 \times 6 = n$; $n =$
(by halving)	$2 \times 6 = n$; $n =$	$2\frac{1}{2} \times 6 = n$; $n =$
(by adding sixes)	$5 \times 6 = n$; $n =$	$7 \times 6 = n$; $n =$
(by subtracting sixes)	$3 \times 6 = n$; $n =$	$4 \times 6 = n$; $n =$

From $20 \times 6 = 120$, derive:	From $100 \times 6 = 600$, derive:
$40 \times 6 = n$; $n =$	$200 \times 6 = n$; $n =$
$10 \times 6 = n$; $n =$	$50 \times 6 = n$; $n =$
$21 \times 6 = n$; $n =$	$130 \times 6 = n$; $n =$
$19 \times 6 = n$; $n =$	$99 \times 6 = n$; $n =$

Adding 1 or more sixes

$5 \times 6 = 30$, $9 \times 6 = (5 \times 6) + \square$

$5 \times 6 = 30$, $9 \times 6 = 30 + (n \times 6) = \square$

$6 \times 6 = 36$, $7 \times 6 = (6 \times 6) + \square$

$7 \times 6 = 42$. $9 \times 6 = (7 \times 6) + \square$

$8 \times 6 = 48$, $9 \times 6 = (8 \times 6) + \square$

$8 \times 6 = 48$, $9 \times 6 = 48 + (n \times 6) = \square$

$20 \times 6 = 120$, $21 \times 6 = (20 \times 6) + \square$

$20 \times 6 = 120$, $24 \times 6 = 120 + (n \times 6)$

$20 \times 6 = 120$, $27 \times 6 = 120 + (n \times 6)$

$50 \times 6 = 300$, $51 \times 6 = (50 \times 6) + \square$

$50 \times 6 = 300$, $53 \times 6 = 300 + (n \times 6) = \square$

Doubling One Factor; Keeping the Other Constant

4 x 6 = 24, 8 x 6 = 24 + (n x 6);
10 x 6 = 60, 20 x 6 = (10 x 6) + (   );
40 x 6 = 240, 80 x 6 = 240 + (n x 6);
60 x 6 = 360, 120 x 6 = 360 + (n x 6) = 720;
240 x 6 = (n x 6) + 720 = □; etc.

The product of 40 x 6 is how many times as large as the product of 20 x 6? Explain. If we double one factor, what happens to the product?

How many sixes are added to 30 sixes to get 60 sixes?

Other Patterns

If 10 x 6 = 60, then 20 x 6 = (n x 6) + (n x 6)
and 30 x 6 = (□ x 6) + (n x 6)

If 5 x 6 = n, then 10 x 6 = (□ x 6) + (□ x 6)
and 15 x 6 = (□ x 6) + (? x 6)

Encourage children to find a product in a variety of ways. Record as children explain.

9 sixes = 18 + 18 + n = ? (3 sixes + 3 sixes + 3 sixes)

9 sixes = 30 + n = ? (5 sixes + 4 sixes)

9 sixes = 24 + 24 + n = ? (4 sixes + 4 sixes + 1 six)

9 sixes = 60 - n = ? (10 sixes - 1 six)

24 x 6 = 60 + 60 + n (10 sixes + 10 sixes + 4 sixes)

24 x 6 = 72 + n (12 sixes + 12 sixes)

24 x 6 = 120 + n (20 sixes + 4 sixes)

123 x 6 = 600 + 120 + n (100 sixes + 20 sixes + 3 sixes)

123 x 6 = 360 + 360 + n (60 sixes + 60 sixes + 3 sixes)

$704 \times 6 = (700 \times 6) + n$

$704 \times 6 = (700 \times n) + 24$

$704 \times 6 = (700 \times n) + ?$

$704 \times 6 = (n \times 6) + 24$

$704 \times 6 = 4200 + n$

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. Write a mathematical sentence to solve the following problems. Draw an array if necessary.

    Ann has two necklaces and 4 bracelets. In how many ways can she match her necklaces and bracelets?

    Four children play the piano and 2 children play the violin. Make an array to show how many different duets can be formed consisting of a pianist with a violinist.

    How many matchings can you make of a set of 3 with a set of 6? A set of 4 with a set of 6? A set of 5 with a set of 6?

    How many dots are there in an array if the two sets being matched have 6 elements and 7 elements, respectively?

2. Interpreting Symbols; Evaluating Concepts

    Write each of the following in as many ways as you can.

$$7 + 7 + 7 + 7 = 28 \qquad \begin{array}{r} 3 \\ \underline{\times 5} \\ 15 \end{array} \qquad 5 \text{ fours} = 20$$

$$4 \times 6 = 24$$

Which of these can be written as multiplication facts?

$$2 + 2 + 2 + 2 \qquad 3 + 3 + 3 + 4$$

$$4 + 4 + 4 + 3 \qquad 5 + 4 + 5 + 4$$

Write these as multiplications.

$$\begin{array}{r} 7 \\ 7 \\ \underline{14} \end{array} \qquad \begin{array}{r} 6 \\ 12 \\ 6 \\ \underline{18} \end{array} \qquad \begin{array}{r} 8 \\ 4 \\ 4 \\ \underline{16} \end{array} \qquad \begin{array}{r} 3 \\ 3 \\ 9 \\ \underline{21} \end{array} \qquad \text{etc.}$$

3. One as a factor; zero as a factor

Write these as multiplication facts.

1 + 1 + 1 + 1 + 1 = 5

0 + 0 + 0 = 0

1 + 1 + 1 + 1 = 4

0 + 0 + 0 + 0 = 0

1 + 1 + 1 + 1 = □ x 1

1 + 1 + 1 + 1 + 1 + 1 = 6 x □

0 + 0 + 0 + 0 + 0 = □ x 0

0 + 0 + 0 = 3 x □

Fill in the missing numerals. Then write these as additions.

7 x 0 = ?

4 x 0 = ?

3 x 0 = ?

6 x 1 = ?

9 x 1 = ?

5 x 1 = ?

Since 6 x 1 = 6, then 1 x 6 = ?

Since 8 x 1 = 8, then 1 x 8 = ? etc.

Since 6 x 0 = 0, then 0 x 6 = ?

Since 4 x 0 = 0, then 0 x 4 = ?

4. Adding Groups

Use the same patterns and equation forms illustrated below for the more difficult facts as soon as groups of children achieve automatic response to easier facts.

Find numerals to make the following sentences true.

Adding 3's

2 x 3 = n + 3
3 x 3 = n + 3
4 x 3 = n + 3
5 x 3 = n + 3
6 x 3 = n + 3

4 x 3 = n
6 x 3 = n + 6
8 x 3 = n + 6
10 x 3 = n + 6
12 x 3 = n + 6

3 x 3 = n
5 x 3 = n + 6
7 x 3 = n + 6
9 x 3 = n + 6
11 x 3 = n + 6 etc.

Doubling and Adding 3's

3 x 3 = n
6 x 3 = n + 9
7 x 3 = n + 3

3 x 3 = n
6 x 3 = n + 9
9 x 3 = n + 9
12 x 3 = n + 9 etc.

Practice

5 x 4 = 8 + 8 + n
9 x 4 = 16 + n + 4
7 x 4 = n + 12 + 4
8 x 4 = 24 + n

7 x 5 = 15 + □ + 5
5 x 5 = □ + 5
9 x 5 = 15 + □ + 15
6 x 5 = 25 + n

The suggested drill for adding 10 "sixes" may be applied to any of the "tables." Continue to read as "sixes."

10 x 6 = n
20 x 6 = n + 60 = ?
30 x 6 = n + 60 = ?
40 x 6 = n + 60 = ?

13 x 6 = n + 18 = ?
23 x 6 = n + 18 = ?
33 x 6 = n + 18 = ?
43 x 6 = n + 18 = ?

14 x 6 = 60 + n = ?
24 x 6 = 120 + n = ?
34 x 6 = 180 + n = ?
44 x 6 = 240 + n = ?

17 x 6 = 60 + n = ?
27 x 6 = 120 + n = ?
37 x 6 = 180 + n = ?
47 x 6 = 240 + n = ?

Doubling One Factor

2 x 4 = 8
4 x 4 = n
8 x 4 = n
16 x 4 = n

3 x 4 = 12
6 x 4 = n
12 x 4 = n
24 x 4 = n

7 x 4 = 28
14 x 4 = n
28 x 4 = n

9 x 9 = 81
18 x 9 = n
36 x 9 = n

□ x 4 = 8
□ x 4 = 16
□ x 4 = 32

□ x 4 = 12
□ x 4 = 24
□ x 4 = 48

12 x 6 = 36 + n = □
14 x 6 = 42 + n = □
16 x 6 = 48 + n = □
18 x 6 = 54 + n = □

20 x 6 = 60 + n = □
40 x 6 = 120 + n = □
80 x 6 = 240 + n = □

60 x 6 = 180 + n = □
120 x 6 = 360 + n = □

Halving One Factor

4 x 8 is half as much as ? x 8
5 x 7 is half as much as ? x 7
3 x 5 is half as much as ? x 5
2 x 9 is half as much as ? x 9

5 x 3 = (10 x 3) - n = ?
5 x 5 = (10 x 5) - n = ?
5 x 6 = (10 x 6) - n = ?
5 x 4 = (10 x 4) = n = ?

□ x 3 = 30
□ x 3 = 15

□ x 4 = 40
□ x 4 = 20

□ x 5 = 50
□ x 5 = 25

Arriving at a Product in Various Ways

15 x 6 = 30 + 30 + n = ? (5 sixes + 5 sixes + 5 sixes)
15 x 6 = 60 + n = ? (10 sixes + 5 sixes)
15 x 6 = 42 + n + 6 = ? (7 sixes + 7 sixes + 1 six)
15 x 6 = 120 - n = ? (20 sixes - 5 sixes)

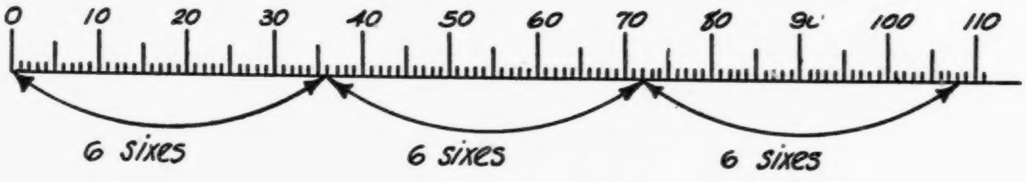
Since 7 x 6 = 42
and 14 x 6 = n
then 15 x 6 = □

Since 10 x 6 = 60
and 5 x 6 = n
then 15 x 6 = □

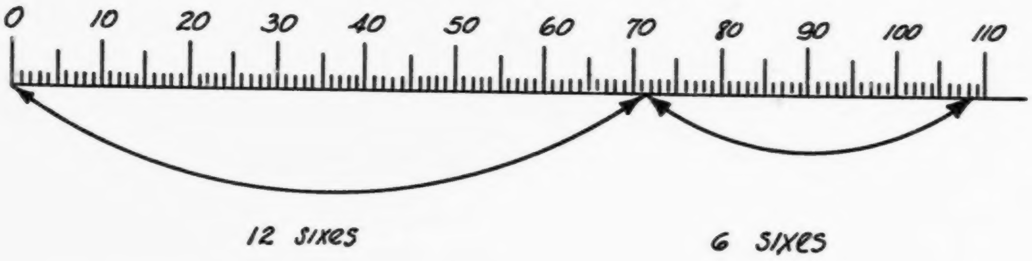
5. Using the Number Line

Provide children with rexographed sheets. (5-10 number lines on each sheet) Children use separate lines to indicate their solutions to problems, as: 13 sixes, 21 sixes, etc.

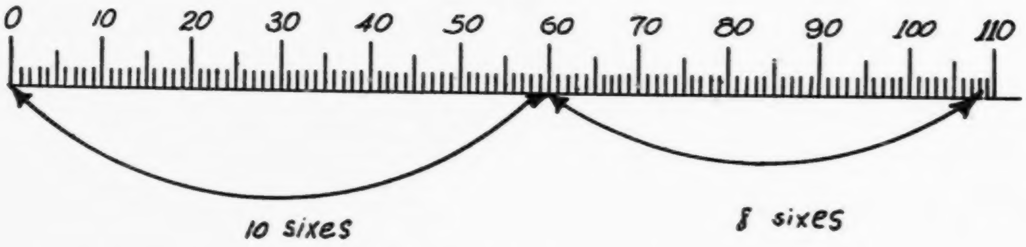
The lines below illustrate how various pupils might arrive at the product for 18 sixes.


0
10
20
30
40
50
60
70
80
90
100
110
6 sixes
6 sixes
6 sixes

6 sixes + 6 sixes + 6 sixes = 18 sixes = 36 + 36 + 36
18 sixes = 108


0
10
20
30
40
50
60
70
80
90
100
110
12 sixes
6 sixes

12 sixes + 6 sixes = 18 sixes = 72 + 36 = 108


0
10
20
30
40
50
60
70
80
90
100
110
10 sixes
8 sixes

10 sixes + 8 sixes = 18 sixes = 60 + 48 = 108

6. Write as many multiplication facts as you can to make each of these open sentences true.

□ x Δ = 12
□ x Δ = 18
□ x Δ = 16
□ x Δ = 24

□ x □ = 9
□ x □ = 49
□ x □ = 25

7. **Fill in the missing numerals to make the open sentence true.**

$\square \times 4 = 6 \times 4 + 4$

$9 \times 3 = 6 \times 3 + \square$

$3 \times \square = 2 \times 9 + 9$

$7 \times 5 = 6 \times 5 + \square$

$3 + 3 + 3 + 3 + 3 = \square \times 3$

$7 \times 4 = 8 \times 4 - \square$

$6 \times 3 = 3 \times \square$

$\square \times 7 = 7 \times 2$

$4 \times 5 = 2 \times \square$

$8 \times \square = 4 \times 4$

$\square \times 4 = 3 \times 8$

OPERATIONS

UNIT 16 - DIVISION OF WHOLE NUMBERS: FACTS AND EXTENSIONS BEYOND THE FACTS

NOTE TO TEACHER

In division we continue to operate on two numbers to obtain another number. Division is the operation of finding one factor when the other factor and the product are known.

$$\text{factor} \times \square = \text{product}$$

For example:

$2 \times 3 = \square$ requires the operation of multiplication for solution.

$\square \times 3 = 6$
or
$2 \times \square = 6$
require the operation of division for solution.

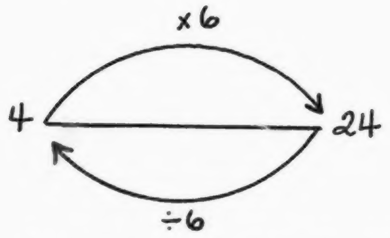
The relationship of the arithmetic operations is shown in the diagram below.

$$\begin{tikzcd}
\text{Addition} \arrow[r] \arrow[d, leftrightarrow] & \text{Multiplication} \arrow[d, leftrightarrow] \\
\text{Subtraction} \arrow[r] & \text{Division}
\end{tikzcd}$$

Properties of Division

Inverse Operations

"Dividing by a number" is the inverse operation of "multiplying by that number."



$$4 \times 6 = 24$$
$$24 \div 6 = 4$$

We develop and drill division facts by relating them to associated multiplication facts.

Commutative Property

Commutativity is not a property of division.

$$8 \div 4 \neq 4 \div 8$$

Distributive Property

In division the dividend can be renamed and the Distributivity of Division with respect to Addition can be applied.

For example:
$$48 \div 8 = (40 \div 8) + (8 \div 8)$$

However the divisor cannot be renamed to apply the Distributive Property of Addition.

For example:
$$48 \div 8 \neq (48 \div 6) + (48 \div 2)$$

Closure

The set of whole numbers is not closed with respect to Division.

For example:

$8 \div 4$ is a whole number [2]

$8 \div 3$ is not a whole number

Division is not Associative

For example:

$(8 \div 4) \div 2 = 1$

but $8 \div (4 \div 2) = 4$

TEACHING SUGGESTIONS

Objectives: To help children understand that a division undoes its related multiplication.
To reinforce and / or develop division facts.

Procedures

Drill division facts by relating them to multiplication facts.

Facts may be recorded with words and numerals, or with numerals only. For example:

? sixes = 24 or $6\overline{)24}$ Children read $6\overline{)24}$ as "how many sixes in 24?" or "24 divided by 6".

To help children find quotients they do not know, encourage them to begin with known quotients.

To find the number of sixes in 42:
Since 6 sixes = 36, and 42 = 36 + 6,
Therefore, there are 7 sixes in 42 (1 more six)

or

Since 5 sixes = 30, and 42 = 30 + 12
Therefore, there are 7 sixes in 42 (2 more sixes)

Provide practice in adding a number of sixes to the dividend.
Adapt these suggestions for sixes to other division facts.

1. Present the following patterns and relationships.
Have children complete the open sentences.

To find	Use Known fact	Then double
$6\overline{)24}$	2 x 6 = 12	☐ x 6 = 24
$6\overline{)48}$	4 x 6 = 24	☐ x 6 = 48
$6\overline{)36}$	3 x 6 = 18	☐ x 6 = 36
$6\overline{)120}$	10 x 6 = 60	☐ x 6 = 120
$6\overline{)1200}$	100 x 6 = 600	☐ x 6 = 1200

2. Using the Distributive Property

Have children find quotients:

$6\,\overline{)72}$ as $\overset{n\,+\,n}{6\overline{)36 + 36}} = \square$

$6\,\overline{)96}$ as $\overset{n\,+\,n}{6\overline{)48 + 48}} = \square$

$6\,\overline{)120}$ as $\overset{n\,+\,n}{6\overline{)60 + 60}} = \square$

Since 6 sixes = 36
Then $\square$ sixes = 42 Why? [36 + 6 = 42]
And $\square$ sixes = 48 Why?

Since 8 sixes = 48
Then $\square$ sixes = 56 Why?

Since 10 sixes = 60
Then $\square$ sixes = 66

Since 100 sixes = 600
Then $\square$ sixes = 606

Adding sixes
2 sixes = 12
$\square$ sixes = 24 Why?
$\square$ sixes = 36 Why?
$\square$ sixes = 48 Why?

Renaming the dividend; Quotient: 10 + a set of sixes

sixes in 120 = $6\overline{)60}$ + $6\overline{)\;n}$ sixes in 120 = 10 + $\square$

sixes in 78 = $6\overline{)60}$ + $6\overline{)\;n}$ sixes in 78 = 10 + $\square$

sixes in 96 = $6\overline{)60}$ + $6\overline{)\;n}$ sixes in 96 = 10 + $\square$

Mixed Patterns
sixes in 78 = $6\overline{)60}$ + $6\overline{)\;n}$ = $\square$
sixes in 78 = 10 + n = $\square$

sixes in 78 = $6\overline{)36}$ + $6\overline{)36}$ + $6\overline{)\;n}$

3. Children find the missing factor in the following sentences.

   $n \times 7 = 35 \quad n = ?$ $\qquad n \times 8 = 72 \quad n = ?$
   $n \times 7 = 49 \quad n = ?$ $\qquad n \times 8 = 80 \quad n = ?$
   $n \times 7 = 63 \quad n = ?$ $\qquad n \times 8 = 88 \quad n = ?$

4. Test children for automatic response to division facts after drill and / or development.

   Use test results to organize children into groups for further drill.

EVALUATION and / or PRACTICE

SUGGESTED EXERCISES

1. Answer the following:

   In the sentence $6 \times n = 42$, 6 is a known factor of the product 42.

   What is "n" called? [unknown factor]
   What operation can be used to find "n" [division]
   What number will replace "n" to make the sentence true? [7]
   Write this sentence in another form. [$6\overline{)42}$]

2. For each multiplication fact below write a related division fact.

   $6 \times 7 = 42$ $\quad \left[42 \div 7 = 6 \text{ or } \begin{array}{r} 6 \\ 7\overline{)42} \end{array} \right]$
   $9 \times 8 = 72$ $\quad \left[72 \div 8 = 9 \text{ or } \begin{array}{r} 9 \\ 8\overline{)72} \end{array} \right]$

3. Rewrite each multiplication sentence as a sentence involving division.

   $6 \times 5 = n$ $\quad [6 \times n = 30 \text{ or } 30 \div n = 6]$
   $4 \times 9 = n$ $\quad [n \times 9 = 36]$

4. Complete the following chart:

Mathematical Sentence	Operation Used	Unknown Factor
n x 9 = 54	÷	?
9 x □ = 63	?	7
?	÷	12

5. Write each of the following in as many ways as you can, using words.

$3\overline{)27}$ 24 ÷ 6

6. Rewrite these division examples using ÷ sign.

$4\overline{)20}$ $9\overline{)18}$ $7\overline{)56}$

At another time, the teacher might ask the children to rewrite the division example from ÷ to) form.

7. Circle the numeral that shows what is to be divided. At another time, the teacher might ask the children to circle the numeral that shows the number of equivalent sets.

$8\overline{)32}$ 64 ÷ 8 = 8 Sixes in 54 are 9

8. Write the related division fact for each of the following: (A)
Write the related multiplication fact for each of the following: (B)

	A
2 x 6 = 12	$\overset{2}{6\overline{)12}}$
4 x 6 = ?	?
8 x 6 = ?	?
9 x 6 = ?	? etc.

B	
$\overset{3}{6\overline{)18}}$	3 x 6 = 18
$\overset{?}{2\overline{)12}}$	?
$\overset{?}{4\overline{)24}}$	?

9. Circle each of the following that can be divided evenly by 4.

10 11 17 14 16 19 15 8 13 6
12 18 20 31 24 29 32 28 30 35 36

10. Complete each of the following:

$4\overline{)\,32\,} = 4\overline{)\,16\,}^{\,n} + 4\overline{)\,16\,}^{\,n} = n$

$4\overline{)\,56\,} = 4\overline{)\,28\,}^{\,n} + 4\overline{)\,28\,}^{\,n} = n$

$4\overline{)\,72\,} = 4\overline{)\,36\,}^{\,n} + 4\overline{)\,36\,}^{\,n} = n$

etc.

$4\overline{)\,28\,} = 4\overline{)\,20\,}^{\,n} + 4\overline{)\,8\,}^{\,n} = n$

$4\overline{)\,56\,} = 4\overline{)\,40\,}^{\,n} + 4\overline{)\,16\,}^{\,n} = n$

$4\overline{)\,60\,} = 4\overline{)\,40\,}^{\,n} + 4\overline{)\,20\,}^{\,n} = n$

etc.

$3\overline{)\,24\,} = 3\overline{)\,12\,} + 3\overline{)\,n\,} = ?$

$4\overline{)\,64\,} = 4\overline{)\,32\,} + 4\overline{)\,n\,} = ?$

$3\overline{)\,54\,} = 3\overline{)\,27\,} + 3\overline{)\,n\,} = ?$

$3\overline{)\,39\,} = 3\overline{)\,30\,} + 3\overline{)\,?\,} = n$

$5\overline{)\,65\,} = 5\overline{)\,50\,} + 5\overline{)\,?\,} = n$

$4\overline{)\,92\,} = 4\overline{)\,80\,} + 4\overline{)\,?\,} = n$

11. Circle the dividend in each of the following:

$25 \div 5 = 5$ 35 divided by 7 equals 5 ? threes = 27

$\overset{7}{6\overline{)\,42\,}}$ There are 8 nines in 72.

Variations: Circle the divisor. Circle the quotient.

12. What is the largest remainder you can have when you divide by 5? By 4? By 9?

13. For each of the following numerals write the closest smaller number which can be divided by 7 without a remainder.

20 36 23 62 50 47 10

14. Apply these patterns to sixes, sevens, eights and nines.

Doubling the Dividend

? sevens = 14
? sevens = 14 + 14
etc.

? sevens = 28
? sevens = 28 + 28
etc.

? sevens = 21
? sevens = 21 + 21
etc.

Halving the Dividend

$7\overline{)28} = 7\overline{)14} + 7\overline{)14} = n$ $7\overline{)14} = 1 + n$

$7\overline{)56} = 7\overline{)28} + 7\overline{)28} = n$ $7\overline{)42} = 3 + n$

$7\overline{)42} = 7\overline{)21} + 7\overline{)21} = n$ $7\overline{)56} = 4 + n$

$7\overline{)98} = 7\overline{)49} + 7\overline{)49} = n$ etc.

* Enrichment Activities (Optional)

Explore some of the properties of division.

Give an example to show that division is distributive with respect to addition when the dividend is renamed.

Use the same example to show that the Property of Distribution of Division over Addition does not apply when the divisor is renamed.

Give an example to show that division is not Commutative; is not Associative; that Closure does not apply to division; that dividing by a number is the inverse operation of multiplying by that number.

SETS; NUMBER; NUMERATION

UNIT 17 - NUMBER: "BETWEENNESS"

NOTE TO TEACHER

To make reasonable estimates children should understand the range or limits within which estimates can be made.

Estimating is a form of "mental computation" to arrive at the approximate value of a mathematical computation.

To help in estimating sums, remainders, products and quotients children should understand the meaning of "betweenness", i.e., when a point on a line lies between two other points on the same line. When the points are associated with numbers on a number line children deal with both whole and fractional numbers.

Estimation involves the application of some of the properties of the fundamental operations and helps children to find answers to exercises by "mental computation".

Experience situations for estimating may be found in newspapers, periodicals, tables of statistics, time lines, graphs, etc.

TEACHING SUGGESTIONS

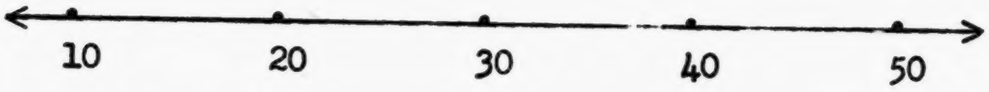
Objective: To help children understand the concept of "betweenness" for numbers and points on a line.

Procedures

1. Draw a line on the chalkboard. Label two points as shown.

Ask children to mark a point between A and B and label it. Discuss the position of this third point.

2. Draw a number line on the chalkboard. Mark off points and label as shown below.



Direct children to:

Name the point between 10 and 30, between 30 and 50.

Mark the point approximately midway between 30 and 40. Name it.

Mark the point approximately midway between 10 and 20. Name it.

Find a point between any points that are now on the line. Name it as accurately as you can.

3. Draw a line segment. Label points as shown.

Have children:

Mark off the mid-point between points 1 and 2 and label it.

Find a point that is midway between 1 and 1 $\frac{1}{2}$;

between 1 $\frac{1}{2}$ and 2. Label these points.

Find points midway between each of the points now named on the line segment.

Discuss the position of the points between points 1 and 2.
Ask children:

Which point is halfway between points $1\frac{1}{2}$ and 2?

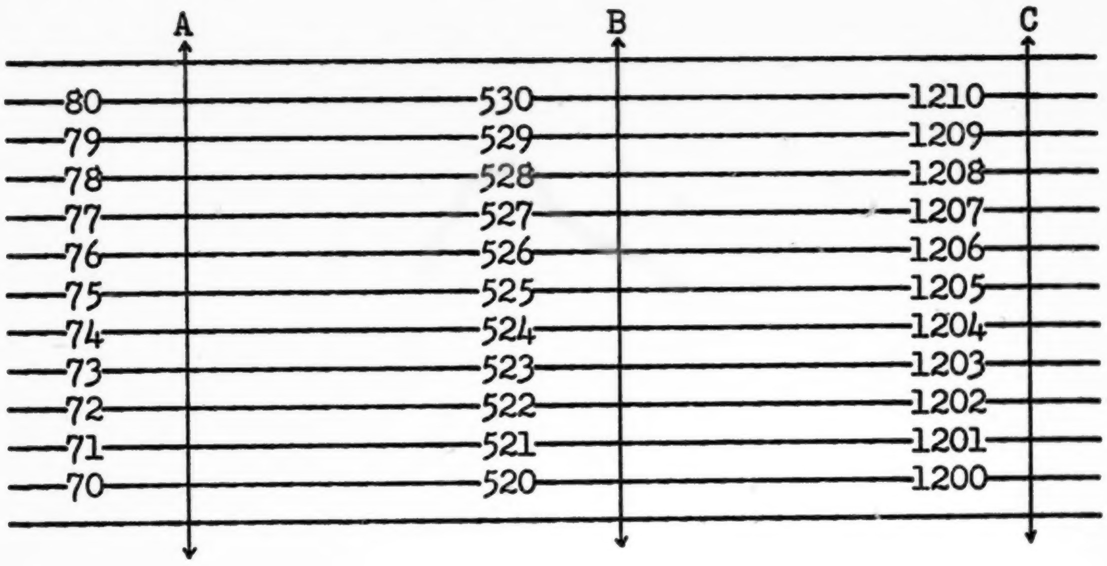
Which of these points is nearest to point 2? [$1\frac{7}{8}$]

Is $\frac{5}{8}$ nearer to point 1 or to point 2?

Is $1\frac{3}{8}$ nearer to point $1\frac{1}{2}$ or to point 2;

to point 1 or to point $1\frac{1}{2}$? ; etc.

4. Draw several vertical lines on the chalkboard. Children use lined paper. Label as illustrated.


A
B
C
80
79
78
77
76
75
74
73
72
71
70
530
529
528
527
526
525
524
523
522
521
520
1210
1209
1208
1207
1206
1205
1204
1203
1202
1201
1200

Ask children:

Which numbers are between 70 and 80?
Which number is halfway between 70 and 80?
Is 78 closer to 75 or 80? [closer to 80]
Is 75 closer to 70 or 80?

Have children note the position of each numeral on the B line in relation to 520 and 530.

Ask children:

Is 523 closer to 520 than to 530?
Is 527 closer to 530 than to 520?
Are 521, 522, 523, 524 closer to 520 than to 530?
Are 526, 527, 528, 529 closer to 530 than to 520?
Is 525 the same distance (half way) from 520 as it is from 530?

Tell children the halfway number (525) may be used as a guide in deciding whether a number is closer to next greater 10 or the next smaller 10.

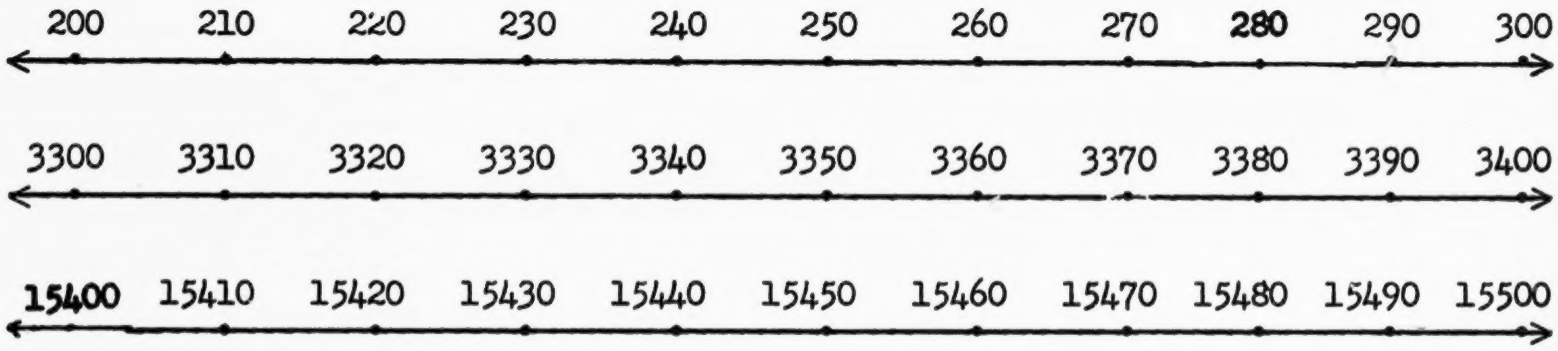
Encourage children to use a variety of terms to express their thinking, e.g.

522 is nearest in value to 520.
522 is closer to 520 than to 530.
522 is between 520 and 525.

Proceed as above for line C.

5. Draw horizontal number lines.
Children should discuss the position of each numeral on the number line in relation to:

the next higher hundred; the next lower hundred.


200 210 220 230 240 250 260 270 280 290 300
3300 3310 3320 3330 3340 3350 3360 3370 3380 3390 3400
15400 15410 15420 15430 15440 15450 15460 15470 15480 15490 15500

EVALUATION and / or PRACTICE

SUGGESTED EXERCISES

1. Which ten comes after 39? Before 41? After 99? After 89? Before 71?
   Which hundred comes after 199? After 799? Before 801?
   Which thousand comes after 2999? After 8999? Before 9001?

2. What is the cost of the following items to the nearest dollar?

   Shoes marked $9.35 Hat marked $8.85 Dress marked $13.75

3. Give the value of each of the following to the nearest ten, the nearest hundred, the nearest thousand.

	Nearest Ten	Nearest Hundred	Nearest Thousand
5169	5170	5200	5000
3228			
12,356			
27,494			

4. 2176 is between □ thousand and Δ thousand.
It is nearest in value to ◇ thousand.

15,429 is between □ thousand and Δ thousand.
It is nearest in value to ◇ thousand.

862 is between □ hundred and Δ hundred.
It is nearest in value to ◇ hundred.

OPERATIONS

UNIT 18 - ADDITION AND SUBTRACTION OF WHOLE NUMBERS: VERTICAL FORMAT

NOTE TO TEACHER

The vertical algorithm is generally used for written computation.

An algorithm is the form used to record the steps of an arithmetic computation. It may take various forms.

At this grade level children should have the ability to rename numbers or to use expanded notation in order to deal with computations requiring exchange.

Estimating sums and remainders precedes written computation. Ways of arriving at reasonable estimates will vary depending upon the ability of the child.

Final solutions should be compared with estimates.

Verification should follow computation.

Teachers and children should be aware of how appropriate properties of operation apply to written computation.

TEACHING SUGGESTIONS

Objective: To develop skill in written computation in addition and subtraction, with emphasis on
Estimating answers
Understanding algorithms

Procedures

1. Test children's ability to rename numbers.

34 ones = ____tens 4 ones
15 tens = ____hundreds ____ tens
19 hundreds = ____ thousand ____ hundreds

We may exchange 93 for ____ tens 13 ones or ____ ones

We may exchange 174 for:

____ tens 4 ones
____ tens 14 ones
one hundred ____ tens 14 ones or for ____ ones

We may exchange 500 for:

____ hundreds 10 tens ____ ones
____ hundreds 9 tens ____ ones
49 tens ____ ones

Fill in the blanks:

8 dollars 6 dimes 2 pennies = ____ pennies
7 dollars 12 dimes 4 pennies = 8 dollars ____ dimes
____ pennies
4 dimes 6 pennies = 3 dimes ____pennies
5 dimes 19 pennies = 6 dimes ____pennies

We may exchange $6.87 for:
6 dollars 7 dimes ____ pennies
5 dollars ____ dimes 7 pennies
5 dollars 17 dimes ____ pennies
____ dimes 7 pennies

2. Reinforce children's ability to rename numbers to help them understand that renaming does not change the value of the original quantity.

Children complete the following:

3 hundreds = 300 or 39 tens = ☐
10 tens = ☐ 17 ones = 17
7 ones = 7 ☐
407

5 dollars = $5.00
17 dimes = ☐
17 pennies = .17
☐

5 dollars = $5.00
☐ dimes = 1.80
7 pennies = .07
$6.87

3. Emphasis should be placed on arriving at estimates of sums and remainders. The following are illustrations of various ways in which children can make estimates from less mature thinking to more mature thinking.

Children should record estimates before they compute the exact sums and remainders.

428
89
276

Add the hundreds. Think of 89 as about 100

Estimate: About 800 (400 + 100 + 300)

or

Add the hundreds, then the tens (600 + 170)

Estimate: About 770

or

Add the hundreds and tens as tens

(43 tens + 9 tens + 28 tens)

Estimate: About 800 (80 tens)

$3.12
.89
1.98

Add the dollars. Think of $.89 as about $1 and $1.98 as about $2.00

Estimate: About $6 ($3 + $1 + $2)

Add the dollars, then the dimes. Think of $.89 as about $.90 and $.98 as about $1.

Estimate: About $6 ($4 + $.10 + $.90 + $1.00)

732
- 367

Subtract the hundreds only. Estimate: about 400
Subtract 400 from 732. Estimate: 332
Subtract 36 (tens) from 73 (tens). Estimate: about 370 (37 tens)

$7.25
- 3.89

Subtract the dollars only. Estimate: about $4
Subtract $4 from $7. Estimate: about $3
Subtract $4 from $7.25. Estimate: about $3.25

Provide practice in estimating only, to develop facility in arriving at reasonable estimates rapidly.

Have children write their own estimate for each of the following:

105 + 89 + 236
239 + 93 + 576
72 + 134 + 681 + 9
315 + 492 + 87 + 73

612 - 298
709 - 188
890 - 287
596 - 129

4. Present problems. Have children estimate, compute, then verify solutions.

Children should check addition by adding in the opposite direction. (Application of the Commutative Property)

Children should check subtraction by adding the number subtracted to the remainder. (Application of the Property of Inverse Operation)

$$\begin{array}{r} 732 \\ \underline{-\,367} \\ 365 \end{array} \qquad \text{Check: } \begin{array}{rl} 365 & \text{(number left after subtraction)} \\ \underline{+367} & \text{(number that had been subtracted)} \\ 732 & \text{(number with which we started)} \end{array}$$

By subtracting the remainder from the minuend.

$$\begin{array}{r} 732 \\ \underline{-\,367} \end{array} \qquad \text{Check: } \begin{array}{r} 732 \\ \underline{-\,365} \end{array}$$

5. Continue to develop skill in addition and subtraction.

Provide practice in addition, sums in tens, then in hundreds.
With 2-place numerals -
With 3-place numerals -
With numerals representing quantities of money - sums through \$9.99

Suggested addition exercises

$$\begin{array}{rrrrrrrr} 23 & 64 & 56 & 45 & 321 & 434 & \$3.86 & \$2.35 \\ 14 & 86 & 72 & 86 & 439 & 244 & 2.98 & .47 \\ 30 & 29 & 49 & 37 & \underline{128} & 191 & \underline{1.53} & 3.21 \\ 31 & \underline{17} & \underline{37} & 59 & & \underline{36} & & \underline{1.28} \\ \underline{58} & & & \underline{73} & & & & \end{array}$$

Introduce addition with two 3-place numerals - sums in the thousands - 1 and 2 exchanges - with two numerals representing quantities of money - sums through \$20 or more.

$$\begin{array}{rrrrr} 834 & 725 & 751 & \$8.89 & \$9.50 \\ \underline{449} & \underline{531} & \underline{876} & \underline{4.05} & \underline{8.75} \end{array}$$

Subtraction

Continue to develop or provide practice in subtracting from minuends through 999; from minuends through $9.99 - 1 and 2 exchanges - 1 zero in the minuend.

80	56	387	464	807	520	148	$3.03	$8.30
- 23	- 37	- 130	- 356	- 664	- 349	- 63	- 1.25	-5.95

Some children may be able to compute from left to right and should be encouraged to do so. Sums and remainders, therefore, are recorded from left to right. This addition may be thought through as:

23	23, 33, 37, 67, 97, 98
14	Children then verify the sum by computing beginning with the ones column.
30	
31	

56	This subtraction may be thought through as:
- 37	56, 26, 19

387	This subtraction may be thought through as:
- 130	387, 287, 257

6. Be sure that children:

Record estimates (or exact sums and remainders) before they compute with pencil and paper.

Compare sums and remainders with estimates.
Check sums and remainders.

Include problem solving in the development of the addition and subtraction processes.

7. Present addition and subtraction problems using a variety of directions and indicating the processes.

Children may find solutions to the following problems

Find the total for 208, 89, 483, 87, 136

Add 8 + 7 + 3 + 4 + 6 + 2 + 9 + 7 = n

Add

352	$56.50
176	9.79
489	12.84

Find the sum: $\begin{array}{r} 5128 \\ \underline{3641} \end{array}$ $6.03 + $1.98 + $.72

Subtract 219 from 458.

Find the difference between 276 and 721.

609 minus 360 = n

From 7425 subtract 5023.

Take $4.38 from $8.00

How much more than 762 is 851?

$$\begin{array}{r} \$47.79 \\ \underline{-\ \ 8.76} \end{array}$$

Underline the example you would use to find the value of "n" in
383 + n = 567

$$\begin{array}{r} 567 \\ \underline{+\ 383} \end{array} \qquad \begin{array}{r} 567 \\ \underline{\times\ 383} \end{array} \qquad \begin{array}{r} 567 \\ \underline{-\ 383} \end{array}$$

8. Include "verbal" problem solving in addition to the items suggested above.

9. Include more exercises from textbook and / or workbook.

10. Terminology

    The teacher and children should use mathematical terms pertaining to addition and subtraction.

    The following list suggests terms to be used rather than such expressions as "answer", "borrow", "carry", "left over", etc.

    add
    addition
    addend
    sum
    total
    plus

    subtract
    subtraction
    difference
    remainder
    minus

    place value
    estimate
    exchange
    regroup

OPERATIONS

UNIT 19 - ADDITION OF FRACTIONAL NUMBERS: COMMON FRACTIONAL FORM; HORIZONTAL FORMAT

NOTE TO TEACHER

Refer to Unit 15 for the development of concepts and comparisons for halves, fourths, eighths and thirds.

Before children can add fractional numbers they must know:

> **Fractions with like denominators can be added by adding the numerators.**
>
> **Fractions with unlike denominators can be added by changing them to equivalent fractions with like denominators.**

Addition of fractional numbers presents no problem when children understand that any fractional number may be renamed in many ways.

Encourage children to express sums of fractions with the smallest denominators. For example, $\frac{4}{8} = \frac{1}{2}$

Properties of Operations With Fractions

Properties of addition apply to fractional numbers as well as to whole numbers.

Addition of fractional numbers is Commutative For example:

$$\frac{2}{3} + \frac{1}{3} = \frac{3}{3}$$

$$\frac{1}{3} + \frac{2}{3} = \frac{3}{3}$$

Addition of fractional numbers is Associative.
For example:

$$\frac{1}{4} + \left(\frac{3}{4} + \frac{1}{2}\right) = \left(\frac{1}{4} + \frac{3}{4}\right) + \frac{1}{2}$$

$$\frac{1}{4} + \frac{5}{4} = \frac{4}{4} + \frac{2}{4}$$

$$\frac{6}{4} = \frac{6}{4}$$

Zero is the identity element for addition of fractional numbers. For example:

$$0 + \frac{3}{4} = \frac{3}{4}$$

$$\frac{3}{4} + 0 = \frac{3}{4}$$

Objective: To develop skill in adding related fractions; halves, fourths, eighths.

TEACHING SUGGESTIONS

1. Reinforce counting forward with fractional numbers. Use a number line and a ruler graduated in eighths. Note: Skill in counting forward increases children's ability to add "mentally".

   Ask children to complete the chart below.

   Use number line. Fill in missing numerals.

Begin with	Size of Interval	Number of Moves	End with
0	$2\frac{1}{2}$	5	☐
2	$\frac{1}{2}$	9	☐
$3\frac{1}{2}$	$2\frac{1}{2}$	4	☐
$13\frac{1}{2}$	$1\frac{1}{4}$	☐	$19\frac{3}{4}$
☐	$\frac{1}{2}$	30	117

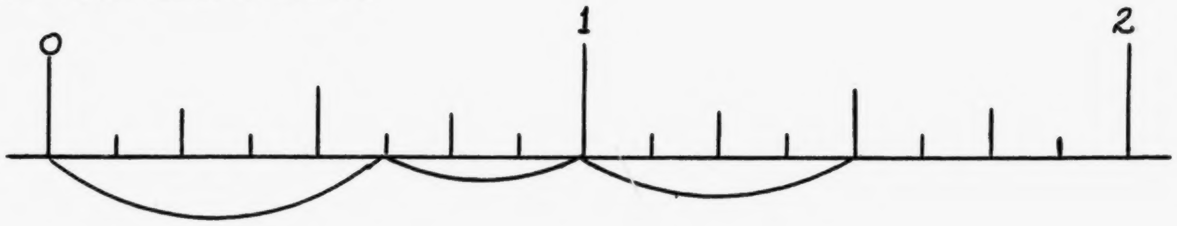
2. When adding fractions mentally children may think forward from the entire first addend and add that part of the second addend needed to reach the next whole number. (Applying The Associative Property).

Problem: We need 2 pieces of shelving paper, one $\frac{5}{8}$ yd. and the other $\frac{7}{8}$ yd. How much shelving paper shall we buy?

Children think forward to complete the whole

$$\frac{5}{8}+\frac{7}{8}=\frac{5}{8}+\frac{3}{8}+\frac{4}{8}=1+\frac{4}{8}=1\frac{1}{2}$$

Check with number line.



3. Reinforce addition of fractions with like denominators. Children may use number lines and rulers where necessary.

Fraction + Fraction

$\frac{5}{8}+\frac{3}{8}=n$

$\frac{6}{8}+\frac{4}{8}=1+n$

$\frac{3}{4}+\frac{3}{4}=1+n$

$\frac{3}{2}+\frac{2}{2}=1+n$

Mixed Form + Whole Number

$1\frac{1}{4}+1=2+n$

$3\frac{1}{8}+1=4+n$

$4\frac{1}{2}+3=7+n$

$2\frac{1}{8}+2=4+n$

Mixed Form + Fraction

$1\frac{1}{4}+\frac{1}{4}=n$

$3\frac{1}{8}+\frac{3}{8}=n$

$4\frac{1}{2}+\frac{1}{2}=n$

$6\frac{1}{4}+\frac{2}{4}=n$

Mixed Form + Mixed Form

$1\frac{1}{4}+1\frac{1}{4}=n$

$4\frac{1}{2}+5\frac{1}{2}=n$

$3\frac{1}{8}+7\frac{3}{8}=n$

$6\frac{1}{8}+2\frac{5}{8}=n$

$3\frac{1}{8} + 2\frac{3}{8} = 5\frac{1}{8} + n = ?$

$3\frac{7}{8} + 2\frac{3}{8} = 5\frac{7}{8} + n = ?$

4. Extend to the addition of related fractions with unlike denominators.

Halves and Fourths	Halves and Eighths	Fourths and Eighths
$\frac{1}{2} + \frac{1}{4} = n$	$\frac{1}{2} + \frac{3}{8} = n$	$\frac{1}{4} + \frac{1}{8} = n$
$\frac{1}{2} + \frac{3}{4} = n$	$\frac{3}{2} + \frac{5}{8} = n$	$\frac{3}{4} + \frac{5}{8} = n$
$\frac{3}{4} + \frac{3}{2} = n$	$\frac{7}{8} + \frac{1}{8} = n$	$\frac{3}{8} + \frac{1}{4} = n$
$\frac{3}{4} + \frac{1}{2} = n$	$\frac{3}{8} + \frac{5}{2} = n$	$\frac{5}{8} + \frac{3}{4} = 1 + n$

5. Present the following exercises for practice

a. $\frac{3}{4} + \frac{1}{2} = 1 + n$

$\frac{6}{8} + \frac{1}{4} = n$

$\frac{6}{8} + \frac{3}{4} = 1 + \frac{n}{8}$

$\frac{1}{2} + \frac{5}{8} = 1 + n$

$1\frac{3}{4} + \frac{1}{2} = n$

$3\frac{3}{8} + \frac{1}{4} = n$

$2\frac{6}{8} + \frac{3}{4} = n$

$3\frac{1}{2} + \frac{3}{8} = n$

$21\frac{3}{4} + 32\frac{1}{2} = n$

$13\frac{3}{8} + 23\frac{1}{4} = n$

$52\frac{6}{8} \div 41\frac{3}{4} = n$

$2\frac{7}{8} + 1 = n$

$2\frac{7}{8} + 1\frac{3}{4} = n$

$2\frac{7}{8} + 1\frac{3}{4} = 3\frac{7}{8} \div n = ?$

b. Add-Use number line where necessary.

$2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} = n$

$1\frac{1}{4} + 2\frac{1}{2} + 3\frac{3}{4} = n$

$32\frac{7}{8} + \frac{3}{4} + 3\frac{1}{2} = n$

$82\frac{1}{8} + 1\frac{3}{4} + 2\frac{1}{2} + 3\frac{3}{8} = n$

c. Supply the missing numerals:

$16 + \square + \square = 23$

$11 = \square + \square$

$\frac{\square}{4} + \frac{\square}{4} = 10$

$\frac{\square}{8} + \frac{\Delta}{8} = 10$

6. Children complete the following. They **should explain the property involved.**

$\frac{3}{4} + \frac{3}{8} = \frac{3}{4} + \left(\frac{\square}{8} + \frac{1}{8}\right)$

$\frac{3}{4} + \frac{1}{2} = \frac{1}{2} + \frac{3}{\square}$

$5\frac{3}{4} + \frac{5}{8} = 5 + \left(\frac{\square}{8} + \frac{5}{8}\right)$

$0 + \frac{2}{3} = \frac{2}{3} + \square = n$

$\frac{3}{8} + \square = \frac{3}{8}$

$\frac{7}{8} + \frac{1}{4} = \frac{1}{4} + \square$

OPERATIONS

UNIT 20 - SET OF FRACTIONAL NUMBERS: SUBTRACTION; COMMON FRACTIONAL FORM; HORIZONTAL FORMAT

NOTE TO TEACHER

When fractional numbers are subtracted the denominators, just as in addition, must be the same.

Since we are dealing with a set of positive numbers, the first fraction must be greater than the second, when subtracting.

For example: Since $\frac{3}{4} > \frac{1}{4}$ we may subtract $\frac{1}{4}$ from $\frac{3}{4}$ and have a result which is a positive number.

Subtraction of fractional numbers may also be thought of as finding a missing addend.

For example:

$$\text{Addition:} \quad \text{Addend} + \text{addend} = \square$$

$$\frac{3}{4} + \frac{1}{4} = \square$$

$$\text{Subtraction} \begin{cases} \frac{3}{4} + \square = \frac{4}{4} \\ \square + \frac{1}{4} = \frac{4}{4} \end{cases} \qquad \begin{array}{rl} \frac{4}{4} & \text{(Sum)} \\ -\frac{1}{4} & \text{(Addend)} \\ \hline \frac{3}{4} & \text{(Addend)} \end{array}$$

$$\text{Subtraction: Addend} + \square = \text{Sum}$$

Subtracting a number is the inverse operation of adding that number. Therefore the solution to a subtraction problem involving fractional numbers may be verified by applying the inverse operation, addition.

Equivalent Fractions

Equivalent fractions are fractions that name the same fractional number. For example:

$$\frac{1}{2} = \frac{2}{4}, \qquad \frac{1}{2} = \frac{3}{6}$$

The renaming of fractions to their equivalents has been taught to children in previous grades through the use of experience materials and representative materials. In a later Unit of Grade 5 the Fundamental Property of Fractions will be developed.

The Fundamental Property of Fractions: If the numerator or denominator of a fraction is multiplied or divided by the same non-zero number, the resulting fraction is equivalent to the original fraction.

$$\frac{a}{b} = \frac{a \times c}{b \times c}, \qquad c \neq 0$$

Ability to find equivalents for fractional numbers (renaming fractions) is essential.

For example:

To subtract $8\frac{3}{4} - 3\frac{7}{8}$ it is essential that children know that $\frac{3}{4}$ must first be changed to the equivalent fraction $\frac{6}{8}$.

At this time children can solve $8\frac{6}{8} - 3\frac{7}{8} = n$ by applying the Associative Property for Addition.

$3\frac{7}{8} = 3 + \frac{7}{8}$ which will be thought of as:

$3 + \left(\frac{6}{8} + \frac{1}{8}\right)$.

Then to solve $8\frac{6}{8} - 3\frac{7}{8} = n$ children can think:

$5\frac{6}{8} - \frac{7}{8} = n$ $\left(\text{after subtracting } 3 \text{ from } 8\frac{6}{8}\right)$

$5\frac{6}{8} - \frac{6}{8} + \frac{1}{8} = n$ $\left(\text{after renaming } \frac{7}{8} \text{ as } \frac{6}{8} + \frac{1}{8}\right)$

$5 - \frac{1}{8} = n$ $\left(\text{after subtracting } \frac{6}{8} \text{ from } 5\frac{6}{8}\right)$

$4\frac{8}{8} - \frac{1}{8} = n$ $\left(\text{after renaming } 5 \text{ as } 4\frac{8}{8}\right)$

$4\frac{8}{8} - \frac{1}{8} = 4\frac{7}{8}$ $\left(\text{solution, } 4\frac{7}{8}\right)$

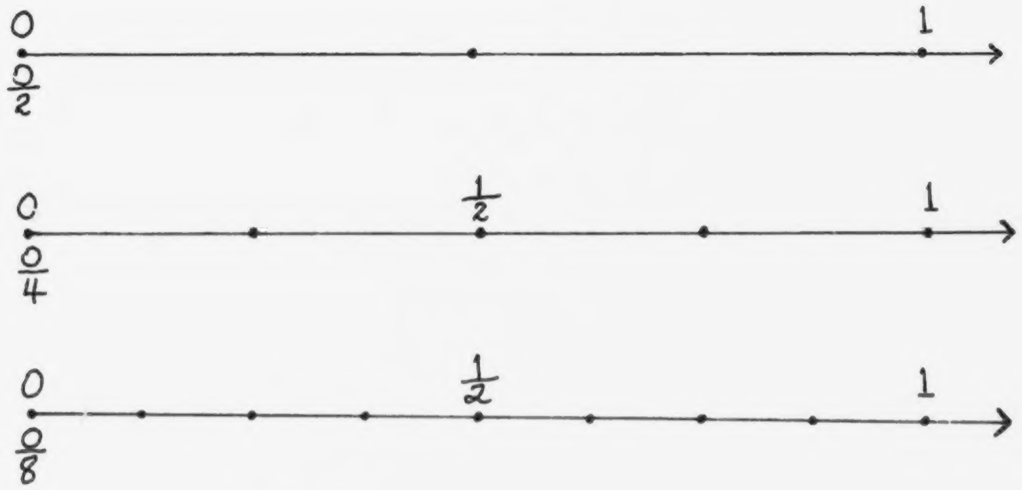
Objectives: To reinforce subtraction of fractional numbers involving halves, fourths, eighths

Meaning of equivalent fractions
Renaming numbers - Equivalents
Inequalities
Counting
Finding the missing addend

TEACHING SUGGESTIONS

Equivalents

1. Use number rays to reinforce equivalents.



Have children:

Assign numbers to the points on the rays above
Assign another number name to the point named by "1" on each of the rays to correspond with the way the ray has been divided. Record the new name below the point. $\left[\frac{2}{2}, \frac{4}{4}, \frac{8}{8}\right]$

Assign another name to the point named by " $\frac{1}{2}$ " on each of the rays.

Record the new name below the point.

Children record the set of equivalent fractions for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$

$$\left[\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \text{etc.}\right]$$

2. Discuss the notations $\frac{0}{2}$, $\frac{0}{4}$, $\frac{0}{8}$. To what whole number is each equivalent? [Zero]

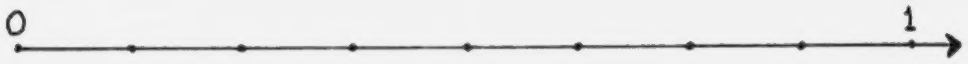
3. Children:
   Record another name for $\frac{8}{8}$, $\frac{4}{4}$, $\frac{2}{2}$

   Explain $\frac{1}{1} = 1$

4. Extend number rays to include more than one unit and follow same procedure.

Inequalities

Present a number ray. Divide one unit into halves, then fourths, then eighths.



Have children find the solution sets for the following:

The points show that the unit on the ray is divided into _____ parts.

a. $\square > \frac{1}{2}$　　c. $\frac{\square}{8} > \frac{3}{4}$

b. $\frac{3}{8} < \square$　　d. $1\frac{1}{2} \; \square \; \frac{8}{8}$

a. $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, 1

b. $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, 1

c. $\frac{7}{8}$, $\frac{8}{8}$,

d. >

Counting

1. Reinforce counting backward by subtracting various fractional numbers. Use number lines.

    Direct children to count backward from:

    $4\frac{1}{2}$, by halves to 2

    $5\frac{1}{8}$, by eighths to 2

    $2\frac{1}{4}$, by eighths to 1

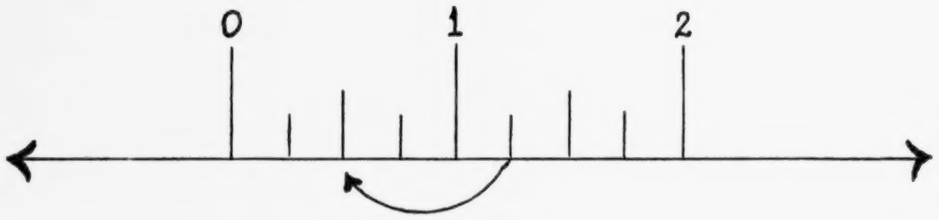
    $5\frac{3}{4}$, by $\frac{3}{4}$ to $4\frac{1}{2}$

    Children explain in their own words how they know they have counted by halves, eighths, etc.

2. Present a subtraction problem involving fractional numbers.

    Children should solve it by counting backward on a number line.

Problem: The recipe for a pie calls for $1\frac{1}{4}$ cups of flour. It tells us to use $\frac{3}{4}$ cup for the crust. How much is left for filling?



$1\frac{1}{4} - \frac{3}{4} = n$

3. Ask children to evaluate each of the following mentally: Check some using the number line.

$2\frac{1}{4} - 1 = n$
$4\frac{1}{2} - 2 = n$
$5\frac{1}{8} - 2 = n$
$5\frac{3}{4} - 3 = n$

$1\frac{7}{8} - \frac{3}{8} = n$
$9\frac{5}{8} - \frac{2}{8} = n$
$5\frac{3}{4} - \frac{1}{4} = n$
$6\frac{3}{4} - \frac{3}{4} = n$

$5\frac{3}{4} - 1\frac{3}{4} = n$
$7\frac{5}{8} - 3\frac{5}{8} = n$
$4\frac{1}{2} - 1\frac{1}{2} = n$
$6\frac{7}{8} - 4\frac{7}{8} = n$

$5\frac{3}{4} - 1\frac{1}{4} = n$
$6\frac{7}{8} - 5\frac{5}{8} = n$
$9\frac{5}{8} - 3\frac{2}{8} = n$
$4\frac{2}{4} - 1\frac{1}{4} = n$

$2\frac{1}{8} - 1 = n$
$2\frac{1}{8} - \frac{1}{8} = n$
$2\frac{1}{8} - 1\frac{1}{8} = n$
$2\frac{1}{8} - \frac{2}{8} = n$

$3\frac{1}{3} - 1\frac{2}{3} = n$
$4\frac{1}{4} - 2\frac{3}{4} = n$
$8\frac{3}{8} - 3\frac{7}{8} = n$
$4\frac{1}{8} - 2\frac{5}{8} = n$

Applying the Associative Property for Addition

1. Reinforce renaming fractional numbers and mixed forms to prepare for

applying the Associative Property for Addition. For example:

Rename $9\frac{6}{8}$ means $9 + \square$

$$9\frac{6}{8} = 8 + \frac{\square}{8}$$

Rename $17\frac{3}{4}$ means $17 + \square$

$$17\frac{3}{4} = 16 + \frac{\square}{4}$$

2. Solve for n in the following exercises. Use the Associative Property of Addition where necessary.

$$1\frac{5}{8} - \frac{1}{4} = n$$

$$\frac{7}{8} - \frac{1}{2} = n$$

$$\frac{5}{8} - \frac{1}{2} = n$$

$$38\frac{7}{8} - 23\frac{3}{4} = n$$

$$17\frac{5}{8} - 12\frac{3}{4} = n$$

$$8\frac{1}{2} - 4\frac{3}{4} = n$$

$$4\frac{3}{8} - 2\frac{1}{2} = n$$

Finding the Missing Addend

1. Find the missing addend:

$37 + \square = 50$ $\quad$ $129 = \square + 109$ $\quad$ $3415 + n = 3525$

Ask the following questions:

What is the sum in each of the problems above?

What are the other numbers in the equation called?

[addend]

What operation can you use to find the missing addend in each case?

[subtraction]

2. Present an exercise such as: $8\frac{3}{4} + 5\frac{1}{8} = \square$

Ask children to:

Find the sum
Name the addends

3. Present the exercise in a word problem: $9\frac{1}{4} + \square = 16\frac{5}{8}$

Children:

Make the open sentence true.

Tell how they found the missing addend.

Record the equation to show subtraction. $\left[16\frac{5}{8} - 7\frac{3}{8} = 9\frac{1}{4}\right]$

Name the addends in the subtraction above. $\left[7\frac{3}{8},\ 9\frac{1}{4}\right]$

Name the sum. $\left[16\frac{5}{8}\right]$

Write each of the subtraction exercises below as an addition exercise:

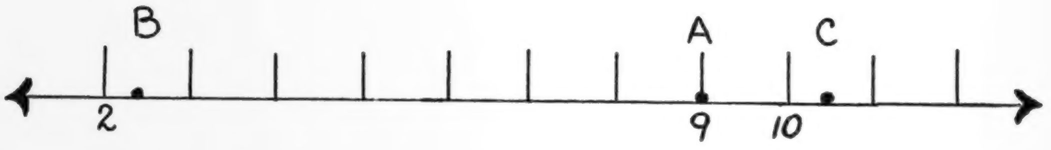
$4\frac{5}{8} - 1\frac{3}{8} = 3\frac{1}{4}$ $\left[4\frac{5}{8} = 3\frac{1}{4} + 1\frac{3}{8}\right]$

$1\frac{7}{8} - \frac{3}{2} = \frac{3}{8}$ $\left[\frac{3}{8} + \frac{3}{2} = 1\frac{7}{8}\right]$

4. Discuss the following. Children find solutions.

We have three points, A, B, C, on a line.
Which two of the three points are farthest apart?
How far apart?

A. 9 B. $2\frac{1}{2}$ C. $10\frac{1}{2}$



Assign other numbers to points A, B, C. Proceed as above.

A.	B.	C.
$10\frac{1}{2}$	$6\frac{1}{2}$	$8\frac{1}{2}$
$\frac{8}{4}$	$\frac{9}{4}$	$\frac{10}{4}$
$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$
5	$5\frac{3}{8}$	$5\frac{3}{4}$

5. **Additional practice exercises**

$5\frac{1}{4} + \square = 5\frac{3}{8}$

$5\frac{1}{4} + \square = 14$

$5\frac{1}{4} + \square = 11\frac{1}{2}$

$5\frac{1}{4} + 1\frac{\square}{\triangle} = 7\frac{1}{8}$

$7\frac{5}{8} - 2\frac{3}{4} = 5\frac{5}{8} - n$

$8\frac{1}{2} - 4\frac{3}{4} = 4\frac{1}{2} - n$

$5\frac{3}{4} - 3\frac{7}{8} = 2 \quad - n$

GEOMETRY AND MEASUREMENT

UNIT 21 - MEASUREMENT: CAPACITY; CONSERVATION; EQUIVALENTS

NOTE TO TEACHER

In Grade 5 continue to emphasize:
A Principle of Conservation in Science
That all Measurements are approximate.

In science a Principle of Conservation refers to the fact that certain properties of things do not change even when conditions about them change. For example, one quart is always one quart whether the quart is distributed in a quart jar or in a gallon jar; one quart is one quart whether it is distributed in a quart milk bottle or in a mayonnaise jar of a one quart capacity.

Understanding conservation may help children to understand equivalents among measures.

Children should also understand that two different substances may have the same weight even though they have different volumes. A pound of feathers occupies more space than a pound of iron **but both weigh one pound.**

Objectives: To reinforce concepts of conservation.
To organize relationships among measurements.
To help children understand capacity.

TEACHING SUGGESTIONS

1. Use classroom experiences such as:

Preparing for Health Day
- Discuss and plan healthful menus
- Discuss amounts of basic foods needed in daily diet

Planning for a Halloween Party

Measure ingredients when preparing food
Estimate quantity of juice or cider needed - using a cup, glass, pint or quart container
(32 cups, or 8 quarts, or 2 gallons, or 4 half gallons)
Double or triple the amount given in recipes

2. Reinforce equivalents among the number of cups, pints, quarts and gallons. (e.g. 2 pints = 1 quart, etc.)

3. Introduce the fluid ounce

   Materials needed:

   Standard measuring spoons
   1 ounce glass
   Perfume bottle
   Water

   Children compare capacities in fluid ounces using measuring spoon; in fluid ounces with standard cup.

   Discuss uses of fluid ounce.
   Discuss the principle of conservation

   Note the need for a unit of measure between the spoon and the cup.

4. Provide practice in changing from larger units to smaller units; smaller units to larger units.

   For example:

   $2\frac{1}{2}$ quarts = 2 quarts and 1 pint, or 5 pints, or 10 cups
   12 pints = 6 quarts, or 1 gallon and 2 quarts, or $1\frac{1}{2}$ gallons, or 1 gallon and 4 pints

   Since 8 pints = 1 gallon, then 1 pint = ? gallon

   Children should note that when changing from smaller units to larger units the number of units is fewer; when changing from larger units to smaller units the number of units is greater. Try to have students discover this and state generalization.

5. Have children organize relationships among measurements including fractional units.

2 cups = 1 pint	1 cup = $\frac{1}{2}$ pint
2 pints = 1 quart	1 pint = $\frac{1}{2}$ quart
4 quarts = 1 gallon	1 quart = $\frac{1}{4}$ gallon
8 pints = 1 gallon	1 pint = $\frac{1}{8}$ gallon

6. Introduce concepts of dry measure.

   Discuss:

   Need for containers for dry measure.
   Products sold in units of dry measure.
   [berries, apples, vegetables, etc.]

   Similarity of names for some units of dry measure and some units of liquid measure.
   [pints, quarts]

7. Have children explore different units used on products found in supermarkets; in books; etc.

8. Introduce terms: peck, bushel.

   Have children bring in containers, such as bushel, etc.

9. Use transparent containers of the same size.
   Fill one container with beads that are 1" in diameter, the other with beads that are $\frac{1}{2}$" in diameter.

   Children note that the number of objects and the weight of objects may vary although the capacity is the same.

10. Children organize "table" of dry measure:

    2 pints = 1 quart
    8 quarts = 1 peck (pk.)
    4 pecks = 1 bushel (bu.)

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Choose the correct unit of measure.

    An oil dealer delivers oil in

    pints, quarts, gallons

2. Name two liquid products measured in quarts;
    two dry products measured in quarts.

3. Are there more green peas or more potatoes in a bushel? Why?

4. Solve the following equations:

    2 quarts = □ gallon

    2 pints = □ gallon

    2 gallons = □ quart

5. Additional practice exercises may be found in textbooks.

OPERATIONS

UNIT 22 - SET OF WHOLE NUMBERS: MULTIPLICATION; HORIZONTAL FORMAT

NOTE TO TEACHER

To multiply "mentally", children must have the following background:

Ability to rename numbers using expanded notation, for example,

79 = 70 + 9; 246 = 200 + 40 + 6

Ability to add "mentally" (left to right computation) numbers through 999, for example, 320 + 81 = ☐

Ability to multiply "mentally" whole decade and whole hundred numbers with one factor through 9. For example,

9 x 30 = 270; 9 x 300 = 2700.

Automatic response to multiplication facts.

Steps in developing understanding and skill in multiplying "mentally" are:

Teacher should present a problem situation familiar to the children.

Children then should determine that the problem calls for multiplication.

Teacher records the problem as a mathematical sentence.

Children then compute "mentally" from left to right.

They should arrive at products in a variety of ways.

They record only the products.
Teacher records their thinking on the board. For example:

For $4 \times 68 = \square$

Child May Think	Teacher Should Write on Chalkboard
4 sixties are 240	$(4 \times 60) + (4 \times 8) = n$
4 eights are 32	$240 + 32 = 272$
240 and 32 are 272	

(Regrouping one factor and applying the Distributive Property)

or

Child May Think	Teacher Should Write on Chalkboard
2 sixty-eights are 136	$(2 \times 68) + (2 \times 68) = n$
2 more sixty-eights are 136	$136 + 136 = 272$
136 and 136 equal 272	

(Regrouping the other factor)

For $3 \times 258 = \square$

Child May Think	Teacher Should Write on Chalkboard
3 two hundreds are 600	$3 \times 258 = n$
3 fifties are 150	$600 + 150 + 24 = 750 + 24$
600 and 150 are 750 and	$= 774$
24 more are 774	

or

2 two hundred fifties are 500	$3 \times 258 = 500 + 250 + 24$
and another 250 would be 750	$= 774$
Add 24 more and that would be 774	

For $2 \times 637 = \square$

Child May Think	Teacher Should Write on Chalkboard
2 six hundreds are	$2 \times 637 = 1200 + 74 = 1274$
12 hundred and 74 more	
(2 thirty-sevens)	
equals 1274	

Objective: To develop skill in mental multiplication by a one-digit factor. To develop understanding why the horizontal algorithm works as an application of the Distributive property.

TEACHING SUGGESTIONS

The teacher should present exercises or problems whose numbers are within the capabilities of the children.

One Factor a Two Digit Numeral; 2 and 3 Digit Numerals in the Product;

For example, 6 x 27 = □ 4 x 324 = □

Before multiplying whole decade numbers, practice should be given in adding, in sequence and out of sequence with whole decade numbers.

For example:

1. Additions

Add 40 to successive multiples of 40; 40 + 40 = □ 80 + 80 = □, etc.
Add 80 to successive multiples of 40; 40 + 80 = □ 120 + 80 = □, etc.
Add 80 to successive multiples of 80; 80 + 80 = □ 160 + 80 = □, etc.

Double 40 and its multiples in and out of sequence.

40 + 40 = □ 80 + 80 = □ 40 + 40 = □ 160 + 40 = □, etc.

2. Multiplications

Multiplying in Sequence 2 x 40 = □ 3 x 40 = □ 4 x 40 = □ etc.

Doubling One Factor 2 x 40 = □ 4 x 40 = □ 8 x 40 = □ etc.
3 x 40 = □ 6 x 40 = □

Doubling and Adding 40's 3 x 40 = □ 6 x 40 = □ 7 x 40 = □
3 x 40 = □ 6 x 40 = □ 8 x 40 = □

From time to time record the children's thinking.

For 8 x 40 record:

(4 x 40) + (4 x 40) = 160 + 160 = 320
or
(5 x 40) + (3 x 40) = 200 + 120 = 320
or
(3 x 40) + (3 x 40) + (2 x 40) = 120 + 120 + 80 = 320

3. Extend to multiplications in which one factor is a 2 digit number.

Present one set of exercises at a time

Illustrative series:

Since $3 \times 60 = 180$
then $3 \times 67 = n$

Since $4 \times 90 = 360$
then $4 \times 96 = n$

$2 \times 63 = \square + 6 = n$
$4 \times 63 = \square + 12 = n$
$8 \times 63 = \square + 24 = n$

$3 \times 63 = \square + 9 = ?$
$6 \times 63 = \square + 18 = ?$

Since $2 \times 80 = 160$
then $4 \times 80 = n$
$4 \times 86 = n$

Since $3 \times 60 = 180$
then $6 \times 60 = n$
$6 \times 62 = n$

$2 \times 24 = \square$
$4 \times 24 = \square + \square = ?$

$2 \times 37 = \square$
$4 \times 37 = \square + \square = ?$ etc.

$3 \times 21 = \square$
$6 \times 21 = \square + \square = ?$

$3 \times 54 = \square$
$6 \times 54 = \square + \square = ?$

A One Digit Factor x a Three Digit Factor

Suggested Types of Exercises

Children add whole hundreds before multiplying with whole hundreds.

Additions

Add 400 to successive multiples of 400;
$400 + 400 = \square$ $\quad$ $800 + 400 = \square$ etc.

Add 400 to multiples of 400 out of sequence:
$1200 + 400 = \square$ $\quad$ $2800 + 400 = \square$ etc.

Add 800 to multiples of 400 out of sequence:
$1200 + 800 = \square$ $\quad$ $2800 + 800 = \square$ etc.

Double 400 and its multiples in and out of sequence:
$400 + 400 = \square$ $\quad$ $800 + 800 = \square$ etc.

2. Multiplications

Multiplying in Sequence: $2 \times 400 = \square$ $\quad$ $3 \times 400 = \square$ $\quad$ $4 \times 400 = \square$

Doubling the Multiplier: 2 x 400 = □ 4 x 400 = □ 8 x 400 = □
3 x 400 = □ 6 x 400 = □

Doubling and Adding Sets of 400 3 x 400 = □ 6 x 400 = □ 7 x 400 = □
3 x 400 = □ 6 x 400 = □ 8 x 400 = □

From time to time record the children's thinking.

Use Equations - Apply Distributive Property with respect to Addition.

Adding in Sequence

2 x 400 = □ + 400
3 x 400 = □ + 400
4 x 400 = □ + 400

Doubling

2 x 400 = □
4 x 400 = □ + □
8 x 400 = □ + □

3 x 400 = □
6 x 400 = □ + □

Doubling and Adding Groups

2 x 400 = □
4 x 400 = □ + □ = n
5 x 400 = □ + 400 = n

3 x 400 = 1200
6 x 400 = □ + □ = n
7 x 400 = □ + 400= n

4 x 400 = 1600
8 x 400 = □ + □ = n
9 x 400 = □ + 400 = n

3 x 400 = □
6 x 400 = □ + □ = n
9 x 400 = □ + 1200 = n

3. Extend to multiplication with more difficult numbers.

Adding Groups

4 x 200 = 800
4 x 223 = 800 + n = n
4 x 254 = 800 + n = n
4 x 285 = 800 + n = n

2 x 839 = n + 78 = n
5 x 212 = n + 60 = n
4 x 342 = n + 168 = n
8 x 105 = n + 40 = n
3 x 521 = n + 63 = n

Doubling One Factor

2 x 316 = □ +32 = n
4 x 316 = □ +64 = n
8 x 316 = □ + 128 = n

3 x 316 = □ + 48 = n
6 x 316 = □ + 96 = n

Varied Patterns Using Same Factor

2 x 749 = 1400 + n = □
2 x 749 = n + 18 = □
2 x 749 = n + 98 = □
2 x 749 = 1500 - n = □

$4 \times 243 = 800 + n + 12 = \square$
$4 \times 243 = 486 + n = \square$
$4 \times 243 = n + 12 = \square$

Present one set of equations at a time. Illustrative series:

Since $2 \times 400 = 800$
Then $2 \times 448 = n$

Since $2 \times 400 = 800$
Then $2 \times 484 = n$

Since $2 \times 400 = 800$
Then $4 \times 400 = n$
$4 \times 405 = n$

Since $3 \times 400 = 1200$
Then $6 \times 400 = n$
$6 \times 411 = n$

Since $4 \times 400 = 1600$
Then $8 \times 400 = n$
$8 \times 402 = n$

Since $3 \times 400 = 1200$
Then $3 \times 425 = n$

Since $4 \times 400 = 1600$
Then $4 \times 408 = n$

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

Solve the following equations:

1. **Adding in Sequence**

$2 \times 30 = \square + 30$
$3 \times 30 = \square + 30$
$4 \times 30 = \square + 30$
through
$9 \times 30 = \square + 30$

Doubling and Adding Groups

$2 \times 30 = \square$
$4 \times 30 = \square + \square = ?$
$8 \times 30 = \square + \square = ?$
$9 \times 30 = \square + 30$

Doubling Groups

$2 \times 30 = \square$
$4 \times 30 = \square + \square$
$8 \times 30 = \square + \square$
$3 \times 30 = \square$
$6 \times 30 = \square + \square$

$3 \times 30 = 90$
$6 \times 30 = \square + \square = ?$
$7 \times 30 = \square + 30$

2. Find the missing number (□) and then solve for "n".

2 x 32 = □ + 4 = n
2 x 69 = □ + 18 = n
2 x 57 = □ + 14 = n

3 x 23 = □ + 9 = n
3 x 58 = □ + 24 = n
3 x 34 = □ + 12 = n
3 x 62 = □ + 6 = n etc.

2 x 37 = 60 + □ = n
2 x 27 = 40 + □ = n
2 x 78 = 140 + □ = n

3 x 43 = 120 + □ = n
3 x 36 = 90 + □ = n
3 x 64 = 180 + □ = n
3 x 59 = 150 + □ = n etc.

Increase the Difficulty

4 x 36 = □ + 24 = n
3 x 52 = □ + 6 = n
5 x 29 = □ + 45 = n
4 x 83 = □ + 12 = n

2 x 37 = □ + 14 = n
3 x 37 = □ + 21 = n
4 x 37 = □ + 28 = n
5 x 37 = □ + 35 = n - through -
9 x 37 = □ + 63 = n

3. Since 2 x 300 = 600 then
2 x 309 = 600 + n
2 x 330 = 600 + n
2 x 345 = 600 + n
2 x 360 = 600 + n
2 x 375 = 600 + n

Since 2 x 300 = 600 then
2 x 321 = 600 + n
2 x 342 = 600 + n
2 x 384 = 600 + n

Since 4 x 300 = 1200 then
4 x 304 = ?
4 x 311 = ?
4 x 325 = ?

Since 4 x 300 = 1200 then
4 x 309 = 1200 + n
4 x 318 = 1200 + n
4 x 336 = 1200 + n

4. Evaluate the following exercises. Tell how they are alike. How are they different? Explain the property or properties involved.

(5 x 4) x 3 = n
5 x (4 x 3) = n
(4 x 5) x 3 = n
4 x (5 x 3) = n
(3 x 4) x 5 = n
3 x (4 x 5) = n

OPERATIONS

UNIT 23 - SET OF WHOLE NUMBERS: MULTIPLICATION; VERTICAL FORMAT

NOTE TO TEACHER

The same properties of the operation of multiplication which children have used when multiplying using a horizontal format algorithm are also used when using a vertical format algorithm.

The vertical algorithm for multiplication depends especially upon the application of the Distributive Property of Multiplication with respect to Addition.

Children may use a variety of algorithms in computing products.

The numbers children should be given will be determined by:

1. Ability to multiply in horizontal format.
2. Understanding of Place Value
3. Ability to express numbers in expanded notation
4. Understanding of the Distributive Property

Some children should be given more difficult computations such as:

5 x 89, 7 x 256, 6 x 629, 8 x 609, etc.;

Others, less mature, simpler computations such as:

6 x 24, 5 x 142, 2 x 697, 3 x 307, etc.

Multiplication situations should be presented in different ways.

For example:

5 x 807 = n
Multiply 5 by 807
Multiply 5 and 807

The factors are 5 and 807. Find the product.
Find the product of 5 and 807.
Find the value of 807 nickels.

$$\begin{array}{r} 807 \\ \underline{\times\ 5} \end{array}$$

Problems may be read in a variety of ways.

Problem	May Be Read As:
5 x 847 = n	5 eight hundred forty sevens
$\begin{array}{r} 847 \\ \underline{\times\ 5} \end{array}$	5 times 847 847 taken 5 times 847 multiplied by 5

Numerals may be read in various ways.
Children should decide on the most convenient way for a specific purpose

4235 may be read as:
Forty two hundred thirty five

or as:
4 thousand 2 hundred thirty five.

Products may be estimated in a variety of ways.

Problem: 5 x 847 = n

Children may think:

5 x 800 = 4000	5 x 800 = 4000
5 x 40 = 200	5 x 50 = 250

Children record estimate

5 x 847 > 4200; or 5 x 847 < 4250;
or, the product of 5 x 847 is between 4200 and 4250.

TEACHING SUGGESTIONS

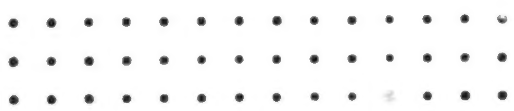
Objective: To develop skill in the techniques of multiplication, using the vertical format.

1. Reinforce understanding of the meaning and method of the vertical algorithm using arrays.

   Present an exercise: $3 \times 14 = n$

   Have children:

   Draw an array to represent the multiplication.

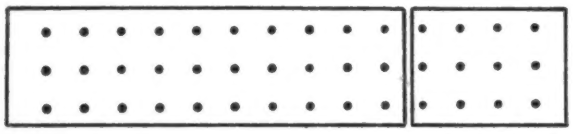


   Express 14 in expanded notation. [$14 = (10 + 4)$)

   Ask how they can apply the Distributive Property for Multiplication with respect to Addition to arrive at a product.

   $$[\ 3 \times 14 = (3 \times 10) + (3 \times 4)\ ]$$

   Emphasize this application of the Distributive Property using the array.



(3 x 10) + (3x4)

   Present the problem in vertical format

$$\begin{array}{r} 14 \\ \underline{\times\ 3} \end{array}$$

   Children should apply the Distributive Property and find the product.

$$\begin{array}{rl} 14 & \\ \underline{\times\ 3} & \\ 12 & (3\times4) \\ \underline{30} & (3\times10) \\ 42 & (3\times14) \end{array} \quad \text{or} \quad \begin{array}{rl} 14 & \\ \underline{\times\ 3} & \\ 30 & (3\times10) \\ \underline{12} & (3\times4) \\ 42 & (3\times14) \end{array} \quad \text{or} \quad \begin{array}{lcr} 10 & + & 4 \\ \underline{\times} & & \underline{3} \\ 30 & + & 12 = 42 \end{array}$$

2. Present exercises involving no exchange, one exchange and the two exchanges.

$$\begin{array}{r} 32 \\ \underline{\times\ 3} \end{array} \qquad \begin{array}{r} 29 \\ \underline{\times\ 4} \end{array} \qquad \begin{array}{r} 71 \\ \underline{\times\ 6} \end{array} \qquad \begin{array}{r} 45 \\ \underline{\times\ 6} \end{array}$$

$$\begin{array}{r} 143 \\ \underline{\times\ 2} \end{array} \qquad \begin{array}{r} 128 \\ \underline{\times\ 4} \end{array} \qquad \begin{array}{r} 182 \\ \underline{\times\ 4} \end{array} \qquad \begin{array}{r} 149 \\ \underline{\times\ 4} \end{array}$$

3. Develop skill in multiplying larger numbers, one factor through 9; the other factor through 999.

Have children first estimate, then compute, then compare the product with the estimate.

For example, 4 x 921 = n

a. Estimation: 4 x 900 = 3600

b. Computation:

```
  921
  x 4
 ----
    4 ( 4x1 )
   80 ( 4x20 )
 3600 ( 4x900)
 ----
 3684  (4x921)
```

Product may be read as:
36 hundred 84 or as:
3 thousand 6 hundred 84

c. Comparison:

3684 with 3600

4. Introduce the Concise Form for the algorithm.

```
      A                    B              C

     312                  312            312
     x 3                  x 3            x 3
    ----                 ----           ----
       6 ( 3x2)             6            936
      30 ( 3x10 )          30
     900 ( 3x300)         900
    ----                 ----
     936 ( 3x312)         936
```

Present Forms A, B, then C.

Discuss differences in recording.

Compare products. Children note that Place Value of the digits in the short form is the same as in the long form.

Refer to form C. Children record product in expanded notation.
[900 + 30 + 6]

Compare Forms A, B, C again.
Note the economy in the use of the short form.

5. Extend to exercises involving Exchange.

   Present forms A and B below simultaneously. If necessary demonstrate with arrays or with squared material first.

A

$$\begin{array}{rl} 216 & \\ \times\ 3 & \\ \hline 18 & (\ 3\times 6\ ) \\ 30 & (\ 3\times 10\ ) \\ 600 & (\ 3\times 200\ ) \\ \hline 648 & \end{array}$$

B

$$\begin{array}{r} 216 \\ \times\ 3 \\ \hline 648 \end{array}$$

Ask children:

Consider Form B. Why is "8" in ones place?

Why is 4 in tens place in the product in Form A?

What did we do to get 4 in tens place in Form B?

Where did we get the number added? [18 = 10 + 8]

Why is 6 in Form B in hundreds place?

6. Extend to exercises involving two and more exchanges.

   Children may check by applying the Distributive Property.

$$\begin{array}{r} 937 \\ \times\ 4 \\ \hline 3748 \end{array}$$

$$\begin{array}{rcrcr} 4 & \times & 7 & = & 28 \\ 4 & \times & 30 & = & 120 \\ 4 & \times & 900 & = & 3600 \\ \hline 4 & \times & 937 & = & 3748 \end{array}$$

7. Suggested practice exercises

$$\begin{array}{r} 419 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 635 \\ \times\ 7 \\ \hline 35 \\ 210 \\ 4200 \\ \hline \end{array} \qquad \begin{array}{r} 302 \\ \times\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 892 \\ \times\ 6 \\ \hline \end{array}$$

etc.

8. Apply to problem situations to provide more practice.

OPERATIONS

UNIT 24 - DIVISION OF WHOLE NUMBERS

$$\begin{tikzcd}\text{Addition} \arrow[r] \arrow[d, leftrightarrow] & \text{Multiplication} \arrow[d, leftrightarrow] \\ \text{Subtraction} \arrow[r] & \text{Division}\end{tikzcd}$$

NOTE TO TEACHER

In Multiplication two known the problem is to find their product.

Factor x Factor = □

Another interpretation of multiplication with whole numbers involves rectangular arrays of elements,

x x x x x

x x x x x

x x x x x

where we know the number of elements in each row and the number of elements in each column product is the number of elements in the array.

In Division two interpretations should be emphasized to parallel the two interpretations of multiplication:

1. When one factor and the product are known and the problem is to determine the missing factor

2. Where the total number of elements in the array is known and the number of rows (or columns) is known. The problem is to determine the number of columns (or rows).

In Division an array can not always be set up with the same number of elements in each row.

For example, 25 ÷ 4 set up in an array will have a remainder.

Given an array of 25 elements with 4 elements in a row the problem is to find the number of rows of 4.

This process may be shown as follows:

25	elements in the array
- 4	one row of 4 elements removed
21	elements remaining
- 4	another row of 4 elements removed
17	elements remaining
- 4	
13	elements remaining
- 4	
9	elements remaining
- 4	
5	elements remaining
- 4	
1	element remaining

By adding the number of times 4 elements have been removed we see that the quotient is 6 and the remainder is 1.

When children divide they remove from the dividend as many equivalent sets as they can.

Gradually they are encouraged to refine the process and remove a greater number of sets at one time.

Children express the remainder as a fractional number when a verbal problem indicates that it is reasonable to do so. It would not be reasonable to divide children into fractional parts.

They record a division with a remainder as:

$$\begin{array}{l} \;\;\;6\ r\ 1 \\ 4\overline{)25} \end{array}$$

Relate division to problem situations based on the children's experience. When situations are presented involving dollars and cents, children think in terms of the number of cents, \$1.54 as 154 cents.

Suggested Problem Situations

Postage stamps purchased

Have \$.72. Need four-cent stamps. How many stamps can be purchased? (Fours in 72)

P.T.A. Cake Sale

86 home baked cookies on hand. Packaged 5 in each waxed bag. How many bags filled with cookies? (Fives in 86)

Cup cakes sold at 7 cents each. \$1.54 collected. How many cup cakes sold? (Sevens in 154)

Picture Postcards in album

Trip to Washington D.C. - 138 cards bought. 6 on each page in the album. How many pages completely filled? (Sixes in 138)

Dance Formations

184 children in gym. Sets of 8 children formed. How many sets formed? (Eights in 184)

Suggested Provision for Differences in Ability

Less Mature	More Mature
Deal with divisors of 3, 4, 5 and perhaps 6 at first.	Deal with all divisors including 6, 7, 8, 9.

Record as few or as many computations as they need, by using groups smaller than 10, or groups of 10.	Are encouraged to shorten their computation by using multiples of 10.
Later deal with divisors of 6, 7, 8, 9 - quotients in the tens, twenties and thirties, e.g., $8\overline{)97}$ $6\overline{)198}$ $7\overline{)158}$	Deal with more difficult division situations - quotients in the fifties, sixties, seventies. $4\overline{)289}$ $6\overline{)321}$ $3\overline{)204}$ $7\overline{)448}$
Are encouraged to shorten the computation by using multiples of 10.	

Objectives: To help children:

Use algorithms to find quotients
Refine the techniques of the Operation of Division.

TEACHING SUGGESTIONS

Evaluation

Before division is introduced, the teacher finds out whether the children:

Understand the meaning of the numbers to be used.
Understand the meaning of subtraction and multiplication.
Have acquired skill in the operations of multiplication and subtraction.
Have achieved automatic response to the multiplication, subtraction and division facts they will use.
Have the ability to multiply numbers through 9 by 10 and multiples of 10.

Dividends through 999: Divisors through 9

1. No Remainder

Present a problem situation such as:
192 children are arranged into sets of 8 for a dance.
How many sets will there be?

Record as a mathematical sentence. $n \times 8 = 192$
Discuss the action involved (separating 192 into subsets of 8).

Record the symbols on the chalkboard. $8\overline{)192}$
(Read as "How many eights in 192?")

Discuss with the children the solution of the problem by removing successive sets of eight.

$$\begin{array}{r} 192 \\ \underline{-\ 8} \\ 184 \\ \underline{-\ 8} \\ 176 \\ \underline{-\ 8} \\ \text{etc.} \end{array}$$

Encourage children to see that it would be more efficient to remove several sets of 8 at a time. Each child may choose the number of sets of 8 he will take out.

Select one of the suggestions, e.g., 5 eights, and use it to develop the division algorism.

After the product (40) has been recorded, question children about its meaning.

Elicit from them that there are still 152 children to be divided into sets of 8.

Children continue to remove sets of 8 until zero is the remainder.

Total the number of eights and record the quotient.

$$\begin{array}{rrl} & 8\overline{)192} & \\ 5 \text{ eights} = & \underline{40} & \\ & 152 & \text{to be divided} \\ 6 \text{ eights} = & \underline{48} & \\ & 104 & \text{to be divided} \\ 10 \text{ eights} = & \underline{80} & \\ & 24 & \text{to be divided} \\ \underline{3} \text{ eights} = & \underline{24} & \\ 24 \text{ eights} & 0 & \end{array}$$

Solution: 24 sets of children.

Solve the same problem beginning with other sets of eights. The illustrations below show some possible variations.

? eights = 192

$$\begin{array}{rr} & 8\overline{)192} \\ 8 \text{ eights} = & \underline{64} \\ & 128 \\ 8 \text{ eights} = & \underline{64} \\ & 64 \\ \underline{8} \text{ eights} = & \underline{64} \\ 24 \text{ eights} & 0 \end{array}$$

Solution-24 eights

$\square \times 8 = 192$

$$\begin{array}{rr} & 8\overline{)192} \\ 5 \times 8 = & \underline{40} \\ & 152 \\ 10 \times 8 = & \underline{80} \\ & 72 \\ \underline{9} \times 8 = & \underline{72} \\ 24 \text{ eights} & 0 \end{array}$$

Solution-24 eights

$\square\ 8 = 192$

$$\begin{array}{rr} & 8\overline{)192} \\ 10 \times 8 = & \underline{80} \\ & 112 \\ 10 \times 8 = & \underline{80} \\ & 32 \\ \underline{4} \times 8 = & \underline{32} \\ 24 \text{ eights} & 0 \end{array}$$

Solution-24eights

Solve other problems with children in a variety of ways. Then present problems for each child to solve in his own way.

2. Remainder

Present a problem situation:

We are making booklets for Social Studies. Seven pieces of colored paper are needed for each booklet. How many booklets can we make if we have 200 sheets of paper?

Follow the same procedure suggested for division without remainders.

$\square \times 7 = 200$

$$\begin{array}{rcccr} & & & & 7\overline{)200} \\ 7 & \times & 7 & = & 49 \\ & & & & \overline{151} \\ 10 & \times & 7 & = & 70 \\ & & & & \overline{81} \\ 10 & \times & 7 & = & 70 \\ & & & & \overline{11} \\ \underline{1} & \times & 7 & = & \underline{7} \\ 28 \text{ sevens} & & & & 4 \end{array}$$

Solution: 28 booklets and 4 extra sheets

Through questioning, lead the children to conclude:
That another set of seven cannot be formed;
That the 4 represents four sheets of paper left from the 200 (the dividend).

Present other divisions without and with remainders.

Develop a More Concise Form

1. Present a problem situation such as: 139 books collected for the Veteran's Hospital, to be tied in bundles of 6 books each. How many bundles will there be?

   As each partial quotient is recorded in A, B, and C compare all forms, asking such questions as:

   What does the 7 represent? What does the 8 represent?
   What is the 97? What is the 49? etc.

A

$$\begin{array}{rcr} & & 6\overline{)139} \\ 7 \times 6 & = & \underline{42} \\ & & 97 \\ 8 \times 6 & = & \underline{48} \\ & & 49 \\ \underline{8 \times 6} & = & \underline{48} \\ 23 \text{ sixes} & & 1 \end{array}$$

B

$$\begin{array}{rcl} 6\overline{)139} & & \\ \underline{42} & = & 7 \times 6 \\ 97 & & \\ \underline{48} & = & 8 \times 6 \\ 49 & & \\ \underline{48} & = & \underline{8 \times 6} \\ 1 & & 23 \text{ sixes} \end{array}$$

C

$$\begin{array}{r|l} 6\overline{)139} & \\ \underline{42} & 7 \\ 97 & \\ \underline{48} & 8 \\ 49 & \\ \underline{48} & \underline{8} \\ 1 & 23 \end{array}$$

Solution: 23 Remainder 1

2. Present other division situations using all forms.
Then encourage the children to use the concise form only.

3. Use 10 or multiples of 10 to arrive at quotients.

Before using 10 or multiples of 10 to find a quotient, reinforce multiplying by whole decade numbers.

10 sixes, 20 sixes, 30 sixes, etc.

Use a familiar problem situation: 139 books collected for the Veteran's Hospital, to be tied in bundles of 6 books each. How many bundles will there be?

At first children find the solution by removing successive groups of 10 sixes.

$$\begin{array}{rr|l} & 23 & \\ & 6\overline{)139} & \\ & \underline{60} & 10 \\ \text{Still to be divided} \rightarrow & 79 & \\ & \underline{60} & 10 \\ \text{Still to be divided} \rightarrow & 19 & \\ & \underline{18} & \underline{3} \\ & 1 & 23 \end{array}$$

Solution: 23 sets and 1 extra book

When children develop facility in multiplying with larger multiples of 10 (20, 30, 40, etc.), they shorten the computation by removing groups of 20 sixes, 30 sixes, 40 sixes, etc.

For the problem suggested above question children and record the proper responses.

Will there be as many as 10 sixes? [Yes; 10 sixes = 60]

Will there be as many as 20 sixes? [Yes; 20 sixes = 120]
Will there be as many as 30 sixes? [No; 30 sixes = 180]

Then how many sets of sixes shall we remove first? [20 sixes]

How many books will that be? [120]

Have we used up all the books?

How many are left to be divided?

Continue until no more sets of six can be removed.

Discuss remainder.

```
     23
  6)139 |
    120 |  20
     19 |
     18 |   3
      1 |  23
```

Solution: 23 sets and 1 extra book.

Present many other divisions. Encourage those children who are able to use the shortened computation (multiples of ten).

```
     35
  8)284 |
    160 |  20
    124 |
     80 |  10
     44 |
     40 |   5
      4 |  35
```

or

```
     35
  8)284 |
    240 |  30
     44 |
     40 |   5
      4 |  35
```

Solution: 35 R 4

```
     63
  6)381 |
    120 |  20
    261 |
    120 |  20
    141 |
    120 |  20
     21 |
     18 |   3
      3 |  63
```

or

```
     63
  6)381 |
    120 |  20
    261 |
    240 |  40
     21 |
     18 |   3
      3 |  63
```

or

```
     63
  6)381 |
    300 |  50
     81 |
     60 |  10
     21 |
     18 |   3
      3 |  63
```

Children will need continuous reference to experience situations.

Many texts contain problems based on children's experience. These should be carefully selected.

Encourage children to shorten the recording by using larger and fewer multiples of ten.

```
              75                             75                          75
From:     6)452 |          Encourage     6)452 |           Advance   6)452 |
            120 |  20                      300 |  50          to       420 |  70
            332 |                          152 |                        32 |
            240 |  40                      120 |  20                    30 |   5
             92 |                           32 |                       ----------
             60 |  10                       30 |   5                     2 |  75
             32 |                          ----------
             30 |   5                        2 |  75
            ----------
              2 |  75
```

Solution: 75 R 2

Dollars and Cents

Present a problem situation such as: How many air mail stamps at $.08 each can be bought for $1.75?

Record and discuss the following forms:

$0.08)$\overline{\$1.75}$	8 cents)$\overline{175}$ cents	8)$\overline{175}$
How many $.08 in $1.75?	8 cents in 175 cents?	eights in 175?

Compute as Shown

```
       21
   8)175 |
     160 |  20
      15 |
       8 |   1
    ----------
       7 |  21
```

Solution: 21 stamps can be purchased.
7 cents left over.

Discuss the meaning of the remainder, 7 and the quotient 21.

Terminology

Discuss the meaning of dividend, divisor, quotient and remainder. Relate each term to the numbers of the specific problem.

139 books are to be tied in bundles of 6. How many bundles will there be?

```
       23
   6)139 |
     120 |  20
      19 |
      18 |   3
    ----------
       1 |  23
```

What does the 139 represent? [The number of books to be divided]

What does the 6 represent? [The number of books in each bundle; the number of each set.]

What does the 23 represent? [The number of equivalent sets; the number of bundles.]

What does the 1 represent? [An extra book; there are not enough books to make another set of 6]

Continue the above type of questioning. Use the terms for division. For example:

Which is the divisor? [6]

What does the 6 tell us? or What is the 6 called? [the divisor]

What does the divisor tell?

GEOMETRY AND MEASUREMENT

UNIT 25 - GEOMETRY: PLANES; SIMPLE CLOSED CURVES

NOTE TO TEACHER

Understanding of the meaning of a plane precedes the understanding of properties of polygons and circles and other plane figures.

A point is an idea just as a number is. It can be represented by a dot, just as a number is represented by a numeral. It has no dimensions.

A line is an idea. Its' representation: has only one dimension, length.

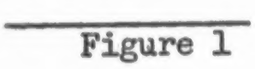
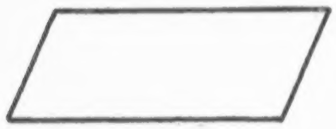


Figure 1

A plane is an idea. Its' representation:



has only 2 dimensions, length and width.

Figure 2

Planes may be thought of as special sets of points in space, which is considered to be the set of all points.

A plane, being a set of points, is a subset of space. (see Figure 2)

A plane can be thought of as extending without limit in space.

The following diagram indicates this.

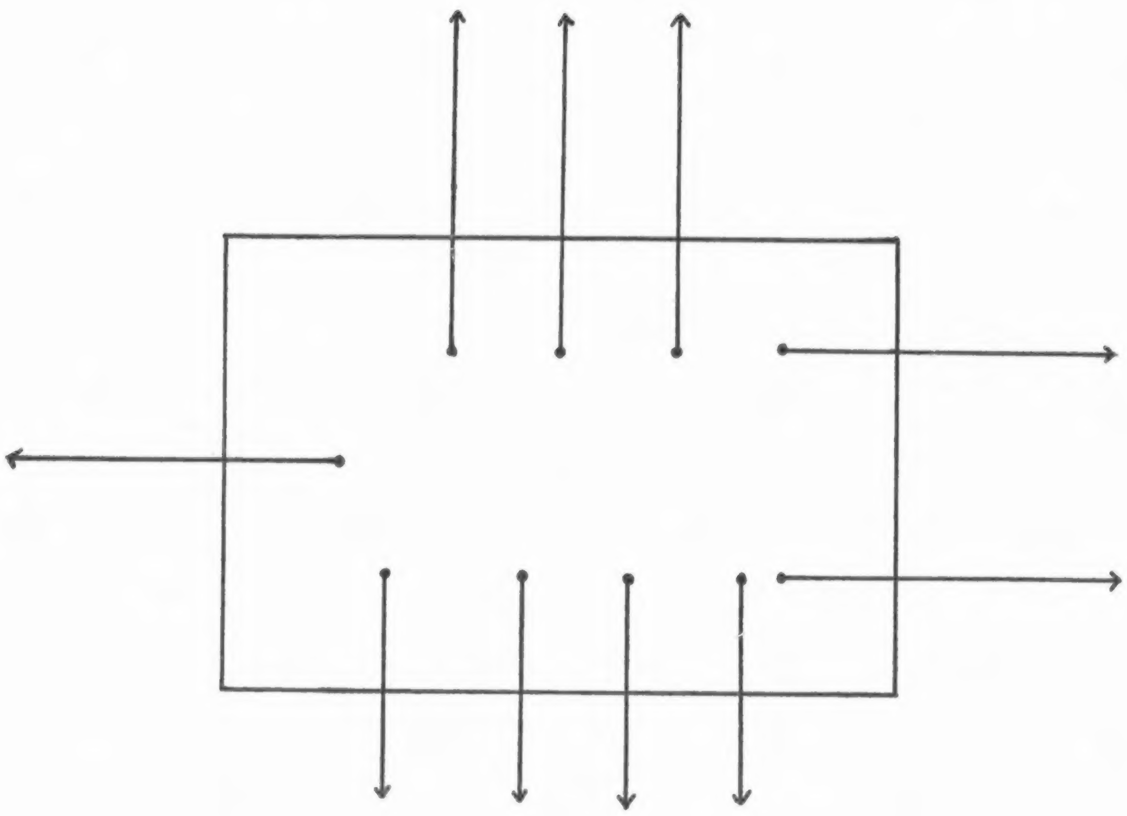
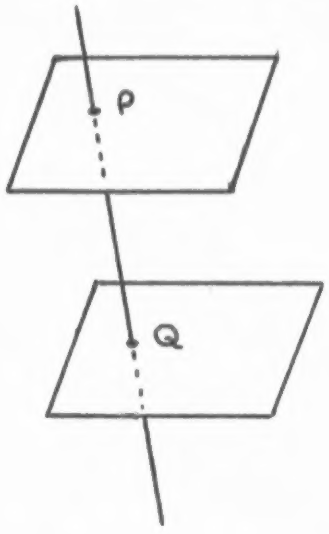
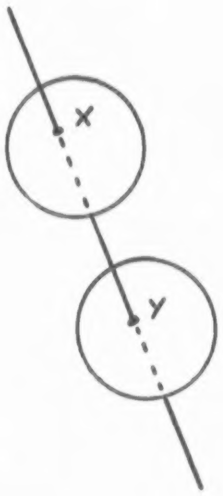


Figure 3 **(A vertical plane)**

Any plane contains an infinite number of lines. Therefore, it contains an infinite number of points. The surface of a table may be thought of as a representation of a segment of a plane.

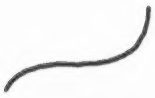
Each diagram below represents segments of 2 planes intersected by a line.

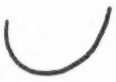


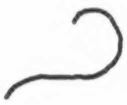


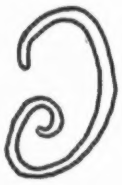
Lines A, B, C, D below are curves in one plane. (plane curves)

A simple plane curve may be represented as a set of points which can be drawn without lifting pencil from paper and without intersecting itself.

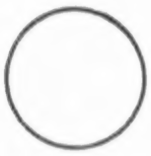

A

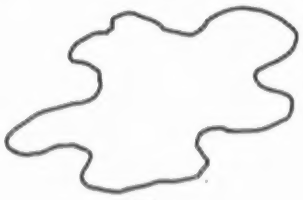

B


C


D

A simple closed plane curve is a curve in a plane whose starting and ending points are the same and which does not intersect itself at any point. For example:


E


F

D, E and F are simple closed curves.

Objective: To develop concepts of planes; simple closed curves.

TEACHING SUGGESTIONS

Planes

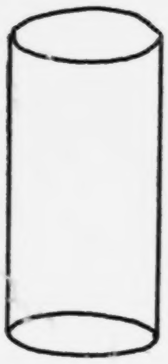
1. Reinforce concepts of:
    - A point
    - Sets of points

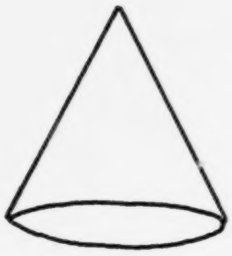
2. Tell children to imagine a pegboard extending without end in all directions. Imagine even more holes then there are in the pegboard. Think of the holes as representing points. All of the points form a set. This set of points on a flat surface is called a plane.

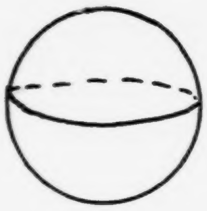
   A plane is therefore a set of points extending indefinitely in all directions.

   Imagine a crushed piece of paper; the ocean; a bumpy road. These are not flat surfaces. These are not in one plane. But these are also surfaces and sets of points.

3. A plane is a special kind of surface. It has a special property that other kinds of surfaces, curved surfaces, such as conical surfaces, cylindrical surfaces, spherical surfaces do not have


Cylindrical
Surface

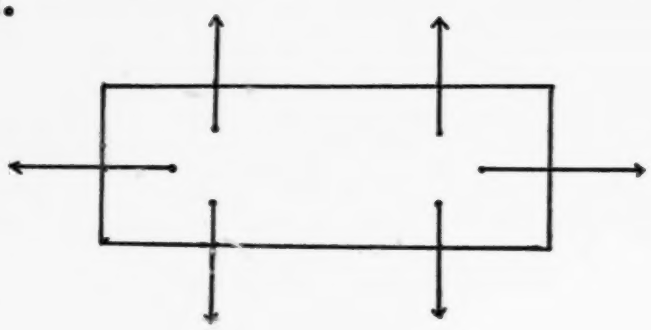

Conical
Surface


Spherical
Surface

   namely: The straight line joining any two points in a plane lies wholly in the plane.

4. Summarize:

   A plane is a set of points.
   A plane is a surface. Other kinds of surfaces are indicated above.
   A plane has a special property. (See underlined statement above.)
   A plane like a set of points can only be imagined.
   A plane has no thickness. It extends indefinitely in all directions except depth.



A plane is a special set of points in space.

Provide each child with a mirror and an index card. Ask children to adjust their mirror so that the card is reflected in the mirror.

They should see that the reflection of the card has negligible thickness, only length and width.

5. Ask children to name different plane surfaces in the classroom.

[desk tops, floor, ceiling, etc.]

6. Tell children that the surfaces mentioned represent only segments of planes. Compare with line segment.

Ask children:

How many lines can be drawn in a plane?
How many points are there in a plane?
How far can a plane be extended?
To describe in their own words the difference between space and a plane; a segment of a plane and a plane.

[A plane is a subset of space; a segment of a plane is a subset of a plane.]

To name some subsets of a plane.

[points, lines, simple closed curves, etc.]

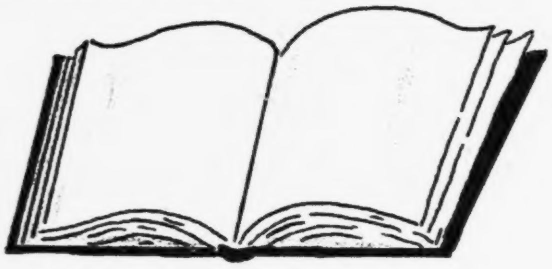
7. Draw a line and represent a point outside the line by a dot. How many planes can contain this line and the point?

[one]

Indicate three points that are not in a straight line. How many planes can contain these 3 points?

[There is only one plane that contains three non-collinear points]

How many planes can be determined by one line?
Explain or illustrate this.



use leaves of a book

What is the intersection of two different planes?

[a line]

8. Develop with children the "linearity" or "flatness" property of a plane surface. Bring into class cylindrical, conical and spherical solids and try to lay a ruler flat on the surface. It can be done:

   a. Sometimes with a cylinder or a cone
   b. Never with a sphere
   c. Always with a plane surface.

   Talk about how a plasterer creates a flat wall by moving a long rectangular instrument in all directions to create the plane surface.

   Do not try to verbalize this definition or property of flatness.

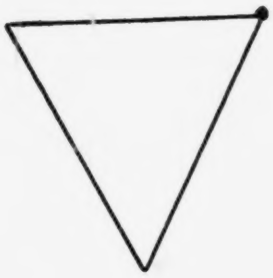
Simple Closed Plane Curves

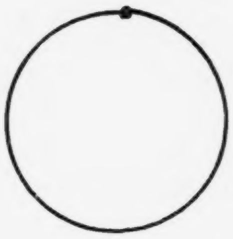
1. Reinforce concept of curve.

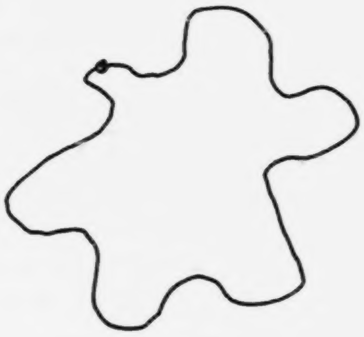
2. Direct children to draw a curve such that:

   The pencil must not be lifted from the paper.
   The starting point and point at which they end must be the same.
   The line must not cross itself at any point.

   For example:

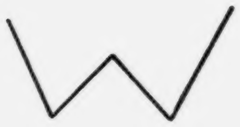


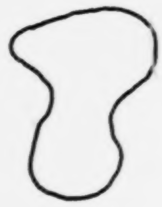


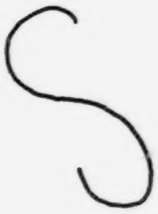


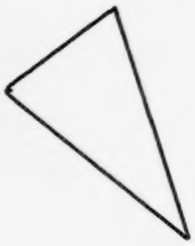
3. Children note that the figures enclose parts of a plane and separate it into an inside and an outside.

4. Draw figures such as the following:









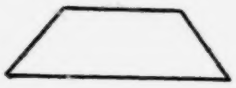
Children discuss differences among curves.

Tell children:

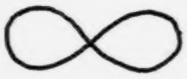
> A closed figure that can be drawn on a plane without lifting the pencil from the paper and that does not intersect itself is called a simple closed figure.

5. Children identify the simple closed curves in the diagrams below.

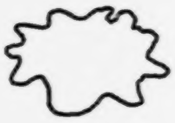
A.



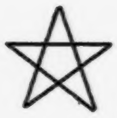
B.



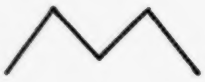
C.



D.

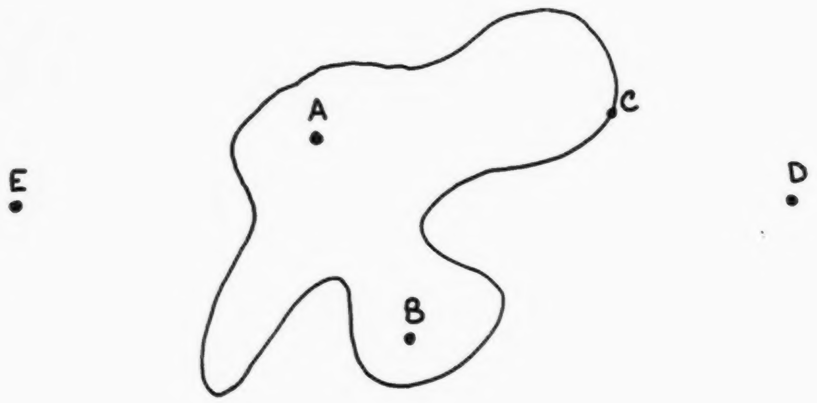


E.



Interior and Exterior of Curves

1. Have children draw a simple closed curve, then place dots representing points anywhere on their papers, and label the dots A, B, C, D, etc.

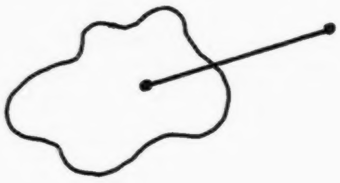


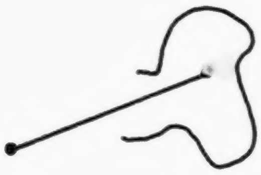
Ask the children to describe the location of point A, point B, point C, etc.

[inside the curve, on the curve, outside the curve]

Introduce the words: interior, exterior, boundary

2. Draw another simple closed curve on the chalkboard.
Have children place a dot on the interior of the curve, another on the exterior, then connect the two dots with a line. They do the same with an open curve.





Ask children:

What separates the interior of the simple closed curve from the exterior?

[the boundary]

Into how many regions does the boundary separate the plane?

[Two regions: the interior of the curve,
the exterior of the curve]

Have children note that the boundary of the two regions is the closed curve.

They note that an open curve that has two endpoints does not separate a plane into two regions.

GEOMETRY AND MEASUREMENT

UNIT 26 - GEOMETRY: RAYS; ANGLES

NOTE TO TEACHER

In **Unit** 4 we defined space as the set of all points.

Physical space considered as the set of all points, means the set of all possible locations in the universe. A moving airplane is constantly occupying different sets of points in space.

We have previously discussed some sets of points as locations in space, as curves, as lines, as line segments, as planes, as other kinds of surfaces.

In this **Unit** we extend concepts of lines.

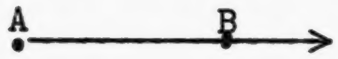
Rays

A point "A" of a line separates the line into two parts.

A ray is a part of a line that is composed of a point, such as A, and all points on that line on only one side of A.

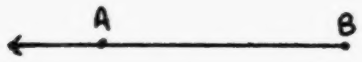
This is a drawing of a ray with endpoint A

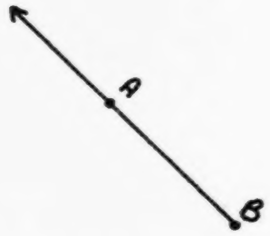
The symbol for ray is $\longrightarrow$. For example,



is symbolized as $\overrightarrow{AB}$

Should the endpoint be B with the ray extending in another direction thus, for



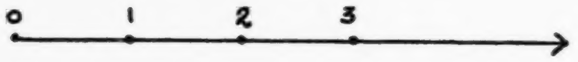


the symbolization would be $\overrightarrow{BA}$.

$\overrightarrow{BA}$ indicates that B is the endpoint.
Every ray has just one endpoint.

Number Ray

Part of a number line is an example of a number ray.



Angle

An angle consists of two rays with a common endpoint called its vertex. The rays are called the sides of the angle.

Objectives: To extend concepts of lines to include rays.

To develop concepts of angles.

TEACHING SUGGESTIONS

Ray

1. Discuss "Living in the Space Age" with children. Use newspaper articles, books, magazines and children's information to talk about Satellites, Astronauts, Landings on the Moon.

   Ask children:

   What information do we need for travel and communication in space?

   [Answers will vary. Some may be: direction, speed, location]

   How can we learn about these things we cannot see?

   [Use imagination, known facts, representations of things, models]

   What do you think space is? [Answers will vary]

2. Tell children that space is considered to be the set of all points everywhere.

   Review concepts of space developed in Unit 4.

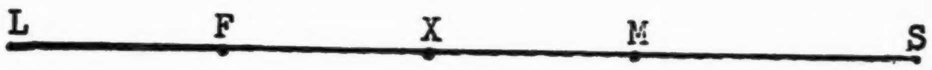
3. Review concept of line and line segment as special sets of points.

4. Direct children to:

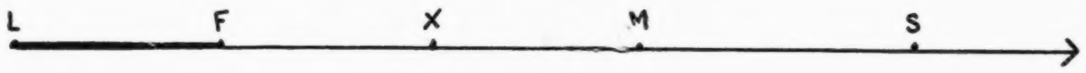
   Draw a line segment. Teacher draws on board.

Label endpoints **L**, F   Name the segment $\overline{LF}$.

Keep L as the endpoint and draw successively longer line segments and label each.



5. Teacher draws a ray with the same endpoint, L, but extending indefinitely.

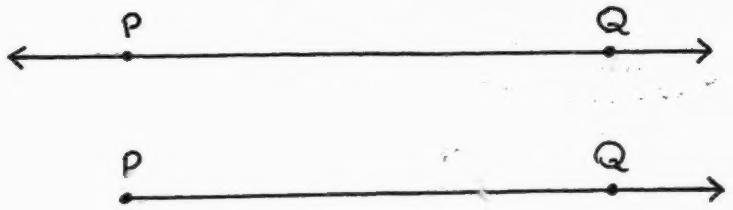


Ask children:

What is the difference between the two drawings?

If we symbolize the first drawing as $\overline{LS}$ can you suggest a way of symbolizing the second drawing?

Teacher may draw the following two figures on the chalkboard.



Ask children:

What do we call the upper drawing?

[line PQ]

How do we symbolize line PQ? [$\overleftrightarrow{PQ}$]

How does the lower drawing differ from $\overleftrightarrow{PQ}$?

Tell children that the lower diagram is called a ray. Discuss the term "ray" in life situations (light ray, x-ray, etc.).

Ask children:

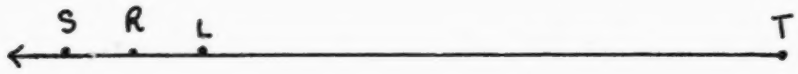
To describe this ray [This ray has an endpoint. It extends indefinitely in one direction only.]

To explain why a ray is a subset of a line.

6. Children draw another line segment with endpoints L and T.

They use T as the endpoint and draw successively longer line segments in the opposite direction.

Ask children to extend the line segment $\overline{TS}$ so that a ray results.



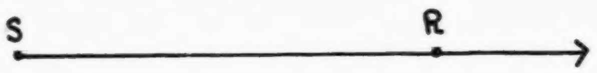
Children note that T is the endpoint and the ray extends indefinitely to the left of T.

7. Help children to verbalize the meaning of a ray.

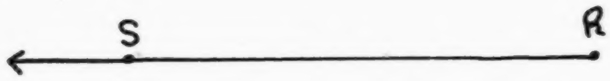
   [A ray has only one endpoint and it extends indefinitely in one direction.]

8. Tell children that the symbol for ray is → .
   The direction of the ray is indicated by the two letters used.

For example, $\overrightarrow{SR}$ indicates that the ray extends to the right of the endpoint, S



$\overrightarrow{RS}$ indicates that the ray extends to the left of endpoint R



$\overrightarrow{SR}$ and $\overrightarrow{RS}$ are different rays having different endpoints. The name of the endpoint is written first. Every ray has only one endpoint.

SUGGESTED PRACTICE EXERCISES

1. Consider this drawing of a ray

   What is the endpoint of $\overrightarrow{DE}$? [D]

   Is $\overrightarrow{DE}$ the same as $\overrightarrow{ED}$? Explain

2. Draw a diagram of a ray. Name it using the symbol for ray.

3. Mark a point on your paper. Label it.

   Draw 5 rays with that endpoint.
   How many rays can be drawn with that endpoint?

4. Draw three different pictures that occupy space.

5. Write what you think space is. Explain

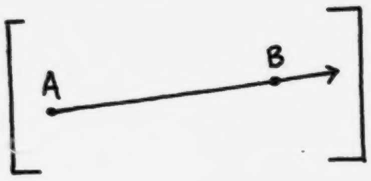
Angles

Direct children:

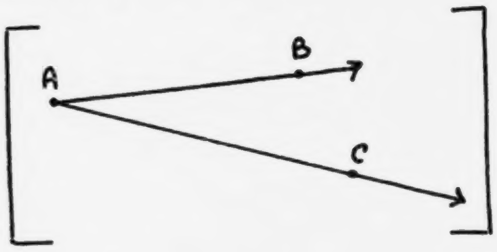
Mark a point on your paper. [•]

Mark it A. [$\overset{A}{\bullet}$]

Use A as an endpoint and draw a ray.



From the same endpoint A draw another ray.



Consider these figures

Note that there are two rays.
Note that the rays have a common endpoint.
Name the two rays.

Tell children that:

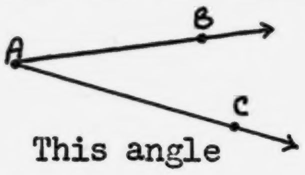
Two rays with a common endpoint form an angle.

Each ray is called a side of the angle.

The common endpoint is called the vertex of the angle.

The symbol for angle is " $\angle$ ".

Angles may be named in several ways. The letter naming the endpoint is placed in the middle.



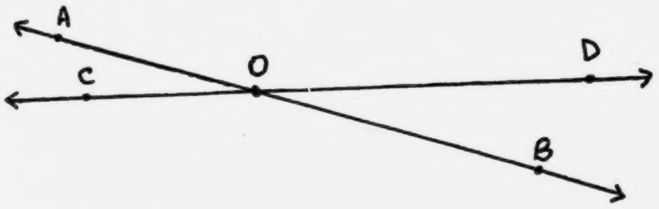
This angle may be named $\angle BAC$ or $\angle CAB$.

Talk about an angle as the union of two rays.

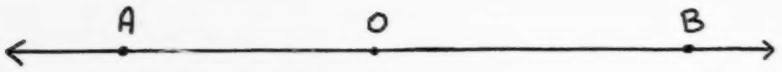
Direct children to:

Move the hands of a clock to indicate angles.
Tell where is the vertex of these angles.
Describe other objects that represent angles.

Name 4 angles that are formed when 2 lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point O.

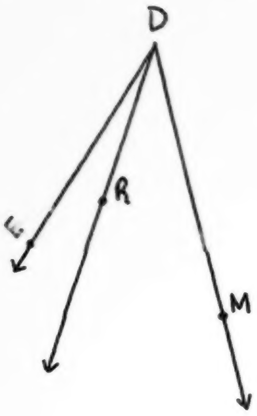


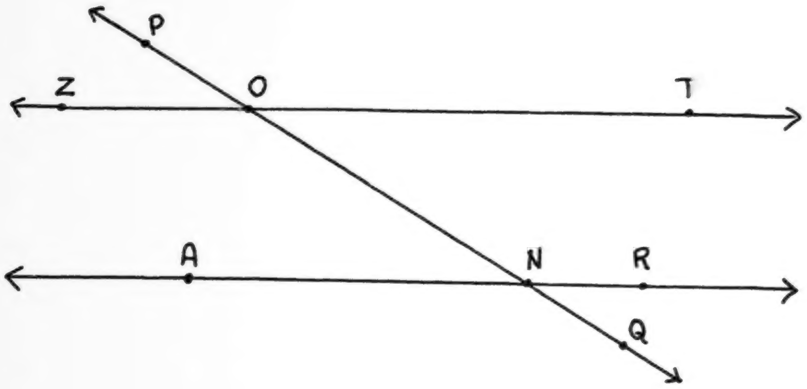
Name the angle formed by the 2 opposite rays meeting at vertex O in the diagram below.



(Here $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are parts of line $\overleftrightarrow{AB}$)

Name all the angles shown in the figures below. Be careful! Find as many angles as you can in each figure.





Direct children to point out the intersection of lines in the classroom that form different angles.

[angles formed by the binding of an open book.]

Tell children that angles shaped like a square corner are called right angles.

Ask children to draw right angles, angles less than right angles (acute), angles greater than right angles (obtuse).

GEOMETRY AND MEASUREMENT

UNIT 27 - GEOMETRY: POLYGONS; *EXPERIMENTAL GEOMETRY

NOTE TO TEACHER

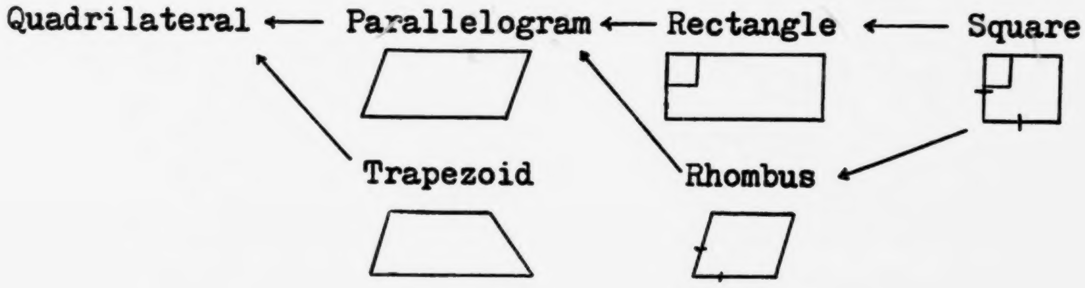
From a study of planes, line segments and simple closed curves we turn next to polygons.

A polygon is a simple closed curve which is a union of line segments. Polygon is the general name given to a family of geometric figures.

A triangle is a polygon consisting of the union of three line segments. We say it has 3 sides.

A quadrilateral is a polygon consisting of the union of four line segments; a pentagon, the union of five line segments; etc.

Each of the members of the set of polygons may again be subdivided by considering the kinds of angles and line segments it contains. Thus among the subset of polygons called quadrilaterals, there are parallelograms and trapezoids; some special parallelograms are rectangles and squares.



Objectives: To help children develop concept of polygons as simple closed curves.

To help children identify some types of quadrilaterals by their characteristics.

TEACHING SUGGESTIONS

1. Polygons

Ask children to:

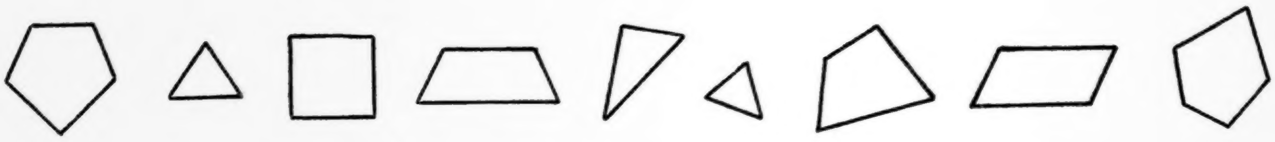
Draw a simple closed curve
Draw simple closed curves which are the union of 3 line segments, 4 line segments, 5 line segments, etc.

Tell children that:

Simple closed figures consisting of line segments are called polygons.
Polygons composed of 3 line segments are called Triangles.
Polygons composed of 4 line segments are called Quadrilaterals.
The line segments are called the sides of the polygon.

Ask children:

What is the name for a polygon containing 3 sides? [Triangle] 4 sides? [Quadrilateral] 5 sides? [Pentagon] etc.
To identify the following polygons and explain



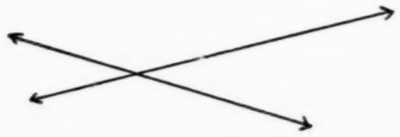
What is the greatest number of sides a polygon can have?
What is the least number of sides a polygon can have? Explain[3]

2. Special Kinds of Quadrilaterals

Parallelograms

Tell children that:

> Parallel Lines are lines in the same plane that do not intersect. They are everywhere equidistant.
>
> Non-parallel Lines in a plane are lines that do intersect. How many points of intersection do 2 non-parallel lines have? [One]



Discuss meaning of parallelogram.

> A Parallelogram is a quadrilateral whose opposite sides are parallel. The opposite sides also have the same length.

Review meaning of rectangle:

> [Rectangles have 4"square corners,"(right angles) and the opposite sides are parallel. A rectangle is a special kind of parallelogram.]

Ask children:

> Is every rectangle a parallelogram? [Yes] Explain.
>
> Is every parallelogram a rectangle? [No] Explain.

Reinforce understanding of:

Right Angle:	A right angle is an angle shaped like a square corner. Name some objects in the room that have right angles.
Square:	A square is a quadrilateral having 4 sides of equal length and 4 right angles.

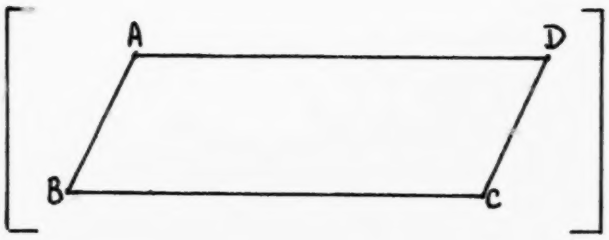
Discuss:

> A square as a special type of rectangle, and a special kind of parallelogram.
> Compare a square with other rectangles.
> When is a rectangle not a square?

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Locate on your paper points A, B, C, D like these below.
   Draw $\overline{AB}$, $\overline{AD}$, $\overline{CB}$ and $\overline{DC}$

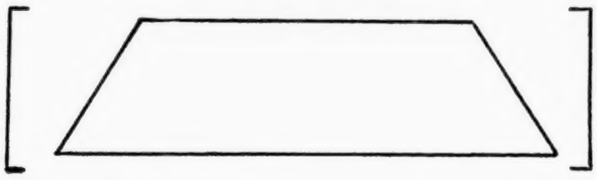


2. Which of these are names for this figure? [a, b, d]

   a. simple closed curve
   b. polygon
   c. triangle
   d. quadrilateral

3. Draw a picture of a quadrilateral without any parallel sides.

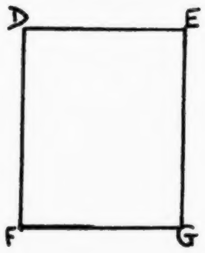
   Draw a picture of a quadrilateral with only two parallel sides.



   We call this a Trapezoid.

4. Name two special types of quadrilaterals. [parallelogram, rectangle]

5. What are the properties of parallelogram DEFG?



[$\overline{DE}$ and $\overline{FG}$ are parallel; $\overline{DF}$ and $\overline{EG}$ are parallel; all angles are right angles; it is a rectangle; etc.]

6. Why is a square a rectangle?

7. Is every square a parallelogram? If this is true, what must be true of the sides of a square?

8. Have children explore ways of drawing a rectangle, a square, a parallelogram.

9. Draw a parallelogram ABCD (as in exercise 1). Measure $\overline{AB}$ and $\overline{DC}$. What seems to be true? Measure $\overline{AD}$ and $\overline{BC}$. What seems to be true?

10. Draw a quadrilateral which is not a parallelogram. Measure pairs of opposite sides. Try to draw one with:

    a. only one pair of opposite sides equal in length
    b. with both pairs equal in length.

    ["a" can be done but for "b" it must be a parallelogram.]

11. Additional exercises may be found in textbooks.

*EXPERIMENTAL GEOMETRY (Optional)

NOTE TO TEACHER

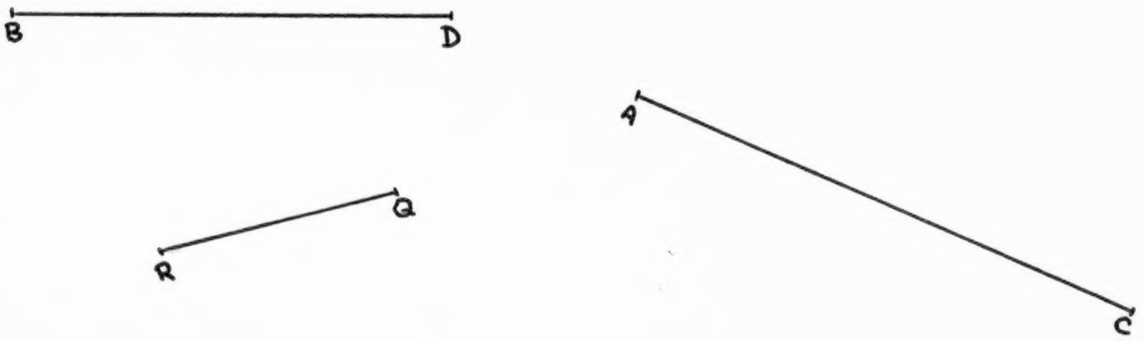
The Geometry in this section which we will call "Experimental Geometry" will be limited to further exploration of polygons.

Its purpose is twofold:
To extend children's reasoning power
To extend the use of measurement in Geometry so that children may discover some additional properties of geometric figures.

TEACHING SUGGESTIONS

Experimenting With Triangles

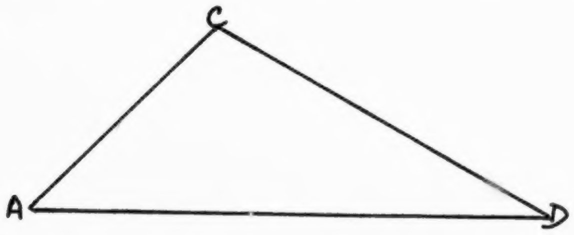
1. Have children draw line segments of varying lengths. For example:



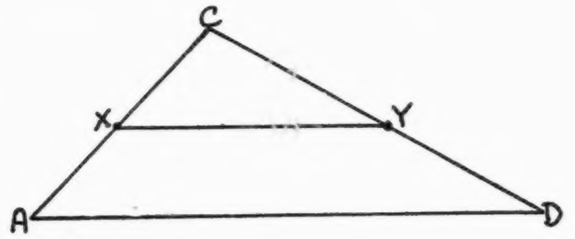
Children should use rulers to find the mid-point of each line. (Practice in dividing a fractional number by 2 may have to be included for some children.)

2. Have children:

Draw any triangle as below, and label the vertices.



Use a ruler to find the mid-point of $\overline{AC}$. Label that point X.
Use a ruler to find the mid-point of $\overline{CD}$. Label that point Y.
Draw $\overline{XY}$

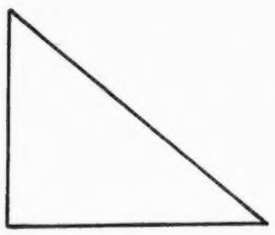


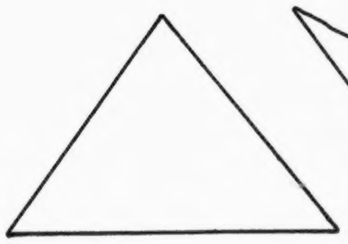
Measure $\overline{XY}$
Measure $\overline{AD}$

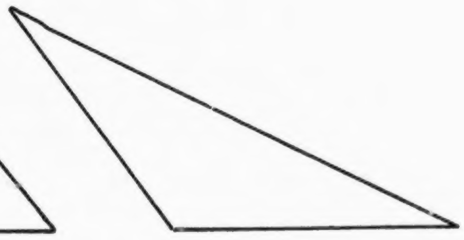
Ask children: What seems to be true about the relationship between $\overline{XY}$ and $\overline{AD}$?

[The measure of the length of $\overline{XY}$ is $\frac{1}{2}$ the measure of the length of $\overline{AD}$.]

3. Children should follow the same procedure using different triangles. For example:


Right Triangle

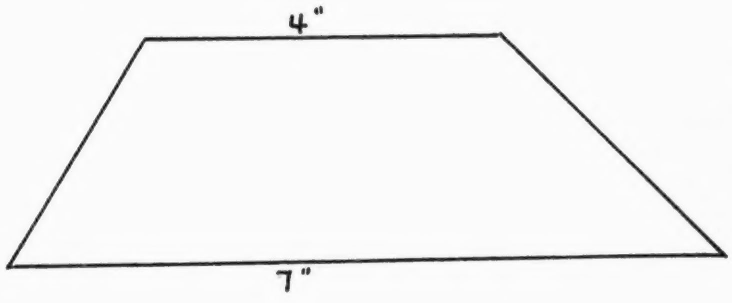

Equilateral Triangle


Obtuse, etc.

They find out whether the same relationship holds true.

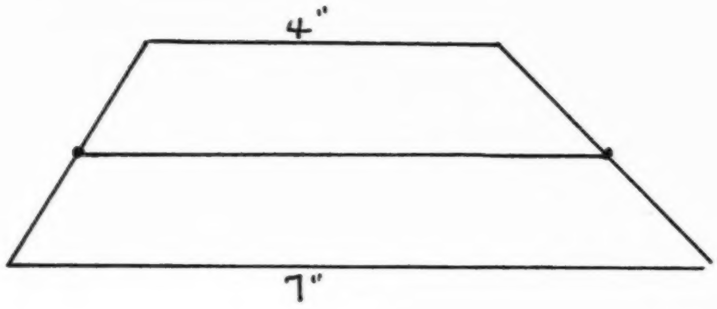
Median of a Trapezoid

1. Reinforce meaning of a trapezoid.
2. Have each child draw a trapezoid whose parallel sides are whole numbers, and mark the length of those sides. For example:



3. Have the children find the mid-point of each of the two non-parallel sides.

   They connect the points. We will call this



   line the center line of the trapezoid.
   Children measure the length of this center line.

4. On the chalkboard, make a chart of the various measurements that the children have obtained.

	Upper Base	Lower Base	Center Line	
John's Trapezoid	1"	2"	$1\frac{1}{2}$"	
Bob's Trapezoid	2"	3"	$2\frac{1}{2}$"	
Mary's Trapezoid	2"	4"	3"	
Ann's Trapezoid	1"	3"	2"	etc.

Question: Can you discover any relationship between the length of the center line and the lengths of the other two sides?

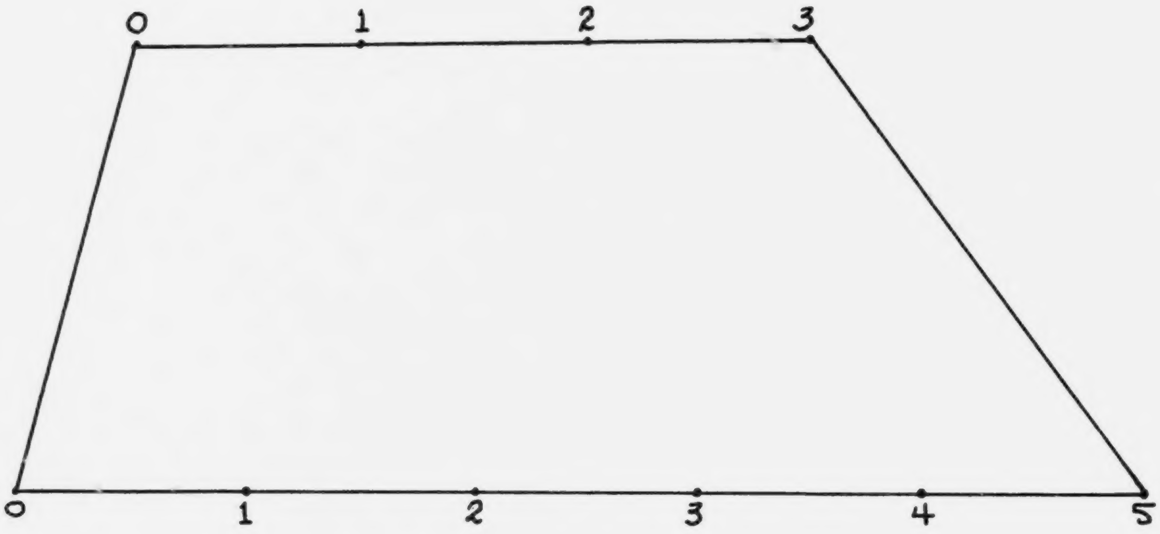
If we add the lengths of the other two sides, what is the relationship of this sum to the length of the center line?

[It is twice as much]

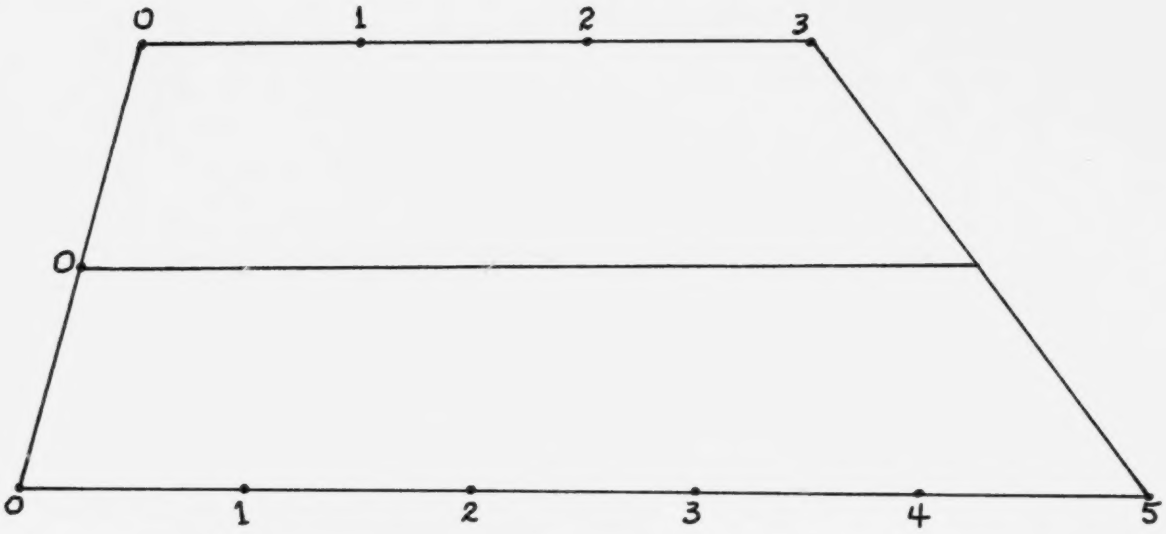
Application of Median of a Trapezoid to Nomogram

Nomogram: One of many mathematical devices used to find the sum of any two numbers.

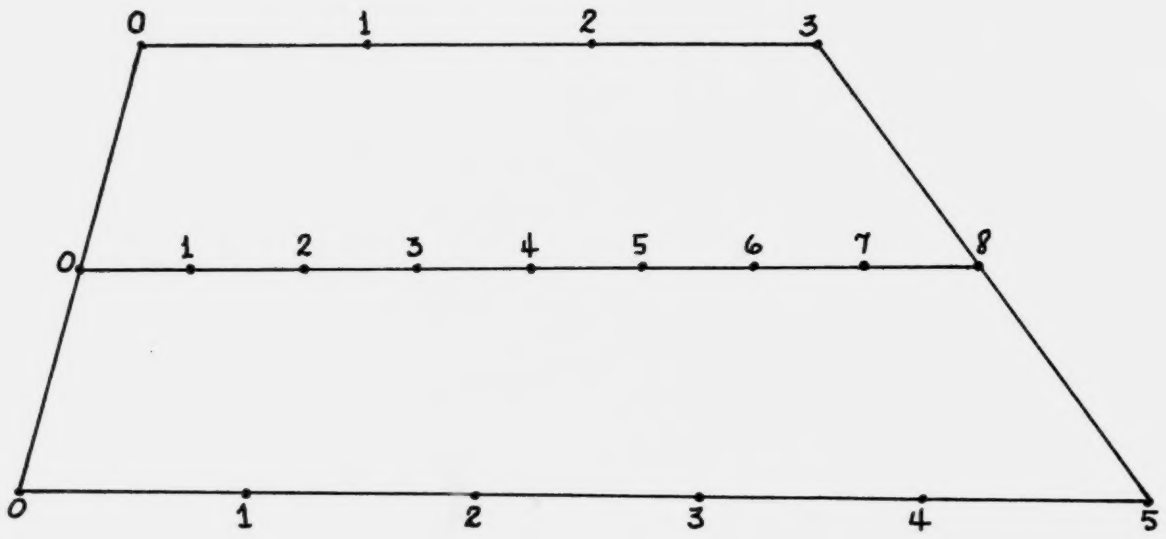
1. Have children mark off equal units on the upper and lower bases of their trapezoids and number the points as on a number line. For example:



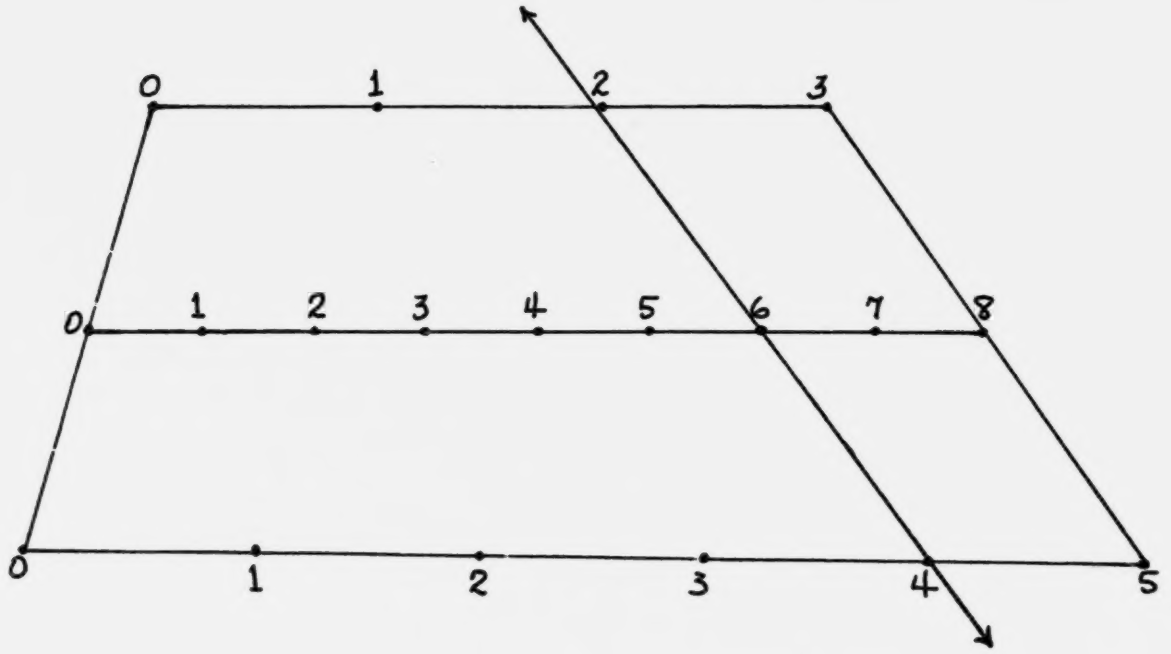
2. Have children find the midpoints and draw the center line.



3. Using a unit that is one half of the unit used on the other two sides, have children mark off the center line. They number the points as on a number line.


0
1
2
3
0
1
2
3
4
5
6
7
8
0
1
2
3
4
5

4. To add 2 and 4, the children locate 2 on the upper line and 4 on the lower line.
They place a ruler or straightedge so that the points 2 and 4 are on a line and are visible.

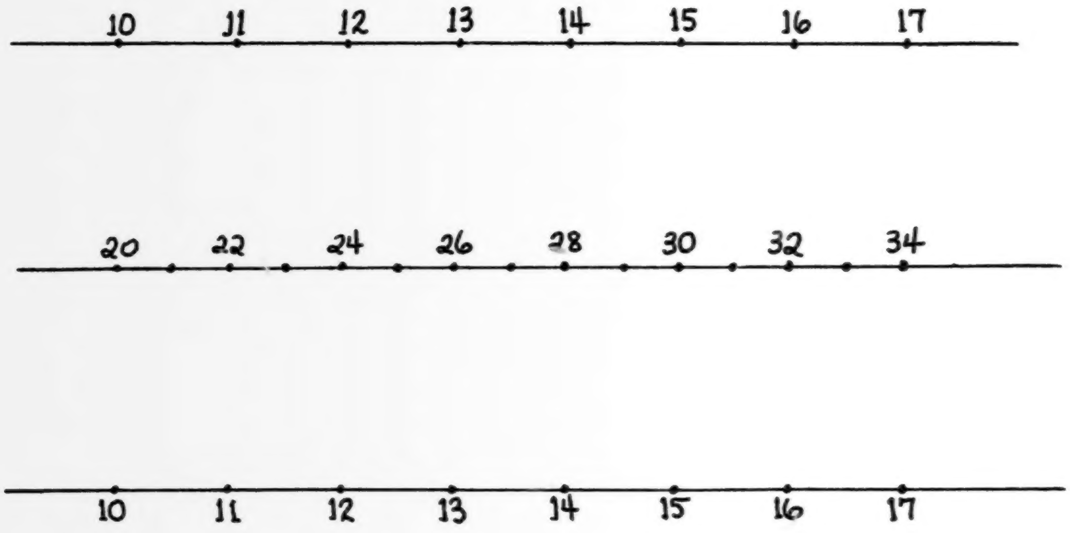

0
1
2
3
0
1
2
3
4
5
6
7
8
0
1
2
3
4
5

What point does the straightedge touch on the center line? [6]

What is the relation of 6 to 2 and 4?

5. Tell children that this device is called a Nomogram.

6. Have them explore the nomogram with other numbers. This may be extended to include larger numbers. For example:


10
11
12
13
14
15
16
17
20
22
24
26
28
30
32
34
10
11
12
13
14
15
16
17

Ask children to:

Use a straightedge to show the sum of 10 and 12.

Show that 10 + 12 = 12 + 10
13 + 16 = 16 + 13 etc.

7. Have children discover how the Nomogram may be used for subtraction.

GEOMETRY AND MEASUREMENT

UNIT 28 - PERIMETER OF POLYGONS

Objectives: To develop concept of perimeter.
To help children derive formula for finding perimeter of rectangles.
To help children derive formula for finding perimeter of polygons of equal sides.

TEACHING SUGGESTIONS

1. Reinforce:

   Rectangles have 4 right angles and the opposite sides are parallel and have the same length.

   Squares have 4 right angles and the 4 sides are parallel and have the same length. Squares are special rectangles.

   Symbols for: foot or feet (') and inch or inches (").
   Terms: length, width, dimensions.

2. A rectangular picture, chart or map to be framed is presented by the teacher.

   Children estimate and then measure the length and width. Elicit from the children various ways of finding the amount of material needed to frame the picture or chart.

3. Introduce the concept Perimeter to mean the distance around. Emphasize that the perimeter of a rectangle means the distance around the four sides.

4. Ask children to estimate the length of picture wire needed to go around the four sides of framed picture. Have them open the wire and measure the length to verify their estimate.

Using the same method, find the perimeter of a table top, of a desk top, of other rectangular surfaces in the room. Compare the estimated perimeters with the measured perimeters.

5. Have children estimate and then measure:

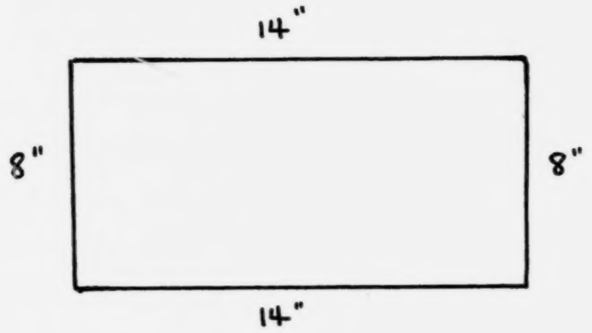
To find the amount of braid to go around a rectangular place mat.

To find the amount of leather binding needed to go around a rectangular foot stool.

To find the amount of crepe paper needed to go around a plant box.

To find the amount of lace needed for a table cloth edging.

Discuss to help children discover that:
The distance around a rectangle (perimeter) may be arrived at in various ways.



A. Adding: (two lengths and two widths, then combining them)

14 in. + 14 in. = 28"
8 in. + 8 in. = 16"
28 in. + 16 in. = 44" (answer)

B. Adding: (length and width and doubling sums)

14 in. + 8 in. = 22 in.
14 in. + 8 in. = 22 in.
22 in. + 22 in. = 44 in.

C. Adding: (1 length and 1 width, then multiplying by 2)

14 in. + 8 in. = 22 in.
2 x 22 in. = 44 in.

D. Multiplying: (length by 2, width by 2. Adding both amounts)

2 x 14 in. = 28 in.
2 x 8 in. = 16 in.
28 in. + 16 in. = 44 in.

Have children tell in their own words how to find perimeter of a rectangle.

Help children to arrive at formulas for Perimeter.

Discuss with children:

If we let P stand for the Perimeter of the rectangle; "l" stand for its length; "w" stand for its width can you make a mathematical sentence to describe the Perimeter?

$$\left[\begin{array}{c} P = l + w + l + w \\ \text{or} \\ P = 2 \times (l + w) \\ \text{or} \\ P = l + l + w + w \\ \text{or} \\ P = 2l + 2w \end{array}\right]$$

6. Extend the above development to find perimeters of:

square - trimming a napkin or handkerchief
triangle - outlining a Christmas tree
hexagon - planning stop signs for safety week
other polygons - making stars, borders, laying out baseball diamonds, planning for carpeting stairs, etc.

Finding Perimeter of Polygons of Equal Sides

A polygon of equal sides is called an equilateral polygon.
Note: By "equal" here, we mean "equal in measure" or length.

Squares

Reinforce understanding of:

Right Angle: A right angle is an angle shaped like a square corner. Name some objects in the room that have right angles.

Square: A square is a quadrilateral having 4 sides of equal length and 4 right angles.

Discuss:

A square as a special type of rectangle.
Compare a square with other rectangles.
When is a rectangle not a square?
Is every rectangle a square? [No] Why?
Is every square a rectangle? [Yes] Why?

Problem: A handkerchief measures 8 inches on each side. How much lace is needed for trim around the handkerchief?

Discuss various ways of finding the perimeter.

$8 + 8 + 8 + 8 = n$ $4 \times 8 = n$

Children find perimeters of other squares.

They arrive at the formula for finding the perimeter of a square.

4 times the number of units of length of a side = the perimeter of a square.

$p = 4 \times s$ or $p = 4s$

Equilateral Triangles, Other Equilateral Polygons

Discuss:

Equilateral Triangle, Rhombus, Pentagon, Hexagon, etc.

Children find perimeter of each type in a variety of ways.

They explain how they would arrive at the formula for finding the perimeter of equilateral polygons.

If "p" represents perimeter, and "s" represents the length of a side, then the perimeter of a:

Triangle is $p = 3 \times s$ or $p = 3s$
Square is $p = 4 \times s$ or $p = 4s$
Pentagon is $p = 5 \times s$ or $p = 5s$
Hexagon is $p = 6 \times s$ or $p = 6s$
Octogon is $p = 8 \times s$ or $p = 8s$ etc.

They arrive at the generalization that when lengths of the sides of a polygon are equal, the perimeter is found by multiplying the length of a side (s) by the number of sides (n).

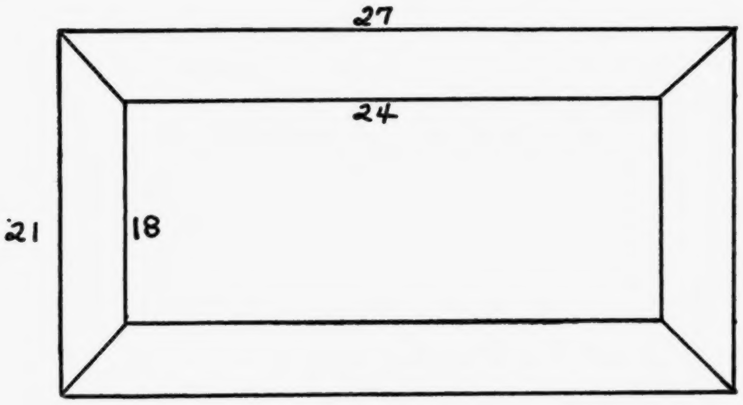
$p = ns$

Children complete the following and explain:

Perimeter of equilateral triangle = □ times length of 1 side [3]
Perimeter of equilateral octogon = □ times length of 1 side [8]
Perimeter of equilateral pentagon = □ times length of 1 side [5]

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Determine the amount of material needed to make a picture frame as shown below.



[21" + 27" + 21" + 27" = 96"]

2. The shape of the baseball diamond in our school yard is square. If one side is 80 ft., about how far does Bob run if he hits a home run? Use formula.

3. Solve the open sentence $P = 3s$ where $s = 10$. Tell what kind of polygon is involved.

4. How much fence is needed to enclose a triangular garden each of whose sides is 7'6".

5. Additional practice exercises in finding perimeter are to be found in textbooks.

SETS; NUMBER; NUMERATION

UNIT 29 - THE BASE-TEN SYSTEM OF NUMERATION EXTENDED

NOTE TO TEACHER

Relate large numbers to children's possible experiences, e.g.,

Seating capacities in various arenas:

Madison Square Garden	15,000	(circus)
	20,000	(boxing)
Yankee Stadium	68,000	
Soldiers' Field	100,000	(seen on television perhaps)
Phila. Municipal Stadium	110,000	

As an aid in reading and understanding large numbers, commas are used to set off groups of three digits, beginning at the right. Each of these groups of digits is called a period. The first period starting at the right, is called the units' period because the number expressed by its digits taken together is that many ones. The second period of a number is called the thousands' period because its digits express the number of thousands.

Objective: To extend understanding of Base Ten System to: hundred thousands; millions

TEACHING SUGGESTIONS

Test for understanding of the Base-Ten System of Numeration through 10,000; place value; reading and writing numbers.

Extend Understanding of Base Ten System to Hundred Thousands

1. Begin with 10,000 - 20,000 and gradually extend the system. Reinforce the fact that each place is ten times the value of the place to its right:

   10,000 = 10 thousands;
   100,000 = 10 ten thousands; also 100,000 = 100 thousands

   Position determines the value of each digit, e.g.,

   44,444

   4 ten thousands = 40,000
   4 thousands = 4,000
   4 hundreds = 400
   4 tens = 40
   4 ones = 4
   44,444

   62,518

   6 ten thousands = 60,000
   2 thousands = 2,000
   5 hundreds = 500
   1 tens = 10
   8 ones = 8
   62,518

2. Have children rewrite the numerals in expanded notation.

   44,444 = 40,000 + 4,000 + _____

   62,518 = 60,000 + n + □ + 8
   etc.

3. Introduce the period chart.

   Record numerals: 142560 142,560

   Ask children:

   Which numeral is easier to read? Why?
   Read that numeral.
   Where is the comma placed?

   Tell children that each group of 3 digits is called a period.

   Present a period chart.

Thousands Period			Units Period		
Hundreds	Tens	Ones	Hundreds	Tens	Ones

Discuss: Ones, Tens, Hundreds in the Thousands Period.
Children note that in reading a number such as 41,235, the 41 is stated first, then the period in which it falls (thousands), then the 235 is stated.

4. Present numerals such as the following:

4
26
523
6,328
28,253
159,417

Have the children read each numeral and identify the periods into which each digit falls. For example: the 6 in 6,328 falls in the thousands period and the 328 in the units period.

Have them note the units period is not stated when reading numerals.

5. Present numerals such as the following:

2075
35240
468506
507436
8057
50289

Direct children to:

Copy these numerals and point off the periods.
Note: To place commas correctly children start at the right (from ones place) and go to the left.

Read the numbers aloud.

Tell or write the meaning of the digit 5 in each of the numbers.

Tell or write the meaning of zero in each of the numbers.

6. Dictate numbers.

Children write the number that is:

100 greater than each of these dictated numbers
1,000 greater than each of these dictated numbers
10,000 greater than each of these dictated numbers
100,000 greater than each of these dictated numbers

Counting

Indicate to the children how far to continue the sequence.
Examples follow:

Counting Forward

10,000, 20,000, 30,000, __, __, __,
21,100, 22,100, 23,100, __, __, __,
25,100, 25,200, 25,300, __, __, __,
30,100, 30,200, 30,300, __, __, __,
18,600, 18,700, 18,800, __, __, __,

23, 46, 69, __, __, __,
373, 381, 389, __, __, __,
414, 428, 442, __, __, __,
120, 240, 360, __, __, __,
102, 204, 408, __, __, __,

600 1200 1800 ___ ___ ___
406 812 1218 ___ ___ ___

11,000 21,000 31,000 ___ ___ ___
14,000 14,500 15,000 ___ ___ ___
100,000 200,000 300,000 ___ ___ ___

Counting Backward

60,000, 50,000, 40,000, ___, ___, ___, 252, 244, 236, ___, ___, ___,
26,100, 25,100, 24,100, ___, ___, ___, 121, 115, 109, ___, ___, ___,
25,600, 25,500, 25,400, ___, ___, ___, 96, 48, 24, ___, ___, ___,
30,600, 30,500, 30,400, ___, ___, ___, 720, 610, 500, ___, ___, ___,
19,400, 19,300, 19,200, ___, ___, ___, 128, 64, 32, ___, ___, ___,

520 480 440 ___ ___ ___
3500 3000 2500 ___ ___ ___

61,000 51,000 41,000 ___ ___ ___
16,500 16,000 15,500 ___ ___ ___
600,000 500,000 400,000 ___ ___ ___

Money

1. Test for understanding of place value and for ability to read, write, make change, regroup, etc., through $99.99.

2. Extend development through $999.99. Include:

Place Value
Regrouping - as for $130.00 - 129 dollars 10 dimes 0 pennies
129 dollars 9 dimes 10 pennies
Reading and Writing - regular and irregular columns
Numbers in Series
Change Making

Develop Understanding of Base Ten System to Millions

1. Test for understanding of our system of numeration through 6 places; place value; reading and writing of numerals; numbers in series.

How many digits are used in our decimal system of numeration? What are they?

Use numerals to express the following: three hundred five thousand three; seventy two thousand one hundred fifty six; ten thousand one.

Use words to express the following: 463,780; 25,021.

Write the numeral that comes after each of the following:

1,099 1,999 10,099

Starting with units place, name the next five places to the left.

State the place value and numerical value of the 4 in each of the following numerals:

411 [hundreds place; 400] 14,115
34 45,231
2,140

What place is the zero holding in each of the following?

40,728 207,354 629,072

Fill in the spaces:

Digit	Place	Numeral
5	Thousands	___[5000]
4	Thousands and Tens	___[4040]
6	Hundred-thousands and Thousands	___[606,000]
3	Ten Thousands and Units	___[30,003]

2. Record: 999,999

Ask children:

What number comes next? [1 million]

Write 1 million as a numeral. [1,000,000]

How many hundred thousands are in 1 million? [10 hundred thousands]

How many thousands are in 1 million? [1000 thousands]

3. Use a place value chart if necessary.

Millions	Hundred Th.	Ten Th.	Thousand	Hundreds	Tens	Ones

Record numerals on place value chart.

Discuss relationships between:

Ten Thousand and One Thousand
One Hundred Thousand and Ten Thousand
One Million and One Hundred Thousand

4. Write a sentence such as the following:

In 1958, the population of New York City was approximately 7891900.

Ask children:

How can the numeral be written so that it would be easier to read? [Leave spaces; use commas; mark off periods]

How are periods named? [Ones or units period; thousands period; millions period]

How are the places in each period named? [Hundreds, tens, ones]

Use a chart as children explain.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O

Which period is not named when a numeral is read? [Units or ones]

5. Have children read the following numerals:

62,471,038 2,596,204 86,003,001

Why is the period farthest to the left the only period that may have one or two digits in it rather than three?

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Write numerals for the following. Use commas to set off periods.

Fifty thousand, fifty.

Four hundred sixty-one thousand, seven hundred sixty eight.

Two million, two hundred.

2. Encircle the numerals where commas are placed incorrectly. Rewrite each to make it correct.

5,371,560 293,45 53,74,357 4,29,543,297

3. Place digits for 5073806 in proper columns. Then read the numeral.

[5 million, 73 thousand, 8 hundred six]

Millions	Hundred Th.	Ten Th.	Thousands	Hundreds	Tens	Ones
[5]	[0]	[7]	[3]	[8]	[0]	[6]

4. Rewrite the numeral inserting commas to mark off periods. [5,073,806]

5. Rewrite the following indicating periods.

5550500 5050005 55005550 555500050

6. Mark the following True or False. If false, make the statement true.

   a. The largest number that can be expressed by a Hindu-Arabic numeral of eight digits is one less than 10,000,000.

   b. There could be some counting numbers that are so large that new symbols would be needed to express them.

OPERATIONS

UNIT 30 - SET OF WHOLE NUMBERS: ADDITION AND SUBTRACTION; FACTS; HORIZONTAL FORMAT; PROPERTIES APPLIED

Objectives: To help children develop skill in adding and subtracting whole numbers.

To make sure children can apply the Commutative and Associative Properties of Addition.

To make sure that children understand that subtracting a number is the inverse operation of adding that same number.

TEACHING SUGGESTIONS

1. Discuss addition as an operation.
   Compare adding a number to a given number, with combining the elements of a set to another set.

2. Test and / or drill where necessary to maintain automatic response to addition and subtraction facts.

   Develop skill in applying addition and subtraction facts to numbers through thousands. Use the horizontal format. Have children record sums and remainders only.

3. Present open sentences to evaluate children's understanding of mathematical properties

   For example,

   a. Children complete the following and explain

   $36 + 53 = 53 + \square$ Why?
   $36 + 53 = 36 + (\square + 3)$ Why?
   $36 + 53 = (30 + \square) + (6 + \triangle)$ Why?

   $89 - 53$ is the inverse operation of $36 + \square$ Why?
   $36 + \square$ is the inverse operation of $89 - \square$ Why?

b. Children find the missing addend "mentally"

$\square + 42 = 69$
$83 = 47 + \square$
$64 + \square = 113$
$122 = \square + 85$

$\square + 35 = 161$
$63 + \square = 281$
$485 = 129 + \square$
$\square + 338 = 565$

$\square + \$56 = \$87$
$\$20 = \square + \$75$
etc.

c. Ask children to find the missing term.

$\square - 23 = 75$
$57 = \square - 35$

$\square - 65 = 87$
$\square - 374 = 233$

$\square - \$24 = \$56$
$\$105 = \square - \$57$

d. Applying the Associative Property of Addition in "Mental Computation"

Ask children to solve the equations below applying the Associative Property.

For example,

$$\begin{aligned} 68 + 43 &= 68 + (40 + 3) \\ &= (68 + 40) + 3 \end{aligned}$$

or

$$\begin{aligned} 68 + 43 &= (60 + 8) + (40 + 3) \\ &= (60 + 40) + (8 + 3) \end{aligned}$$

$32 + 95 = n$ $324 + 59 = n$ $276 + 138 = n$, etc.

They explain their reasoning in each case.

4. Suggested exercises for adding and subtracting "mentally"

a. $360 + 80 = n$
$364 + 80 = n$
$364 + 85 = n$
$364 + 88 = n$

$240 + 130 = n$
$246 + 133 = n$
$246 + 138 = n$
$431 + 259 = n$

b. A similar series for subtraction should be established. For example,

$210 - 50 = n$
$210 - 54 = n$
$218 - 54 = n$
$218 - 59 = n$

$240 - 120 = n$
$376 - 130 = n$
$376 - 134 = n$
$376 - 137 = n$

Both series should be extended to include numbers in the thousands.

Drill should first be given in adding tens to thousands, then hundreds to thousands, etc.

c. Suggested exercises involving dollars and cents

Adding dollars	Adding dollars and cents
$4.16 + $2 = n	$4.20 + $4.20 = n
$23.89 + $5 = n	$3.46 + $2.10 = n
$68.19 + $4 = n	$14.61 + $3.30 = n
	$37.43 + $4.20 = n
	$4.61 + $4.61 = n

Subtracting dollars	Subtracting dollars and cents
$6.16 - $2 = n	$5.56 - $2.10 = n
$28.89 - $5 = n	$8.40 - $4.20 = n
$72.19 - $4 = n	$17.91 - $3.30 = n
	$41.63 - $4.20 = n
	$9.22 - $4.60 = n

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. How many books were borrowed by 5th grade pupils on Monday? Where will this number be placed in the table?

NUMBER OF BOOKS BORROWED BY 5TH GRADES FROM SCHOOL LIBRARY

	Mon.	Tues.	Wed.	Thurs.	Fri.	Total
Class 5-1	9	8	16	12	29	
Class 5-2	6	8	15	10	34	
Class 5-3	3	9	12	13	25	
Class 5-4	14	8	6	8	32	
Total						

How many books were borrowed by 5th grade pupils on Tuesday? on Wednesday? on Thursday? on Friday?

From your answers to 1 and 2, find the total number of books borrowed by 5th grade pupils during the week.

How many books did pupils from class 5-1 borrow during the week?

How many books did pupils from class 5-2 borrow during the week?

How many books did pupils from class 5-3 borrow during the week?

How many books did pupils from class 5-4 borrow during the week? etc.

2. Replace each placeholder with a numeral that will make the statement true.

$54 + 39 = 39 + \square = n$
$54 + 39 = 54 + \square + \Delta = n$
$54 + 39 = 54 + \square - 1 = n$
$54 + 39 = 50 + 40 + 4 - \square = n$

3. Without adding tell how you know that each of the following is a true sentence.

$3679 + 2000 = 2000 + 3679$

$153 + (10 + 6) = (153 + 10) + 6$

$968 = 968 + 0$

4. Further practice exercises may be found in textbooks and other printed material.

OPERATIONS

UNIT 31 - ADDITION AND SUBTRACTION OF WHOLE NUMBERS; VERTICAL FORMAT; PROPERTIES APPLIED

Objective: To maintain skill in:
Renaming numbers
Adding and subtracting whole numbers in vertical format

TEACHING SUGGESTIONS

1. Test children's ability to solve addition and subtraction exercises involving numbers as suggested in Unit 23.

2. Continue to develop or to provide practice in addition with 3-place numerals, sums in the thousands, and with numerals representing quantities of money, addends through $9.99.

   Present problem situations.

   Have children estimate, compute, then compare solution with estimate **and check.**

   Some suggested exercises:

$$\begin{array}{r} 436\\ 325\\ \underline{547} \end{array} \qquad \begin{array}{r} 568\\ 207\\ 472\\ \underline{93} \end{array} \qquad \begin{array}{r} 39\\ 256\\ 947\\ \underline{81} \end{array} \qquad \begin{array}{r} \$2.24\\ 7.65\\ 8.36\\ \underline{.59} \end{array} \qquad \begin{array}{r} \$3.15\\ 4.59\\ 7.28\\ \underline{1.50} \end{array} \qquad \text{etc.}$$

3. Extend renaming numbers to include thousands.
   For example, 4030 may be renamed as:

   a. 3 thousands + 10 hundreds + 3 tens + 0 ones; or
   3 thousands + 10 hundreds + 2 tens + 10 ones; or
   3 thousands + 9 hundreds + 12 tens + 10 ones; or
   39 hundreds + 13 tens + 0 ones

b. Direct children to:
Complete exercises like the following

4321 = ...thousands + ...hundreds + ...tens + . . . ones
4321 = ...hundreds + ...ones
4321 = ...tens + ...ones

54321 = ...ten-thousands + . . . thousands + . . . hundreds +
. . . tens + . . . ones
54321 = ...thousands + ... tens + ... ones

c. Record the following in expanded notation

39 hundreds + 13 tens + 0 ones [3900 + 130 + 0 = 4030]
43 hundreds + 3 ones
24 thousands + 7 tens + 8 ones

4. Provide practice in renaming numbers.
Ask children to complete the following:

2526 = 2000 + 500 + 20 + □
4000 = 3000 + □ + 90 + 10
3802 = □ + 1800 + 0 + 2
6000 = □ hundreds + 10 tens + 0 ones

5. Ask children to estimate sums and remainders, then complete and compare.

Sums:

Continue to emphasize estimating prior to written computation

761 Add hundreds only. Estimate: 1400 + **or**
376 Add hundreds, then tens: (1400 + 180). Estimate: 1580 **or**
459 Add the first two digits: 76 (tens) + 37 (tens) +
45 (tens). Estimate: 1580 (158 tens)

3671 Add thousands only. Estimate: 6000 + or
2298 Add thousands, then hundreds: 6000 + 1300. **Estimate: 7300 + or**
1538 Add the first two digits: 36 (hundreds) + 22 (hundreds) +
15 (hundreds). Estimate: 73 hundred or 7300

Remainders:
6279 Subtract thousands only. Estimate: about 4000.
- 2930 Subtract 3000 from 6000. Estimate: about 3000.
Subtract 3000 from 6279. Estimate: about 3279.
Subtract 29 (hundred) from 62 (hundred). Estimate:
3300 (33 hundred).

From time to time ask children at varying levels of ability to tell how they arrived at their estimates. Mature children should be encouraged to use mature methods of estimation. Children unable to make reasonable estimates should use smaller numbers.

6. Continue to develop skill in addition and subtraction.

Addition with 4-place numerals - 2, 3, 4 addends - sums in the thousands and ten thousands - first 1 exchange, then 2, 3, 4 exchanges; with numerals representing quantities of money - no addend larger than $99.99 - first with 1 exchange, then 2, 3, 4 exchanges.

Some suggested exercises.

4118	298	2814	2376	4625	$34.98	$52.28
2457	3665	2498	8924	537	68.50	19.79
	2033	1573		3869		8.54
		2865		5386		

Extend addition development to include:

5-place numerals - 2, 3, 4 addends - sums in the ten thousands - 1, 2, 3, 4 exchanges

Numerals representing quantities of money - maximum sum $999.99 - 1, 2, 3, 4 exchanges

Some suggested exercises

25,694	$57.96	16,385	
13,835	43.85	42,031	
3,610	25.19	31,400	
20,360		6,193	etc.

Subtraction

Continue to develop or provide practice in subtracting from numbers through 9999; through $99.99 - 1 and 2 exchanges.

Exchanges should include as many as 2 zeros.

Extend development to include subtracting from:

Numbers through 99,999; 1 and 2 exchanges; a maximum of 2 zeros.
Numbers through $999.99; 1 and 2 exchanges

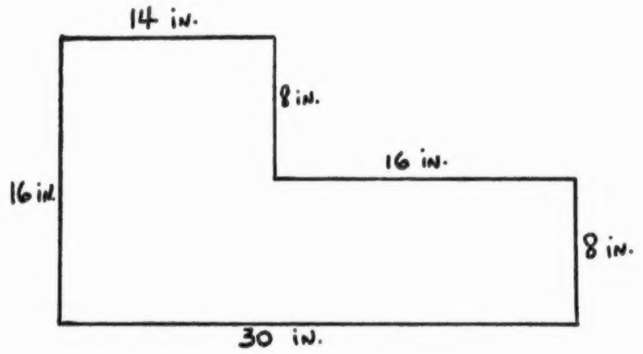
Some suggested exercises

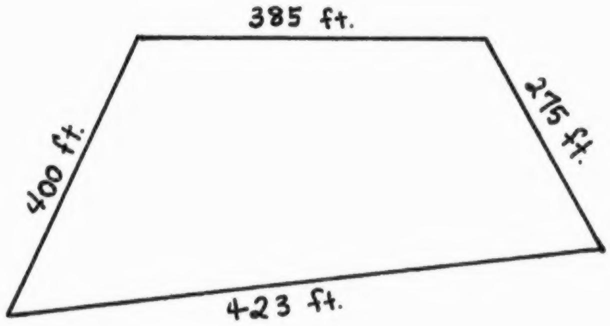
58, 780	15, 759	$600.79
- 42, 352	- 9, 241	- 124.26

EVALUATION and / or PRACTICE

SUGGESTED EXERCISES

1. Find perimeters of polygons by adding.


14 in.
8 in.
16 in.
16 in.
8 in.
30 in.


385 ft.
400 ft.
275 ft.
423 ft.

3. A rectangle measures 8 inches on each of two sides. A third side measures 6 inches. What is the length of the fourth side?

4. Provide for maintenance of computational skills. Refer to texts.

   Children continue to estimate sums and remainders. Verify sums and remainders.

   Select suitable verbal problems from textbooks and from other curriculum areas.

SETS; NUMBER; NUMERATION

UNIT 32 - SET OF FRACTIONAL NUMBERS: SIXTHS; RELATED TO HALVES AND THIRDS; CONCEPTS; COUNTING.

Objectives: To develop understanding of sixths.

To extend understanding of relationships among fractions including inequalities.

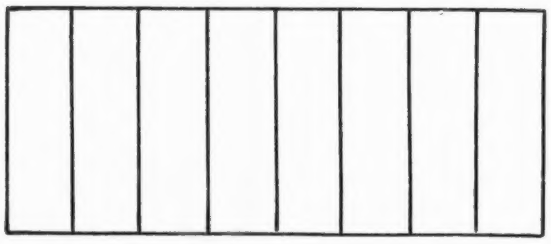
To develop understanding of whole numbers named as fractions.

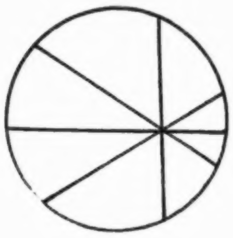
TEACHING SUGGESTIONS

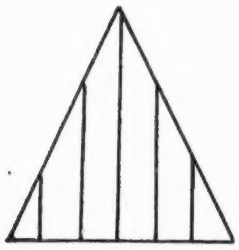
1. Continue to use number lines to reinforce relationships among halves, halves and fourths, halves, fourths and eighths, fourths and eighths.

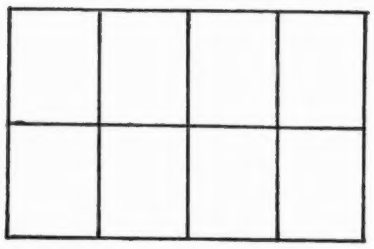
   Suggested exercises for evaluation

   a. Check the diagrams that show which region has been divided into eighths.

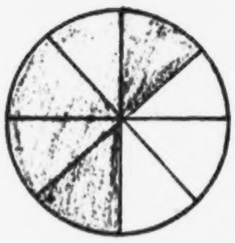


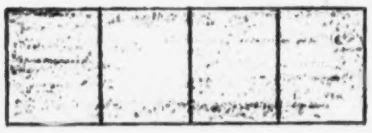


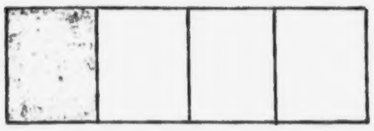


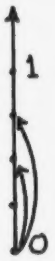


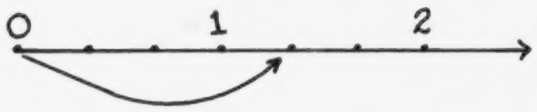
b. Record symbols to indicate the size of the shaded and indicated parts of the diagrams below.











c. Write the fraction that is twice as large as $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$

Arrange in order of size: $\frac{1}{4}$, $\frac{1}{8}$, $\frac{5}{8}$, $\frac{1}{3}$, $\frac{4}{3}$, $\frac{1}{3}$

d. Circle the fractions in each line which have the same value as the first fraction:

$\frac{1}{2}$ - $\frac{2}{3}$ $\frac{2}{4}$ $\frac{4}{8}$

$\frac{1}{4}$ - $\frac{2}{8}$ $\frac{4}{8}$ $\frac{1}{2}$

$\frac{4}{8}$ - $\frac{3}{2}$ $\frac{2}{3}$ $\frac{2}{4}$

e. Show the following on a number line

$\frac{1}{2}$ is the same point as $\frac{2}{4}$ or $\frac{4}{8}$ on the number line

$\frac{6}{8}$ is equivalent to $\frac{3}{4}$ or $\left(\frac{1}{2} + \frac{3}{4}\right)$

$\frac{3}{3}$ is equivalent to 1 whole

$\frac{6}{3}$ is equivalent to 2 wholes

$\frac{7}{3}$ is equivalent to 2 wholes and 1 third $\left(2\frac{1}{3}\right)$, etc.

f. Circle the larger (or smaller) of the following pairs:

$\frac{1}{3}$	$\frac{1}{4}$	$\frac{6}{4}$	$1\frac{1}{4}$
$\frac{3}{8}$	$\frac{1}{4}$	$2\frac{1}{8}$	$\frac{9}{4}$
$\frac{3}{2}$	$\frac{4}{4}$	etc.	

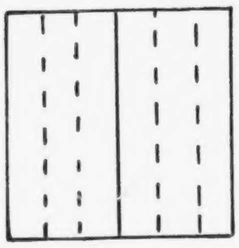
2. Develop concept of sixths

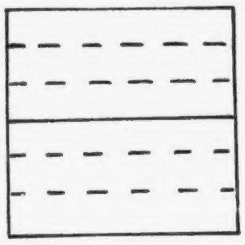
Problem: You have several sheets of paper of the same size. Fold each into 6 equal parts in different ways.

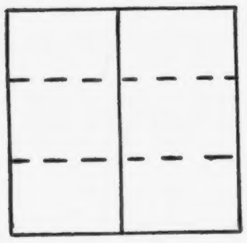
Ask children to describe in their own words how they derived sixths.

Some explanations might be

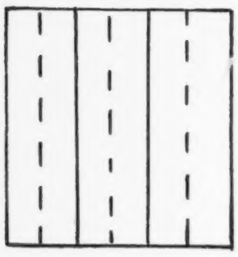
a. Finding halves, then dividing each half into 3 equal parts.

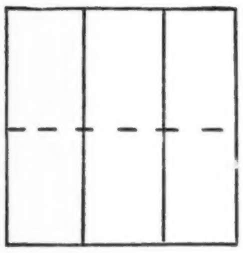


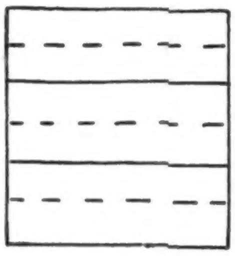




b. Finding thirds, then dividing each third in half



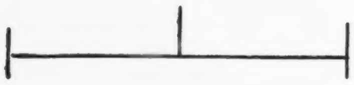




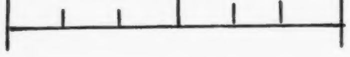
Ask children to identify the name of the part and to tell how they discovered that each part is 1 sixth of its original unit despite difference in shapes.

3. Use number lines to extend concepts of sixths in relation to halves and thirds.

Have children draw a line segment to represent a unit. They divide the unit into 2 equal parts.



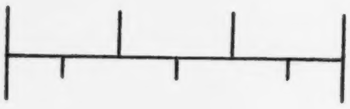
Children suggest ways to indicate sixths on the line segment divided into 2 equal parts.



Discuss dividing the line segment into halves first, then each half into 3 equal parts.

Ask children to draw a line segment to represent one unit. They divide the unit into 3 equal parts.

Children suggest ways to indicate sixths on the line segment divided into 3 equal parts.



Discuss dividing the line into thirds first, then each third into 2 equal parts.

Discuss the following comparisons.

Sixths of the same or of equal wholes are the same size

One sixth derives its name because it is one of 6 equal parts of a whole

There are 6 sixths in a whole

Discuss the following relationships. Use number lines. How are sixths related to thirds and to halves?

$\frac{1}{6}$ is half as large as $\frac{1}{3}$, or $\frac{1}{6} = \frac{1}{2}$ of $\frac{1}{3}$

$\frac{1}{3}$ is twice the size of $\frac{1}{6}$, or $\frac{1}{3} = 2$ one sixths

$\frac{1}{6}$ is $\frac{1}{3}$ the size of $\frac{1}{2}$, or $\frac{1}{6} = \frac{1}{3}$ of $\frac{1}{2}$

$\frac{1}{2}$ is 3 times as large as $\frac{1}{6}$, or $\frac{1}{2} = \frac{1}{6}$ taken 3 times or 3 one sixths

1 whole is 6 times as large as $\frac{1}{6}$, or $1 = \frac{1}{6}$ taken 6 times or 6 one sixths

Have children complete the statements below

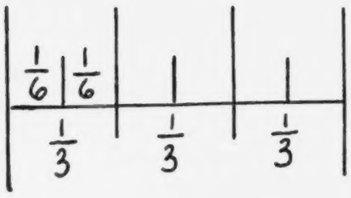
$\frac{1}{6}$ = ☐ of $\frac{1}{2}$

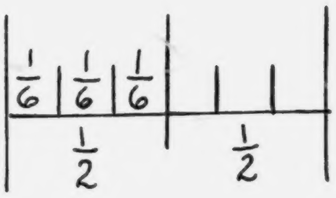
$\frac{1}{2}$ is ☐ times $\frac{1}{6}$

$\frac{1}{3}$ is ☐ times $\frac{1}{6}$

1 is ☐ times $\frac{1}{6}$

4. Emphasize equivalencies between:


Sixths and Thirds
$\frac{1}{6}$
$\frac{1}{6}$
$\frac{1}{3}$
$\frac{1}{3}$
$\frac{1}{3}$


Sixths and Halves
$\frac{1}{6}$
$\frac{1}{6}$
$\frac{1}{6}$
$\frac{1}{2}$
$\frac{1}{2}$

Have children complete the following open sentences.

$$\frac{2}{6} = \frac{\square}{3}$$

$$\frac{3}{6} = \frac{1}{3} + \frac{\square}{6}$$

$$\frac{4}{6} = \frac{\square}{3}$$

$$\frac{5}{6} = \frac{2}{3} + n$$

$$\frac{3}{6} = \frac{\square}{2}$$

$$\frac{4}{6} = \frac{1}{2} + n$$

$$\frac{5}{6} = \frac{1}{2} + n$$

$$\frac{5}{6} = \frac{1}{2} + \frac{?}{3}$$

$$\frac{7}{6} = 1 + n$$

$$\frac{8}{6} = 1 + \frac{n}{6}$$

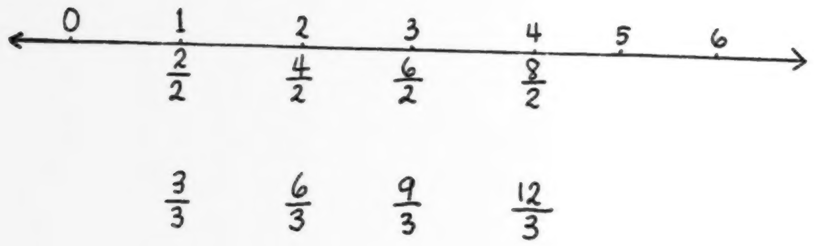
$$\frac{8}{6} = 1 + \frac{n}{3}$$

Ask children to rename one of the fractions in each of the following sets to make the denominators the same.

$\frac{1}{2}$, $\frac{5}{6}$; $\left[\frac{3}{6}\right]$ $\frac{2}{3}$, $\frac{2}{6}$; $\frac{4}{6}$, $\frac{2}{3}$; etc.

5. Extend concepts of equivalent fractions.

On the number line below, rename 1, 2, 3, 4, 5, 6 as halves; as thirds; as fourths; sixths; eighths.



Name other halves in the interval between 0 and 6. $\left[\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \text{etc.}\right]$
Other thirds in the interval between 0 and 6. etc.

What does 2 in the denominator tell us? 3? , 4? , etc.

6. Extend concepts of fractions to include denominators of one.

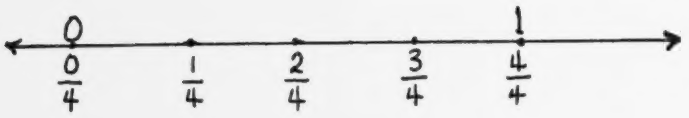
Ask children:

> If 2 units of length are not divided into halves, thirds, fourths, etc. but are left whole, what denominator could be used to show this? [1]
>
> What does the fraction $\frac{6}{1}$ indicate? Why?

Direct children to:

> Write 2 wholes as a fraction with a denominator of 1 $\left[\frac{2}{1}\right]$;
>
> 3 wholes $\left[\frac{3}{1}\right]$; 4 wholes $\left[\frac{4}{1}\right]$; 1 whole $\left[\frac{1}{1}\right]$; etc.
>
> Draw another number line renaming 1, 2, 3, 4 in 5 different ways.

7. Use the number line to answer the questions below.



In the fraction $\frac{0}{4}$, what do the denominator and the numerator tell us?

If the line had been divided into halves, where would you record no halves? [at the zero point] How? $\left[\frac{0}{2}\right]$

What whole number does $\frac{0}{2}$ represent?

If the numerator of any fraction is zero, what whole number does that fraction represent? [0]

8. Compare with other fractions:

Children compare sixths with other fractions.

Extend understanding through use of circular materials, then number lines.

$\frac{1}{6} > \frac{1}{8}$	Why?	$\frac{1}{6} < \frac{1}{4}$	Why?	
$\frac{5}{6} > \frac{1}{2}$	Why?	$\frac{3}{6} < \frac{2}{3}$	Why?	etc.

Ask children to write " > " or " < " between each group of fractional numerals below to make a true statement.

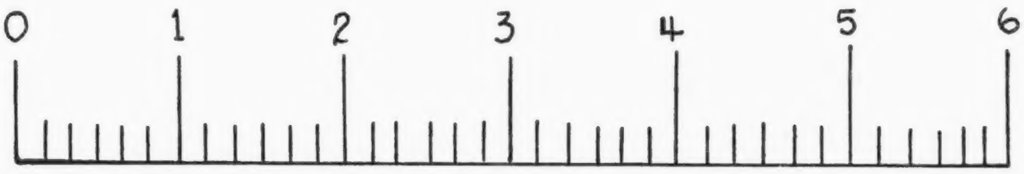
$\frac{2}{6} \square \frac{1}{4}$; $\frac{3}{4} \square \frac{3}{6}$; $\frac{3}{6} \square \frac{1}{8}$; $\frac{1}{2} \square \frac{5}{6}$; etc.

Draw number lines to show that each relation is correct.

9. Counting

Reinforce counting forward and backward by halves, fourths and eighths as suggested in Topic 15.

Ask children to use number lines to count forward and backward by thirds and sixths.



$\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3} \ldots$ Later: $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3} \ldots$

$\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6} \ldots$ Later: $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6} \ldots$

$2\frac{1}{6}, 2\frac{2}{6}, 2\frac{3}{6}, 2\frac{4}{6} \ldots$ Later: $2\frac{1}{6}, 2\frac{1}{3}, 2\frac{1}{2}, 2\frac{2}{3} \ldots$

$3\frac{4}{6}, 4, 4\frac{2}{6}, 4\frac{4}{6}, 5 \ldots$ Later $3\frac{2}{3}, 4, 4\frac{1}{3}, 4\frac{2}{3}, 5 \ldots$

Have children start at any point:

Count backward by thirds and sixths
Count forward and backward by groups of 1 half, 1 third, 1 fourth, 1 sixth, etc. For example:

$0, \frac{2}{3}, \frac{4}{3}, \frac{6}{3}, \frac{8}{3}, \frac{10}{3}, \frac{12}{3}, \ldots$

$0, \frac{2}{3}, 1\frac{1}{3}, 2, 2\frac{2}{3}, 3\frac{1}{3}, 4, \ldots$

10. Terminology

Provide for a growing mathematical vocabulary by introducing and using meaningfully the following terms: fraction, numerator, denominator, fractional numeral, whole number and fraction.

Numerator and Denominator

3 (numerator) - the number of parts considered

4 (denominator) - the number of equal parts into which the whole has been divided.

Whole Number indicates a number of units. 1 7 253 etc.

SETS; NUMBER; NUMERATION

UNIT 33 - SET OF FRACTIONAL NUMBERS: TWELFTHS; NINTHS; SEVENTHS

Objective: To develop the understanding of concept of twelfths, ninths and sevenths.

TEACHING SUGGESTIONS

Twelfths

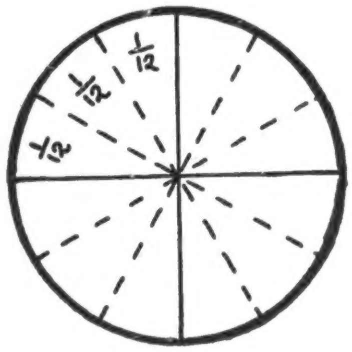
1. Relate to halves and fourths; halves and sixths; thirds and sixths. Use circle and number line diagrams.

2. Suggested problem:

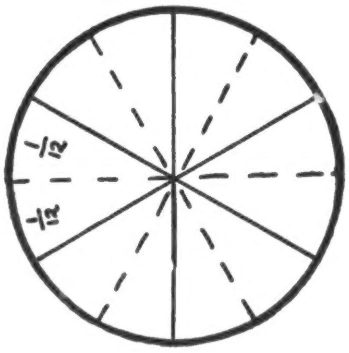
   Mother bought a pie large enough for 12 people. How can she cut it so that each person will receive the same amount?

   Children may draw circles or line segments. By experimenting they will discover various ways to arrive at twelfths. Why are these equal parts called twelfths?

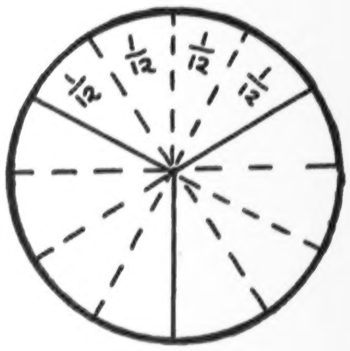
Divide into halves,
then fourths,
then twelfths



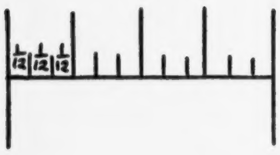
Divide into halves,
then sixths,
then twelfths



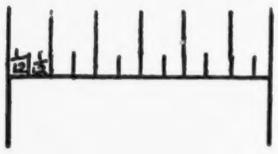
Divide into thirds,
then sixths,
then twelfths



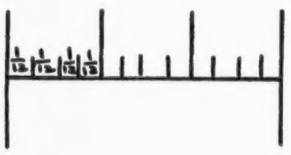
Each fourth is divided into 3 equal parts.



Each half is divided into 3 equal parts, then each sixth is divided into 2 equal parts.



Each third is divided into 2 equal parts, then each sixth is divided into 2 equal parts.



3. Suggested questions to ask:

How did we obtain:

$\frac{1}{12}$ from $\frac{1}{4}$? $\frac{1}{12}$ is what part of $\frac{1}{4}$? $\left[\frac{1}{12} = \frac{1}{3} \text{ of } \frac{1}{4}\right]$

$\frac{1}{12}$ from $\frac{1}{6}$? $\frac{1}{12}$ is what part of $\frac{1}{6}$? $\left[\frac{1}{12} = \frac{1}{2} \text{ of } \frac{1}{6}\right]$

$\frac{1}{12}$ from $\frac{1}{3}$? $\frac{1}{12}$ is what part of $\frac{1}{3}$? $\left[\frac{1}{12} = \frac{1}{4} \text{ of } \frac{1}{3}\right]$

$\frac{1}{12}$ from $\frac{1}{2}$? $\frac{1}{12}$ is what part of $\frac{1}{2}$? $\left[\frac{1}{12} = \frac{1}{6} \text{ of } \frac{1}{2}\right]$

After the pie is cut into 12 pieces:

What part of this pie would 6 pieces be? $\left[\frac{1}{2}\right]$ Why? $\left[\frac{6}{12} = \frac{1}{2}\right]$

What part of this pie would 4 pieces be? $\left[\frac{1}{3}\right]$ Why? $\left[\frac{4}{12} = \frac{1}{3}\right]$

What part of this pie would 3 pieces be? $\left[\frac{1}{4}\right]$ Why? $\left[\frac{3}{12} = \frac{1}{4}\right]$

What part of this pie would 2 pieces be? $\left[\frac{1}{6}\right]$ Why? $\left[\frac{2}{12} = \frac{1}{6}\right]$

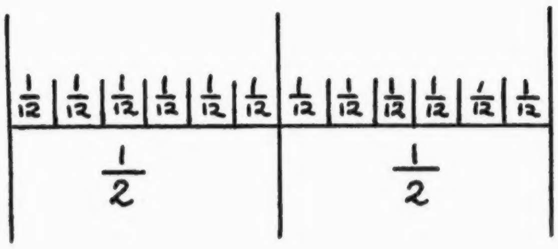
How many pieces are in $\frac{1}{4}$ of the pie? $[3]$ Why? $\left[\frac{1}{4} = \frac{3}{12}\right]$

How many pieces are in $\frac{1}{2}$ of the pie? $[6]$ Why? $\left[\frac{1}{2} = \frac{6}{12}\right]$

4. Equivalent Fractions

Use line diagrams to discover equivalents. Have children complete sentences.

Twelfths and Halves



$\frac{6}{12} = \square$

$\frac{12}{12} = \frac{\square}{2}$

$\frac{12}{12} = \square$

$\frac{6}{12} = \frac{1}{2}$

$\frac{7}{12} = \frac{1}{2} + \square$

$\frac{8}{12} = \frac{1}{2} + \frac{2}{\square}$

$\frac{9}{12} = \frac{1}{2} + \frac{\square}{12}$

$\frac{10}{12} = \square + \frac{4}{12}$

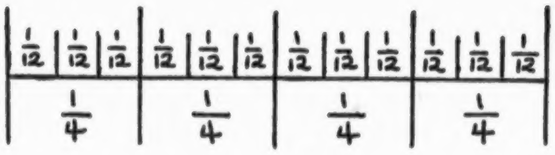
$\frac{11}{12} = \frac{1}{2} + \frac{5}{\square}$

$\frac{12}{12} = \frac{1}{2} + \frac{\square}{12}$

$\frac{12}{\square} = 1$

Since $\frac{6}{12} = \frac{1}{2}$ then $\frac{10}{12} = \frac{1}{2} + \frac{?}{12}$

Twelfths and Fourths



$\frac{3}{12} = \frac{\square}{4}$

$\frac{6}{12} = \frac{\square}{4}$

$\frac{9}{12} = \frac{\square}{4}$

$\frac{12}{12} = \frac{\square}{4}$

$1 = \frac{\square}{12}$

$\frac{4}{12} = \frac{1}{4} + \frac{\square}{12}$

$\frac{5}{12} = \frac{1}{4} + \frac{\square}{12}$

$\frac{7}{12} = \frac{2}{4} + \frac{\square}{12}$

$\frac{8}{12} = \frac{2}{4} + \frac{\square}{12}$

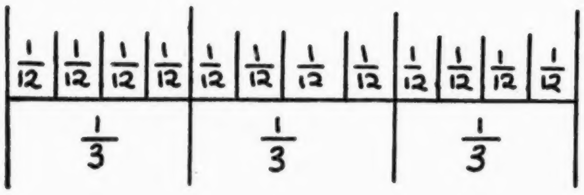
$\frac{10}{12} = \frac{3}{4} + \frac{\square}{12}$

$\frac{11}{12} = \frac{3}{4} + \frac{\square}{12}$

$\frac{12}{12} = \frac{3}{4} + \frac{\square}{12}$

Since $\frac{9}{12} = \frac{3}{4}$ then $\frac{11}{12} = \frac{3}{4} + ?$

Twelfths and Thirds



$\frac{4}{12} = \frac{\square}{3}$
$\frac{8}{12} = \frac{\square}{3}$
$\frac{12}{12} = \frac{\square}{3}$

$\frac{5}{12} = \frac{1}{3} + \frac{\square}{12}$
$\frac{6}{12} = \frac{\square}{3} + \frac{2}{12}$
$\frac{7}{12} = \frac{1}{3} + \frac{\square}{12}$

$\frac{9}{12} = \frac{\square}{3} + \frac{1}{12}$
$\frac{10}{12} = \frac{2}{3} + \frac{\square}{12}$
$\frac{11}{12} = \frac{2}{3} + \frac{\square}{12}$
$\frac{12}{12} = \frac{\square}{3} + \frac{4}{12} = \frac{2}{3} + \frac{\square}{3}$

Since $\frac{4}{12} = \frac{1}{3}$ then $\frac{5}{12} = \frac{1}{3} + ?$

Twelfths and Sixths - Use Line Diagrams

$\frac{2}{12} = \frac{\square}{6}$
$\frac{4}{12} = \frac{\square}{6}$
$\frac{6}{12} = \frac{\square}{6}$
$\frac{8}{12} = \frac{\square}{6}$
$\frac{10}{12} = \frac{\square}{6}$
$\frac{12}{12} = \frac{\square}{6}$

$\frac{3}{12} = \frac{1}{6} + \frac{\square}{12}$
$\frac{5}{12} = \frac{\square}{6} + \frac{1}{12}$
$\frac{7}{12} = \frac{\square}{6} + \frac{1}{12}$
$\frac{9}{12} = \frac{\square}{6} + \frac{1}{12}$
$\frac{11}{12} = \frac{\square}{6} + \frac{1}{12}$

Since $\frac{4}{12} = \frac{2}{6}$ then $\frac{8}{12} = \frac{?}{6}$

Have children insert the correct numerator:

$\frac{1}{2} = \frac{\square}{12}$
$\frac{1}{3} = \frac{\square}{12}$
$\frac{1}{4} = \frac{\square}{12}$
$\frac{1}{6} = \frac{\square}{12}$

$\frac{12}{12} = \frac{\square}{2}$
$\frac{12}{12} = \frac{\square}{3}$
$\frac{12}{12} = \frac{\square}{4}$
$\frac{12}{12} = \frac{\square}{6}$

5. Comparisons and Relationships
Use number lines. Have children discover the following relationships:

$\frac{1}{12}$ is half as large as $\frac{1}{6}$, or $\frac{1}{12} = \frac{1}{2}$ of $\frac{1}{6}$

$\frac{1}{6}$ is twice as large as $\frac{1}{12}$, or $\frac{1}{6}$ = 2 one-twelfths

$\frac{1}{12}$ is 1 fourth as large as $\frac{1}{3}$, or $\frac{1}{12} = \frac{1}{4}$ of $\frac{1}{3}$

$\frac{1}{3}$ is 4 times as large as $\frac{1}{12}$, or $\frac{1}{3}$ = 4 one-twelfths or $4 \times \frac{1}{12}$

$\frac{1}{12}$ is 1 third as large as $\frac{1}{4}$, or $\frac{1}{12} = \frac{1}{3}$ of $\frac{1}{4}$

$\frac{1}{4}$ is three times as large as $\frac{1}{12}$, or $\frac{1}{4}$ = 3 one-twelfths or $3 \times \frac{1}{12}$

$\frac{1}{12}$ is 1 sixth as large as $\frac{1}{2}$, or $\frac{1}{12} = \frac{1}{6}$ of $\frac{1}{2}$

$\frac{1}{2}$ is 6 times as large as $\frac{1}{12}$, or $\frac{1}{2}$ = 6 one-twelfths or $6 \times \frac{1}{12}$

Compare twelfths with halves, fourths, sixths, etc. Have children insert symbols " < ", " > ", between each of the following pairs of fractional numbers:

$\frac{3}{12}$ □ $\frac{1}{2}$; $\frac{3}{4}$ □ $\frac{10}{12}$; $\frac{5}{6}$ □ $\frac{5}{12}$; $\frac{3}{8}$ □ $\frac{3}{12}$; etc.

6. Present a number ray. Ask children to locate the following points on the ray.

$\frac{3}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{9}{12}$; $1\frac{1}{12}$, $1\frac{5}{12}$, $2\frac{2}{12}$, $4\frac{7}{12}$, etc.

Have children find the distance between

$\frac{3}{12}$ and $\frac{9}{12}$; $\frac{4}{12}$ and $\frac{11}{12}$; $\frac{9}{12}$ and $1\frac{1}{12}$; $1\frac{4}{12}$ and $1\frac{5}{12}$

7. Fractions in Series; Using Number Line

Count forward and backward by one-twelfth; then change to equivalent fractions or mixed form.

Count forward and backward by one-twelfth, starting at any point on the line. Count forward and backward by groups of $\frac{2}{12}$, then $\frac{3}{12}$, then $\frac{4}{12}$, etc.

For example:

Counting forward - Groups of $\frac{2}{12}$

$\frac{1}{12}$, $\frac{3}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{9}{12}$, $\frac{11}{12}$ . . . . then $\frac{1}{12}$, $\frac{1}{4}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{3}{4}$, $\frac{11}{12}$ . . .

$\frac{2}{12}$, $\frac{4}{12}$, $\frac{6}{12}$, $\frac{8}{12}$, $\frac{10}{12}$, $\frac{12}{12}$ . . . . then $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, 1, $1\frac{1}{6}$

Counting backward - Groups of $\frac{3}{12}$

$\frac{14}{12}$, $\frac{11}{12}$, $\frac{8}{12}$, $\frac{5}{12}$, $\frac{2}{12}$, ... then $1\frac{1}{6}$, $\frac{11}{12}$, $\frac{2}{3}$, $\frac{5}{12}$, $\frac{1}{6}$

Ask children

When counting by twelfths:

$1\frac{1}{12}$ follows __________ and comes before _______

Find the number that is 2 less than $5\frac{7}{12}$.

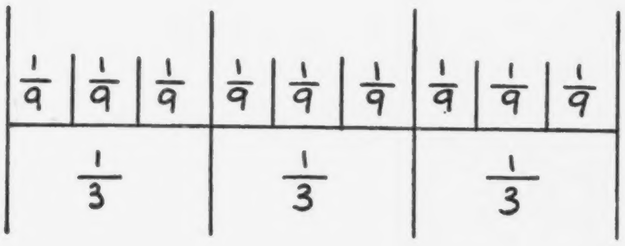
What number is one more than $3\frac{11}{12}$?

How many twelfths larger than $3\frac{1}{2}$ is $4\frac{1}{2}$?

How many twelfths must I add to $2\frac{1}{3}$ to reach the next whole number?

Ninths

1. Reinforce concept of thirds.
2. Children draw a line indicating a whole and discuss ways of dividing it into 9 equal parts. They discover that the line may be divided into thirds, then each third into 3 equal parts. A ninth is 1 third of 1 third.



Sevenths

Children draw a line indicating a whole, then divide it into seven equal parts.

Discuss reason why sixths, twelfths, etc., can be derived from halves, thirds, etc. but sevenths can not. Of what other fractional parts would this be true?

[elevenths, thirteenths, etc.]

EVALUATION and / or PRACTICE

SUGGESTED EXERCISES

1. Complete each of the following.

$\frac{6}{12} = \frac{?}{2}$ $\quad \frac{6}{8} = \frac{?}{4}$ $\quad \frac{9}{12} = \frac{\square}{4}$ $\quad \frac{4}{6} = \frac{\square}{3}$

$\frac{4}{6} = \frac{?}{3}$ $\quad 1\frac{5}{9} = \frac{?}{9}$ $\quad 2\frac{5}{6} = \frac{n}{6}$ $\quad \frac{3}{9} = \frac{?}{3}$

2. Change the following to equivalent fractions with smaller denominators.

$\frac{8}{12} =$ ____ $\quad \frac{6}{8} =$ ____ $\quad \frac{6}{9} =$ ____

3. Change the following to equivalent fractions.

$\frac{1}{2} = \frac{?}{6}$ $\quad \frac{3}{4} = \frac{?}{8}$ $\quad \frac{2}{3} = \frac{?}{9}$ $\quad \frac{2}{3} = \frac{?}{12}$

4. Change these fractions to whole numbers or to mixed form.

$\frac{7}{7} = n$ $\quad \frac{8}{3} = n + \frac{\square}{3}$

$\frac{16}{8} = n$ $\quad \frac{16}{12} = n + \frac{\square}{3}$

5. Fill in the missing blanks in the following series:

$\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ ____, $\quad \frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, ____, $1\frac{5}{6}$.

3, $2\frac{7}{8}$, $2\frac{3}{4}$, ____, $2\frac{1}{2}$.

6. Answer the following questions:

$\frac{1}{4}$ is what part of $\frac{1}{8}$?

$\frac{1}{12}$ is what part of $\frac{1}{2}$?

$\frac{1}{2}$ is how many times as big as $\frac{1}{8}$?

$\frac{3}{4}$ is how many times as big as $\frac{1}{4}$?

$\frac{3}{4}$ is how many times as big as $\frac{1}{8}$?

7. Place a circle around the fraction on each line which has the greatest value:

$\frac{1}{2}$ . . . . . . $\frac{1}{4}$ . . . . . . $\frac{1}{8}$

$\frac{3}{4}$ . . . . . . $\frac{7}{8}$ . . . . . . $\frac{1}{2}$

$\frac{3}{8}$ . . . . . . $\frac{5}{6}$ . . . . . . $\frac{2}{3}$

$\frac{3}{4}$ . . . . . . $\frac{2}{3}$ . . . . . . $\frac{5}{6}$

8. Compare the following. Tell which is larger (smaller).
How much larger (smaller).

$\frac{1}{4}$ and $\frac{1}{8}$ $\qquad$ $\frac{2}{3}$ and $\frac{5}{6}$

$\frac{3}{4}$ and $\frac{3}{8}$ $\qquad$ $\frac{7}{8}$ and $\frac{3}{4}$

OPERATIONS

UNIT 34 - ADDITION OF FRACTIONAL NUMBERS: HORIZONTAL AND VERTICAL FORMAT

Objectives: To develop horizontal and vertical algorithms for addition of fractional numbers with unlike denominators.

To apply the Commutative and Associative Properties of Addition.

TEACHING SUGGESTIONS

1. Reinforce meaning of fractional numbers. For example:

   $\frac{1}{2} + \frac{1}{2} = 1$ Is the sum a fractional number? Explain.

   [Yes. Any whole number can be expressed as a fraction. $1 = \frac{1}{1}$ etc.]

2. Children should understand that the properties of Addition for the set of Whole Numbers are true for Addition for the set of Fractional Numbers.

   Suggested exercises:

   a. Find the sum of any two whole numbers. What kind of number is the sum? [Whole Number]

   b. Find the sum of $\frac{1}{3} + \frac{1}{6}$. What kind of number is the sum? [Fractional Number]

   c. Add: $\frac{1}{8} + \frac{0}{8} = n$

   What do you notice about the sum? [It is the same as the first addend]

   Explain. [$\frac{0}{8} = 0$ therefore adding $\frac{0}{8}$ is the same as adding 0]

d. Use number lines to show that: $\frac{3}{4} + \frac{1}{8} = \frac{1}{8} + \frac{3}{4}$

What property does this illustrate? [Commutative]

e. Show that:

$$\frac{1}{2} + \left(\frac{1}{2} + \frac{1}{4}\right) = \left(\frac{1}{2} + \frac{1}{2}\right) + \frac{1}{4}$$

What property does this illustrate? [Associative]

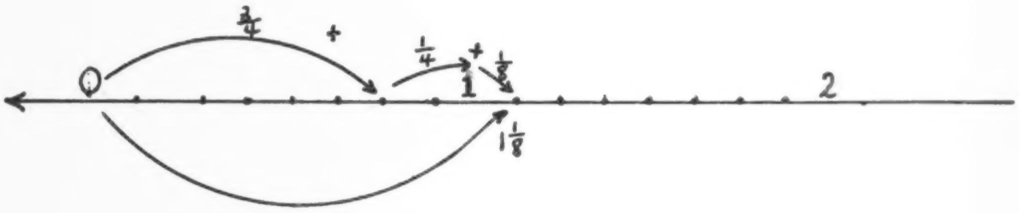
Explain.

Addition of Fractions With Unlike Denominators: Common Denominator Apparent; Horizontal Format

1. Introduce addition exercises through problem solving situations. Use number lines to help children in their thinking. For example:

Problem:

Mary needed 3 fourths of a yard of material for an apron and 3 eighths more for pockets and belt. How much cloth did she need? $\left(\frac{3}{4} + \frac{3}{8} = n\right)$



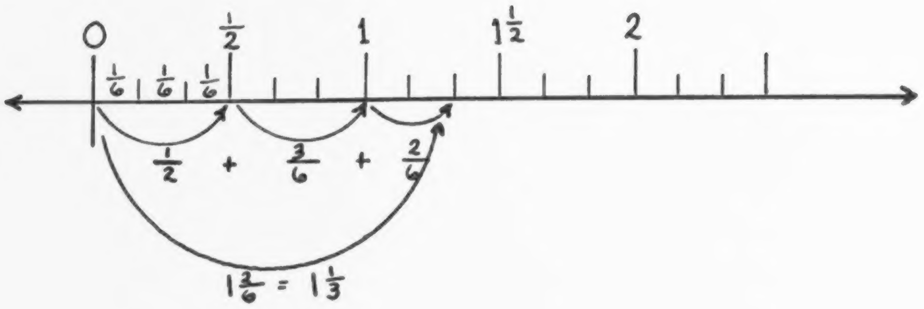
$$\frac{3}{4} + \frac{3}{8} \text{ as } \frac{3}{4} + \left(\frac{2}{8} + \frac{1}{8}\right) \text{; then}$$

$$\text{as } \left(\frac{3}{4} + \frac{1}{4}\right) + \frac{1}{8} \text{; as } 1,\ 1\frac{1}{8}$$

(Note use of Associative Property for Addition)

Have children find the solution for: $\frac{1}{2} + \frac{5}{6} = n$

a.


0
$\frac{1}{2}$
1
$1\frac{1}{2}$
2
$\frac{1}{6}$
$\frac{1}{6}$
$\frac{1}{6}$
$\frac{1}{2}$ + $\frac{3}{6}$ + $\frac{2}{6}$
$1\frac{2}{6} = 1\frac{1}{3}$

b. They should think through to complete the problem as follows:

$$\left(\frac{1}{2} + \frac{3}{6}\right) + \frac{2}{6} = 1\frac{2}{6} \qquad \text{or}$$

$$1\frac{2}{6} = 1\frac{1}{3}$$

or

$$\left(\frac{1}{2} + \frac{1}{2}\right) + \frac{1}{3} = 1\frac{1}{3}$$

Note the use of Associative Property for Addition.
$\frac{5}{6}$ was renamed as $\frac{3}{6} + \frac{2}{6}$.

2. Suggested type of practice exercises to maintain skill.
Use number line to check solutions.

Sixths and Wholes

$3\frac{1}{6} + 1 = n$ $\qquad 2\frac{3}{6} + 4 = n$

$4\frac{5}{6} + 2 = n$ $\qquad 3\frac{4}{6} + 2 = n$

Sixths and Halves

$\frac{2}{6} + \frac{1}{2} = n$ $\qquad 2\frac{1}{2} + \frac{5}{6} = n$

$\frac{1}{2} + \frac{4}{6} = n$ $\qquad 3\frac{1}{6} + 2\frac{1}{2} = n$

Sixths	Sixths and Thirds
$\frac{5}{6} + \frac{1}{6} = n$	$\frac{2}{6} + \frac{1}{3} = n$
$\frac{5}{6} + \frac{5}{6} = n$	$\frac{5}{6} + \frac{2}{3} = n$
$2\frac{1}{6} + \frac{5}{6} = n$	$2\frac{3}{6} + \frac{1}{3} = n$
$4\frac{2}{6} + 2\frac{3}{6} = n$	$3\frac{1}{6} + 2\frac{2}{3} = 5\frac{1}{6} + n = ?$

Encourage a variety of ways of evaluating.

For $5\frac{2}{3} + \frac{5}{6} = n$

$$\begin{aligned} &\left(5 + \frac{2}{3}\right) + \frac{5}{6} \\ &= 5 + \left(\frac{2}{3} + \frac{5}{6}\right) \\ &= 5 + \left(\frac{4}{6} + \frac{5}{6}\right) \\ &= 5\frac{9}{6} \\ &= 6\frac{1}{2} \end{aligned}$$

or

$$\begin{aligned} &5\frac{2}{3} + \left(\frac{2}{6} + \frac{3}{6}\right) \\ &= \left(5\frac{2}{3} + \frac{1}{3}\right) + \frac{1}{2} \\ &= 6\frac{1}{2} \end{aligned}$$

Vertical Format

1. Provide drill in renaming a whole number and a fraction. Include halves, fourths, eighths, thirds, sixths.

Have children complete the following:

$1\frac{5}{4} = 2\frac{\square}{4}$ $9\frac{5}{2} = 11\frac{?}{2}$ $13\frac{9}{8} = 14\frac{?}{8}$

$2\frac{5}{4} = 3\frac{\square}{4}$ $10\frac{4}{2} = n$ $18\frac{16}{8} = n$

$16\frac{5}{4} = 17\frac{\square}{4}$ $13\frac{8}{2} = n$ $10\frac{12}{8} = 11\frac{?}{8}$

$3 = 2\frac{\square}{3}$ $2 = 1\frac{\square}{6}$

$3\frac{1}{3} = 2\frac{\square}{3}$ $2\frac{1}{6} = 1\frac{\square}{6}$

$3\frac{2}{3} = 2\frac{\square}{3}$ $2\frac{2}{6} = 1\frac{\square}{6}$

$1\frac{9}{8} = \square\frac{1}{8}$; $3\frac{17}{8} = \square\frac{1}{8}$ $7\frac{4}{3} = 8\frac{\square}{n}$; $11\frac{8}{6} = 12\frac{\square}{n}$

2. Reinforce or introduce vertical algorithms for addition.

Have children first estimate or arrive at the sum through "mental" computation.

a. Fractions with Like Denominators

Sums Less than 1

$$\begin{array}{r} \frac{2}{8} \\ \frac{1}{8} \\ \underline{\frac{2}{8}} \\ \frac{5}{8} \end{array}$$

Sums Equal to or More than 1

$$\begin{array}{r} \frac{4}{8} \\ \underline{\frac{4}{8}} \\ \frac{8}{8} = 1 \end{array} \qquad \begin{array}{l} \frac{3}{8} \\ \frac{3}{8} \\ \underline{\frac{4}{8}} \\ \frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4} \end{array} \qquad \begin{array}{l} 8\frac{3}{8} \\ 7\frac{1}{8} \\ \underline{15\frac{5}{8}} \\ 30\frac{9}{8} = 31\frac{1}{8} \end{array}$$

b. Fractions with Unlike Denominators

Use known equivalents

Sums Less than 1

$$\begin{array}{ccc} \frac{1}{3} & = & \frac{2}{6} \\ \frac{1}{6} & = & \frac{1}{6} \\ \hline & & \frac{3}{6} = \frac{1}{2} \end{array}$$

Sums Equal to or More than 1

$$\begin{array}{ccc} \frac{7}{8} & = & \frac{7}{8} \\ \frac{3}{4} & = & \frac{6}{8} \\ \hline & & \frac{13}{8} = 1\frac{5}{8} \end{array} \qquad \begin{array}{ccc} \frac{2}{3} & = & \frac{4}{6} \\ \frac{5}{5} & = & \frac{5}{6} \\ \frac{1}{2} & = & \frac{3}{6} \\ \hline & & \frac{12}{6} = 2 \end{array}$$

c. Whole Number and Fraction

$$\begin{array}{r} 27 \\ 13\frac{2}{3} \\ \hline 40\frac{2}{3} \end{array} \qquad \begin{array}{l} \frac{3}{4} \\ 2\frac{1}{4} \\ \hline 2\frac{4}{4} = 3 \end{array}$$

$$\begin{array}{ccc} \frac{4}{6} & = & \frac{4}{6} \\ 5\frac{2}{3} & = & 5\frac{4}{6} \\ \hline & & 5\frac{8}{6} = 6\frac{2}{6} = 6\frac{1}{3} \end{array} \qquad \begin{array}{ccc} 7\frac{3}{4} & = & 7\frac{6}{8} \\ 2\frac{5}{8} & = & 2\frac{5}{8} \\ \hline & & 9\frac{11}{8} = 10\frac{3}{8} \end{array}$$

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Find the sum for each pair of numbers and write its simplest fraction name. Use algorithms.

a. $\frac{3}{4}$, $\frac{1}{2}$ $\left[\frac{5}{4} = 1\frac{1}{4}\right]$ $11\frac{2}{3}$, $\frac{5}{6}$ $\left[12\frac{3}{6} = 12\frac{1}{2}\right]$

$\frac{3}{8}$, $5\frac{1}{4}$ $\left[5\frac{5}{8}\right]$

b. $$\begin{array}{r} \frac{1}{2} \\ \frac{3}{8} \\ \frac{5}{8} \\ \hline \end{array} \quad \left[\frac{12}{8} = 1\frac{4}{8} = 1\frac{1}{2}\right]$$

$$\begin{array}{r} 4\frac{1}{2} \\ 7\frac{3}{4} \\ \hline \end{array} \quad \left[11\frac{5}{4} = 12\frac{1}{4}\right]$$

$$\begin{array}{r} 28\frac{2}{3} \\ 59\frac{5}{6} \\ \hline \end{array} \quad \left[88\frac{3}{6} = 88\frac{1}{2}\right]$$

2. A magic square is one in which you can perform the operation on the numbers vertically, horizontally or diagonally and always get the same number for a result.

Copy the square below.

a. Add the numbers named by the fractions in each column and record the sum for each column. [10]

b. Add the numbers named by the fractions in each row and record the sum for each row. [10]

c. Begin in lower left hand corner. Add the numbers named by the fractions diagonally. Record their sum. [10]

d. Begin in upper left hand corner. Add the numbers named by the fractions diagonally. Record their sum. [10]

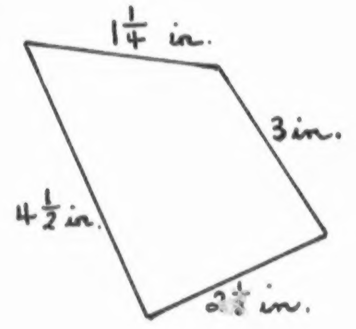
e. Is each sum the same fractional number? What is the number? [10]

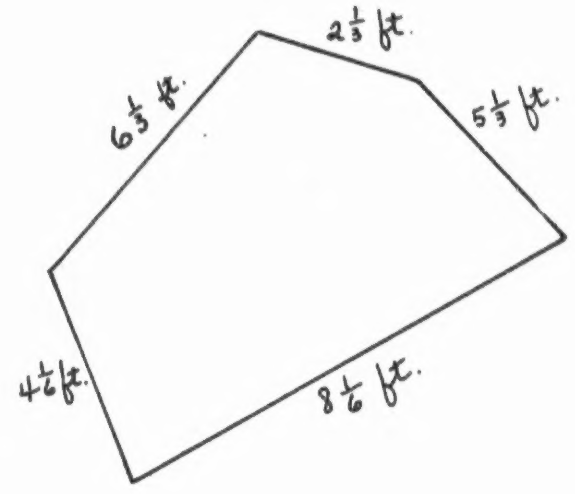
f. Is the square a magic square? [yes]

$2\frac{1}{2}$	$3\frac{3}{8}$	$\frac{1}{2}$	$1\frac{3}{8}$	$2\frac{1}{4}$
$3\frac{1}{4}$	1	$1\frac{1}{4}$	$2\frac{1}{8}$	$2\frac{3}{8}$
$\frac{7}{8}$	$1\frac{1}{8}$	2	$2\frac{7}{8}$	$3\frac{1}{8}$
$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{3}{4}$	3	$\frac{3}{4}$
$1\frac{3}{4}$	$2\frac{5}{8}$	$3\frac{1}{2}$	$\frac{5}{8}$	$1\frac{1}{2}$

3. John traveled $3\frac{1}{2}$ hours by plane and then $\frac{3}{4}$ of an hour by car to get to his grandmother's house. How many hours did he travel?

4. Find the perimeter of the following polygons.


$1\frac{1}{4}$ in.
3 in.
$4\frac{1}{2}$ in.
$2\frac{1}{8}$ in.


$2\frac{1}{3}$ ft.
$6\frac{1}{3}$ ft.
$5\frac{1}{3}$ ft.
$4\frac{1}{6}$ ft.
$8\frac{1}{6}$ ft.

$\left[10\ \frac{7}{8}\text{ in.}\right]$ $\left[26\ \frac{1}{3}\text{ ft.}\right]$

5. Write next to each step in the following algorithm either the property or operation involved. Indicate as well those steps in which renaming a fractional number occurred.

$4\frac{1}{2} + 7\frac{3}{4} = n$

a. $4\frac{1}{2} + 7\frac{3}{4} = \left(4 + \frac{1}{2}\right) + \left(7 + \frac{3}{4}\right)$

b. $= 4 + \left(\frac{1}{2} + 7\right) + \frac{3}{4}$ [associative property]

c. $= 4 + \left(7 + \frac{1}{2}\right) + \frac{3}{4}$ [Commutative property]

d. $= \left(4 + 7\right) + \left(\frac{1}{2} + \frac{3}{4}\right)$ [associative]

e. $= 11 + \left(\frac{2}{4} + \frac{3}{4}\right)$ [rename $\frac{1}{2}$; addition $(4 + 7)$]

f. $= 11 + \frac{5}{4}$ [addition]

g. $= 11 + \left(\frac{4}{4} + \frac{1}{4}\right)$ $\left[\text{renaming } \frac{5}{4}\right]$

h. $= 11 + \left(1 + \frac{1}{4}\right)$ $\left[\text{renaming } \frac{4}{4}\right]$

i. $= (11 + 1) + \frac{1}{4}$ [associative]

j. $= 12 + \frac{1}{4}$ [renaming 11 + 1]

k. $= 12\frac{1}{4}$ $\left[\text{renaming } 12 + \frac{1}{4}\right]$

6. Additional practice exercises may be found in textbooks and other printed material. Include problem situations that involve addition of fractional numbers.

OPERATIONS

UNIT 35 - SUBTRACTION OF FRACTIONAL NUMBERS: HORIZONTAL AND VERTICAL FORMAT

Objectives: To develop horizontal and vertical algorithms for subtraction of fractional numbers with unlike denominators.

To reinforce renaming numbers needed for subtraction with exchange.

To emphasize checking subtraction by adding.

TEACHING SUGGESTIONS

Halves, Fourths, Eighths, Thirds, Sixths

Horizontal Format - Subtraction of Fractional Numbers

1. Reinforce finding equivalent fractions for sixths, eighths, etc.

   For example: $\frac{6}{8} = \frac{3}{4}$; $\frac{7}{6} = 1\frac{1}{6}$; etc.

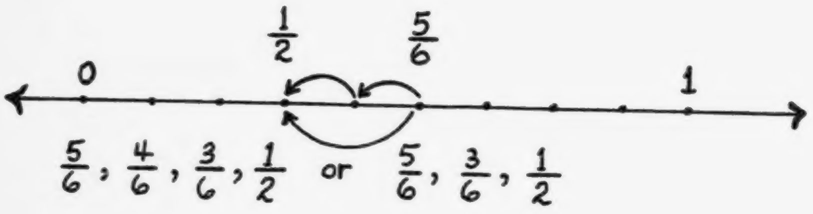
2. When children subtract "mentally" they generally begin with the entire minuend.

   Use number lines often.

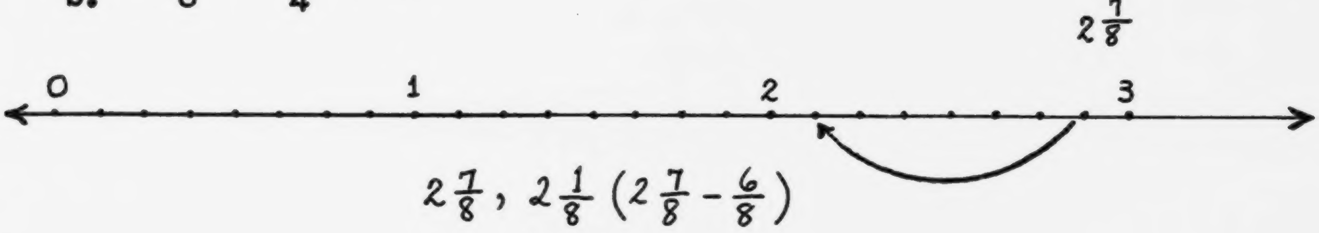
   Present a problem situation such as:

   a. Mary had $\frac{5}{6}$ of a yard of material. If she used $\frac{1}{3}$ of a yard to make a dress for her sister's doll, how much material did she have left?

$\frac{5}{6} - \frac{1}{3} =$


$\frac{1}{2}$
$\frac{5}{6}$
0
1
$\frac{5}{6}, \frac{4}{6}, \frac{3}{6}, \frac{1}{2}$ or $\frac{5}{6}, \frac{3}{6}, \frac{1}{2}$

b. $2\frac{7}{8} - \frac{3}{4} = n$



$2\frac{7}{8}$, $2\frac{1}{8}$ $\left(2\frac{7}{8} - \frac{6}{8}\right)$

3. Have children solve the following open sentences and explain their solutions.

$$3\frac{5}{6} - 1\frac{1}{2} = 2\frac{5}{6} - n = ?$$

$$8\frac{5}{6} - 2\frac{2}{3} = 6\frac{5}{6} - \square = n$$

$$24\frac{3}{4} - 12\frac{1}{2} = 12\frac{6}{6} - \frac{4}{8} = n$$

Have children complete the following:

Sixths	Sixths and Halves	Sixths and Thirds
$2\frac{1}{6} - 1 = \square$	$3\frac{1}{6} - 1 = \square$	$\frac{5}{6} - \frac{1}{3} = n$
$2\frac{1}{6} - \frac{1}{6} = \square$	$3\frac{1}{6} - \frac{1}{2} = \square$	$\frac{9}{6} - \frac{2}{3} = n$
$2\frac{1}{6} - 1\frac{1}{6} = \square$	$3\frac{1}{6} - 1\frac{1}{2} = \square$	$3\frac{4}{6} - \frac{1}{3} = n$
$2\frac{1}{6} - \frac{3}{6} = \square$	$3\frac{5}{6} - 1\frac{1}{2} = \square$	$8\frac{5}{6} - 2\frac{2}{3} = n$
$2\frac{1}{6} - 1\frac{3}{6} = \square$		

Verifying Solutions: Adding and Subtracting as Inverse Operations

Have children solve the additions below and check each by its related subtraction. For example:

$5\frac{2}{3} + \frac{5}{6} = n$ $\left[6\frac{1}{2}\right]$

$\frac{1}{2} - \frac{5}{6} = n$ $\left[5\frac{2}{3}\right]$

$2\frac{1}{8} + \frac{5}{8} = n$ $\left[2\frac{3}{4}\right]$

$2\frac{3}{4} - \frac{5}{8} = n$ $\left[2\frac{1}{8}\right]$

Vertical Format

1. Reinforce or introduce vertical algorithm.

No Exchange

Minuends Less Than 1

$$\begin{array}{r} \frac{7}{8} \\ -\frac{3}{8} \\ \hline \frac{4}{8} = \frac{1}{2} \end{array}$$

$$\begin{array}{rcl} \frac{3}{4} & = & \frac{6}{8} \\ -\frac{1}{8} & = & \frac{1}{8} \\ \cline{1-1}\cline{3-3} & & \frac{5}{8} \end{array}$$

Whole Number from Whole Number and Fraction

$$\begin{array}{r} 12\frac{5}{8} \\ -9 \\ \hline 3\frac{5}{8} \end{array}$$

$$\begin{array}{r} 12\frac{4}{6} \\ -7 \\ \hline 5\frac{4}{6} = 5\frac{2}{3} \end{array}$$

Fraction from Whole Number and Fraction

$$\begin{array}{r} 11\frac{4}{6} \\ -\frac{3}{6} \\ \hline 11\frac{1}{6} \end{array}$$

$$\begin{array}{rcr} 15\frac{7}{8} & = & 15\frac{7}{8} \\ -\frac{3}{4} & = & \frac{6}{8} \\ \cline{1-1}\cline{3-3} & & 15\frac{1}{8} \end{array}$$

Whole Number and Fraction from Whole Number and Fraction

$$\begin{array}{r} 18\frac{5}{6} \\ -9\frac{2}{6} \\ \hline 9\frac{3}{6} = 9\frac{1}{2} \end{array}$$

$$\begin{array}{rcr} 13\frac{7}{8} & = & 13\frac{7}{8} \\ -4\frac{3}{4} & = & 4\frac{6}{8} \\ \cline{1-1}\cline{3-3} & & 9\frac{1}{8} \end{array}$$

With Exchange - Related Denominators

1. Reinforce renaming.

Have children complete the following:

$3 = \frac{\square}{6}$ $\qquad 3 = 2\frac{\square}{6}$ $\qquad 3\frac{1}{6} = 2\frac{\square}{6}$

$15\frac{5}{6} = 14\frac{\square}{6}$ $\qquad 22\frac{3}{4} = 21\frac{\square}{4}$ $\qquad 9\frac{1}{2} = 8\frac{\square}{4}$

2. As children solve problems they should discuss reasons for regrouping:

a. Fraction from Whole number

$6 - \frac{1}{2} = n$ $\qquad 8 - \frac{3}{4} = n$

$$\begin{array}{rcr} 6 & = & 5\frac{2}{2} \\ -\ \frac{1}{2} & = & \frac{1}{2} \\ \hline & & 5\frac{1}{2} \end{array} \qquad\qquad \begin{array}{rcr} 8 & = & 7\frac{4}{4} \\ -\ \frac{3}{4} & = & \frac{3}{4} \\ \hline & & 7\frac{1}{4} \end{array}$$

Have children check their solutions.

b. Whole Number and Fraction from Whole Number

$8 - 3\frac{5}{6} = n$ $\qquad 14 - 8\frac{2}{3} = n$

$$\begin{array}{rcr} 8 & = & 7\frac{6}{6} \\ -\ 3\frac{5}{6} & = & 3\frac{5}{6} \\ \hline & & 4\frac{1}{6} \end{array} \qquad\qquad \begin{array}{rcr} 14 & = & 13\frac{3}{3} \\ -\ 8\frac{2}{3} & & 8\frac{2}{3} \\ \hline & & 5\frac{1}{3} \end{array}$$

Have children check solutions.

c. Whole Number and Fraction from Whole Number and Fraction

Problem: There were $12\frac{1}{4}$ yds. of material in the stockroom. Our class used $6\frac{3}{4}$ yds. for aprons. How much material was left over?

Children may interpret the problem as a mathematical sentence.

$$\left[12\frac{1}{4} - 6\frac{3}{4} = n\right]$$

Estimate:

$n < 6$ and $n > 5$ n is between 5 and 6 $5 < n < 6$

Algorithm:

$$\begin{array}{rcrcl} 12\frac{1}{4} & = & 11\frac{5}{4} & & \\ \underline{-\ 6\frac{3}{4}} & = & \underline{6\frac{3}{4}} & & \\ & & 5\frac{2}{4} & = & 5\frac{1}{2} \end{array}$$

d. Problems involving finding an equivalent fraction and then regrouping.

Problem: We had $3\frac{2}{3}$ yards of material. $\frac{5}{6}$ of a yard was cut off to make a tea apron. How much was left?

Children may interpret verbal problem as the open sentence:

$$3\frac{2}{3} - \frac{5}{6} = n$$

Estimate: $n > 2$

Algorithm:

$$\begin{array}{rcrcrl} 3\frac{2}{3} & = & 3\frac{4}{6} & = & 2\frac{10}{6} & \text{(Renaming and regrouping)} \\ \underline{-\ \frac{5}{6}} & = & \underline{-\ \frac{5}{6}} & = & \underline{-\ \frac{5}{6}} & \\ & & & & 2\frac{5}{6} & \end{array}$$

e. **Problem:** A baby weighed $7\frac{3}{4}$ pounds when he was born. At the end of the month he weighed $9\frac{1}{2}$ pounds. How much did he gain?

$$9\frac{1}{2} - 7\frac{3}{4} = n$$

Estimate: $n < 2$

Algorithm:

$$\begin{array}{rcrcr} 9\frac{1}{2} & = & 9\frac{2}{4} & = & 8\frac{6}{4} \\ \underline{-\ 7\frac{3}{4}} & = & \underline{-7\frac{3}{4}} & = & \underline{-7\frac{3}{4}} \\ & & & & 1\frac{3}{4}\text{ lb.} \end{array}$$

Verifying Solutions

Children Check Subtracting by Adding

Since $15\frac{1}{2} - 7\frac{5}{6} = 7\frac{2}{3}$, $7\frac{2}{3} + 7\frac{5}{6}$ should equal $15\frac{1}{2}$

Continue to test and reinforce subtraction with fractions.

Further practice exercises and verbal problems may be found in textbooks and from situations found in other curriculum books, newspapers, etc.

OPERATIONS

UNIT 36 - MULTIPLICATION OF WHOLE NUMBERS: OBSERVING AND USING PATTERNS

Objectives: Using patterns to:
Develop understanding of the property of one in multiplication
Develop understanding of the property of zero in multiplication
Observe factor-factor-product relationships

TEACHING SUGGESTIONS

Property of "1" in Multiplication (Identity Element)

1. Have children complete each sentence below.

   a. Present open sentences with 1 as the first factor.

   1 x 1 = □
   1 x 2 = □
   1 x 3 = □
   1 x 4 = □

   1 x 24 = □
   1 x 189 = □
   1 x 1,000,000 = □
   1 x n = □ (n stands for any number)

   b. Present open sentences with 1 as the second factor.

   1 x 1 = □
   2 x 1 = □
   3 x 1 = □
   4 x 1 = □

   24 x 1 = □
   189 x 1 = □
   1,000,000 x 1 = □
   n x 1 = □ (n stands for any number)

2. Have children discuss the products. Try to get children to state the generalizations

   a. If one factor is "1", the product is the other factor.

   (1 x 6 = 6 or 6 x 1 = 6)
   (1 x a = a or a x 1 = a)

   b. "1" is a neutral (identity) element in multiplication.

Property of Zero in Multiplication

Zero as a factor.

1. Have children complete each of the following sentences and observe the pattern of products.

a. $1 \times 3 = \square$ $1 \times 2 = \square$ $1 \times 1 = \square$ $1 \times 0 = \square$

$1 \times 0 = \square$ $16 \times 0 = \square$
$2 \times 0 = \square$ $74 \times 0 = \square$
$3 \times 0 = \square$ $123 \times 0 = \square$
$4 \times 0 = \square$ $n \times 0 = \square$ (n stands for any number)

b. $3 \times 1 = \square$ $2 \times 1 = \square$ $1 \times 1 = \square$ $0 \times 1 = \square$

$0 \times 1 = \square$ $0 \times 16 = \square$
$0 \times 2 = \square$ $0 \times 74 = \square$
$0 \times 3 = \square$ $0 \times 123 = \square$
$0 \times 4 = \square$ $0 \times n = \square$ (n stands for any number)

Have children observe the product where zero is a factor.

2. Ask children to state the generalization:

If one factor is zero, the product is zero.
($a \times 0 = 0$ and $0 \times a = 0$)

Comparing Properties of Zero and One in Multiplication

1. Have children complete the sentences below

$0 \times 8 = n$ $8 \times 0 = n$ $\square \times 8 = 0$
$1 \times 8 = n$ $8 \times 1 = n$ $\square \times 8 = 8$

$0 \times 12 = n$ $12 \times 0 = n$ $\square \times 12 = 0$
$1 \times 12 = n$ $12 \times 1 = n$ $\square \times 12 = 12$

2. Ask children to verbalize the role of 0 and 1 in

   a. multiplication

   b. addition

Factor - Factor - Product Relationships

1. Doubling the first factor; keeping the second factor constant

2 x 12 = 24
4 x 12 = 48
8 x 12 = 96
16 x 12 =192

3 x 12 = □
6 x 12 = □
12 x 12 = □
24 x 12 = □

Ask children:

Which factor is kept constant?
Compare each first factor with the one preceding it. What pattern do you see?
If the second factor remains the same, and the first factor is doubled, what happens to the product?
What happens to the product of 2 numbers when the second factor is kept constant and the first factor is tripled? Multiplied by 4? by 5?

Write 4 other equations to show this relationship.

2. Doubling the second factor; keeping the first factor constant.

2 x 12 = 24
2 x 24 = 48
2 x 48 = 96
2 x 96 = 192

4 x 12 = □
4 x 24 = □
4 x 48 = □
4 x 96 = □

Children study the relationships among the first factors; the second factors.

If the first factor remains the same, and the second factor is doubled, what happens to the product?

What happens to the product when the first factor is kept constant and the second is tripled? Multiplied by 4? by 5?

Write 4 other equations to show this relationship.

Children state the generalization:

When one factor is multiplied by a number, the product is multiplied by that same number.

3. Doubling both factors.

a. Present the following:

$$\begin{aligned} 2 \times 5 &= 10 \\ 4 \times 10 &= 40 \\ 8 \times 20 &= 160 \end{aligned}$$

Ask children to tell the relationship between:

The first factors of each equation
The second factors of each equation
The products of each equation

b. Have children compare, complete and compare again

$4 \times 10 = 40$ with $8 \times 20 = 160$
$8 \times 20 = 160$ with $16 \times 40 = 640$
$3 \times 5 = \square$ with $6 \times 10 = \square$
$6 \times 10 = \square$ with $12 \times 20 = \square$

Ask children to study the pattern and discuss

If both factors are doubled, what happens to the product? Tripled? Multiplied by 4? etc.

Ask children to state the generalization:

If both factors are multiplied by 2, the product is multiplied by 4.

If both factors are multiplied by 3, the product is multiplied by 9, etc.

Suggested Practice to Discover Patterns

Apply the following patterns to multiplying by sixes, sevens, eights and nines.

Ask children to explain patterns after each series.

a. Doubling a Factor

10 x 8 = □
20 x 8 = □ + □ = ?
40 x 8 = □ + □ = ?
80 x 8 = □ + □ = ?

b. Adding 5 Eights

5 x 8 = 40
10 x 8 = □ + □ = n
15 x 8 = □ + 40 = n
20 x 8 = □ + 40 = n

c. Adding 10 Eights

10 x 8 = □
20 x 8 = □ + 80 = n
30 x 8 = □ + 80 = n
40 x 8 = □ + 80 = n

d. Doubling and then Adding 1, 2, 3, etc. Eights

Since 6 x 8 = 48
Then 12 x 8 = n
And 13 x 8 = n

Since 10 x 8 = 80
Then 20 x 8 = □
And 22 x 8 = n

Since 20 x 8 = 160
Then 40 x 8 = □
And 43 x 8 = n

e. Doubling

20 x 70 = n
40 x 70 = □ + □ = n
80 x 70 = □ + □ = n

f. Doubling and Adding Groups

3 x 900 = n
6 x 900 = □ + □ = n
7 x 900 = □ + 900 = n

Money: Products Within $99.99

1. Be sure that children have acquired:

   Understanding of place value concepts of dollars and cents.
   Ability to regroup dollars, dimes and pennies.
   Ability to add by groups (mental computation).
   Ability to add with dollars and cents without and with exchange (written computation).

2. Drill additions that relate to multiplication.

3. Multiplying by 2

   2 x $ 2.30 = $ 2.30 + □ = ?
   2 x $ 4.31 = $ 4.31 + □ = ?
   2 x $12.62 = $12.62 + □ = ?

Doubling beginning with cents

$2 \times \$.65 = \$1.30$
$4 \times \$.65 = \square + \square = ?$
$8 \times \$.65 = \square + \square = ?$

Doubling -
dollars and cents

$2 \times \$1.24 = \$2.48$
$4 \times \$1.24 = \square + \square = n$
$8 \times \$1.24 = \square + \square = n$
$6 \times \$4.13 = n$
$6 \times \$4.13 = n + n = \square$

OPERATIONS

UNIT 37 - MULTIPLICATION OF WHOLE NUMBERS: VERTICAL FORMAT; ONE FACTOR THROUGH NINE

NOTE TO TEACHER

Encourage children to devise and present their own methods of arriving at close estimates. Provide practice in estimating and / or arriving at exact products before computing.

Use a variety of directions:

Find the product of 321 and 8.
Multiply 321 by 8.
One factor is 321. The other factor is 8.
Find the product.

Objectives: To develop understanding of multiplication of whole numbers: one factor through 9; the other factor through 9999.

To help children understand that the vertical algorithm for multiplication makes use of the distributive property .

To help children develop skill in multiplying whole numbers involving dollars and cents.

TEACHING SUGGESTIONS

1. Evaluate and / or reinforce ability to multiply: one factor through 9, other factor through 999. For example:

$$\begin{array}{r} 68 \\ \underline{\times\ 8} \end{array} \qquad \begin{array}{r} 728 \\ \underline{\times\ 5} \end{array} \qquad \begin{array}{r} 486 \\ \underline{\times\ 9} \end{array} \quad \text{etc.}$$

2. Extend understanding of multiplication: one factor through 9; other factor through 9999.

Sequence

No exchange $\begin{array}{r}4214\\ \underline{\text{x } 2}\end{array}$

Exchange in Tens Place and / or Units Place

$$\begin{array}{r}1207\\ \underline{\text{x } 4}\end{array} \qquad \begin{array}{r}3274\\ \underline{\text{x } 3}\end{array} \qquad \begin{array}{r}2008\\ \underline{\text{x } 4}\end{array} \qquad \begin{array}{r}1040\\ \underline{\text{x } 7}\end{array}$$

Exchange of Hundreds for Thousands only

$$\begin{array}{r}2632\\ \underline{\text{x } 3}\end{array} \qquad \begin{array}{r}1812\\ \underline{\text{x } 4}\end{array} \qquad \begin{array}{r}2934\\ \underline{\text{x } 2}\end{array}$$

Exchange in One, Two or Three Places

$$\begin{array}{r}1240\\ \underline{\text{x } 5}\end{array} \qquad \begin{array}{r}1432\\ \underline{\text{x } 4}\end{array} \qquad \begin{array}{r}2735\\ \underline{\text{x } 3}\end{array} \qquad \begin{array}{r}1254\\ \underline{\text{x } 6}\end{array}$$

Problem Situations:

a. A plane travels 2143 miles in one round trip. How many miles does it travel in 3 round trips?

$$3 \text{ x } 2143 = n$$

Estimate (Some children arrive at a closer estimate or the exact product.)

2143 is 2000 and 143
3 two thousands = 6 thousand
Estimate: $n > 6000$
or
2143 is over 21 hundred
3 times 21 hundred = 63 hundred
Estimate: $n > 6300$
or
n is between 6000 and 7000

Record the estimate.

Compute

$$\begin{array}{r}2143\\ \underline{\text{x } 3}\\ 6429\end{array}$$

Have children compare product with recorded estimate.

Children should verify solutions:

By Adding	or	by applying the Distributive Property	
2143 2143 2143 6429		2143 x 3 6429	3 x 2000 = 6000 3 x 100 = 300 3 x 40 = 120 3 x 3 = 9 6429

b. 2413
x 3
39

Record partial product for units, tens (39)
Discuss need for regrouping 12 hundred (3 x 400) as 1 thousand 2 hundred

Present similar problems for discussion of regrouping hundreds for thousands.

Children continue to estimate, compute, check.

Gradually increase difficulties. Include more than one exchange.

3. Multiplication Exercises Involving Money.

Suggested Sequence:

No exchange	3 x \$ 1.23 4 x \$ 2.01 2 x \$42.30 etc.
One Exchange.	3 x \$ 2.26 4 x \$12.82
Two Exchanges	7 x \$ 1.38 5 x \$14.60
Three Exchanges	2 x \$ 6.58 4 x \$23.64 7 x \$12.75

a. Have children estimate before they compute. This helps them determine the correct placement of the decimal point in the product.

b. No Exchange

Problem: What is the cost of 2 baseballs at \$2.34 each?

2 x \$2.34 = n

Children record estimate.
n > \$4 (2 x \$2) or as
n > \$4.60 (2 x \$2.30)

Have children compare the addition form with the multiplication form.

$$\begin{array}{r}\$2.34\\ 2.34\\ \hline \$4.68\end{array} \qquad \begin{array}{r}\$2.34\\ \times\ 2\\ \hline \$4.68\end{array}$$

As children multiply they should relate each step to the addition, beginning with the pennies.

Discuss the placement of the decimal point to separate the dollars from the cents.

Compare the product with the estimate.

c. Exchange (Dimes for dollars)

Follow the procedure suggested for "No Exchange"

$$\begin{array}{r}\$12.63\\ 12.63\\ 12.63\\ \hline \end{array} \qquad \begin{array}{r}\$12.63\\ \times\ 3\\ \hline \end{array}$$

Estimate: $n > \$36$ (3x$12)
$n > \$37.50$ (3x$12.50)
$n > \$37.80$ (3x$12.60)

As the product is recorded the teacher should direct the children's attention to the 18 dimes (3 x 6 dimes) which has been regrouped as 1 dollar and 8 dimes.

When understanding is assured, use the multiplication form mainly.

Have children check the product by:

Adding if the multiplier is 2, 3, 4, or 5.

$$\begin{array}{r}\$12.63\\ 12.63\\ 12.63\\ \hline \$37.89\end{array}$$

Applying the Distributive Property

$$\begin{array}{rcr}3 \times \$12 & = & \$36.00\\ 3 \times \$\ .63 & = & 1.89\\ \hline & & \$37.89\end{array}$$

d. Extend to multiplications involving 2 or 3 exchanges.

4. Additional problems will be found in textbooks.

OPERATIONS

UNIT 38 - MULTIPLICATION OF WHOLE NUMBERS: DEVELOPING GENERALIZATIONS

Objective: To help children formulate generalizations for multiplying with 10; 100; 1000 and their multiples.

TEACHING SUGGESTIONS

Multiplying with 10 as One Factor

1. Present the following, recording products as children state them.

A		B		C	
3	10	5	10	8	10
x 10	x 3	x 10	x 5	x 10	x 8

Question children about the product when 10 is one factor. They should note:

Similarity of products in each pair of expressions.
Position of zero in product when 10 is a factor.
Position of digit representing the other factor.

2. Repeat procedure above: One factor 10, the other a two digit number.

A		B		C	
11	10	14	10	32	10
x10	x11	x10	x14	x10	x32

Have children record products and observe:

The zero in the products always occurs in ones place.
The numeral in the other factor appears one place to the left in the product.
When one factor is 10 the product is ten times greater than the other factor.

3. Record a series of examples, such as:

$$\begin{array}{rrrrrl} 16 & 10 & 27 & 10 & 28 & \\ \underline{\text{x}10} & \underline{\text{x}19} & \underline{\text{x}10} & \underline{\text{x}35} & \underline{\text{x}10} & \text{etc.} \end{array}$$

Ask children whether they can arrive at the products in a faster or shorter way.

Record the products as the children express their thinking.

The children observe that when 10 is a factor the same pattern for the product emerges.

4. Have children express the generalizations in their own words.

5. Provide for practice using both the vertical and horizontal forms.

Multiplying with Multiples of 10 as One Factor

1. Reinforce and / or teach finding sums needed to derive unknown products from known products by applying the Distributive Property of Multiplication with respect to Addition.

In order to find a solution using the Distributive Property, children should have the ability to find required sums quickly. For example, To find the product for

20 x 13 = n   children should know that:
10 x 13 = 130 and that another
10 x 13 = 130 Then

130 + 130 will equal 260 or, 20 x 13

Suggested exercises to reinforce finding sums.
Multiples of thirteen are used here. Adapt to other numbers as required.

a. Adding 10 Thirteens to multiples of 10 Thirteens.

130 + 130 = ?
260 + 130 = ? . . . (20 x13) + (10 x 13) or 30 x 13
390 + 130 = ?
520 + 130 = ?
650 + 130 = ?
through
1040 + 130 = ?

b. Doubling multiples of 10 thirteens

$130 + 130 = ?$

$520 + 520 = ? \qquad (40 \times 13) + (40 \times 13) \text{ or } 80 \times 13$

$390 + 390 = ?$

$260 + 260 = ?$

2. As children solve the following exercises the teacher should record some of their thinking. For example,

30×13 as:

$20 \times 13 = 260$

$30 \times 13 = 260 + 130 = 390$

or as

$$\begin{array}{r} 20 \times 13 = 260 \\ \underline{10 \times 13 = 130} \\ 30 \times 13 = 390 \end{array}$$

80×13 as:

$40 \times 13 = 520$

$80 \times 13 = 520 + 520 = 1040$

or as

$$\begin{array}{r} 40 \times 13 = 520 \\ \underline{40 \times 13 = 520} \\ 80 \times 13 = 1040 \end{array}$$

a.

A

$10 \times 13 = \square$

$20 \times 13 = \square + 130 = n$

through

$90 \times 13 = \square + 130 = n$

B

$10 \times 13 = \square$

$20 \times 13 = \square + \square = n$

$40 \times 13 = \square + \square = n$

$80 \times 13 = \square + \square = n$

C

$20 \times 13 = \square + 130 = n$

$40 \times 13 = \square + 260 = n$

$60 \times 13 = \square + 260 = n$

$80 \times 13 = \square + 260 = n$

D

$20 \times 13 = \square + \square = n$

$40 \times 13 = \square + \square = n$

$50 \times 13 = \square + 130 = n$

E

$20 \times 13 = \square + \square = n$

$40 \times 13 = \square + \square = n$

$80 \times 13 = \square + \square = n$

$90 \times 13 = \square + \square = n$

b. Since 10 x 13 = 130
5 x 13 = ?

Since 10 x 13 = 130
15 x 13 = ?
etc.

Since 10 x 13 = 130
20 x 13 = n

Since 20 x 13 = 260
21 x 13 = n

Since 10 x 13 = 130
20 x 13 = ?
30 x 13 = ?
40 x 13 = ?
42 x 13 = ?

Since 100 x 13 = 1300
Then 50 x 13 = ?

Since 10 x 13 = 130
11 x 13 = ?

Since 10 x 13 = 130
13 x 13 = ?
etc.

Since 20 x 13 = 260
40 x 13 = ?

Since 30 x 13 = 390
60 x 13 = ?

Since 10 x 13 = 130
20 x 13 = ?
40 x 13 = ?
80 x 13 = ?
81 x 13 = ?

3. Develop generalization for multiplying with multiples of 10 as one factor; 20, 30, 40, 50 . . . 90

Present the following exercises recording products as children state them.

$\begin{array}{r} 20 \\ \underline{x\ 5} \end{array}$ $\begin{array}{r} 30 \\ \underline{x\ 5} \end{array}$ $\begin{array}{r} 40 \\ \underline{x\ 5} \end{array}$ $\begin{array}{r} 50 \\ \underline{x\ 5} \end{array}$ etc.

Have children observe that:

The zero in the product always occurs in ones place.

The digits in tens and hundreds places are arrived at by multiplying the number of tens and the other factor.

Present the following exercises.

$\begin{array}{r} 20 \\ \underline{x10} \end{array}$ $\begin{array}{r} 20 \\ \underline{x20} \end{array}$ $\begin{array}{r} 20 \\ \underline{x30} \end{array}$ $\begin{array}{r} 20 \\ \underline{x40} \end{array}$ $\begin{array}{r} 20 \\ \underline{x50} \end{array}$ $\begin{array}{r} 20 \\ \underline{x60} \end{array}$ through $\begin{array}{r} 20 \\ \underline{x90} \end{array}$

Children record the products.

They should observe that the zero in the product always occurs in ones place.

They compare the product with the factors in $\begin{array}{r}20\\ \underline{x10}\end{array}$

and in $\begin{array}{r}20\\ \underline{x20}\\ 400\end{array}$

They should see that when one of the factors is a multiple of 10, the digits in the other places are arrived at by multiplying the number of tens.

For example:

	Step 1 Record Zero	Step 2 Multiply by Number of Tens
$\begin{array}{r}20\\ \underline{x30}\end{array}$	$\begin{array}{r}20\\ \underline{x30}\\ 0\end{array}$	$\begin{array}{r}20\\ \underline{x30}\\ 600\end{array}$
$\begin{array}{r}20\\ \underline{x90}\end{array}$	$\begin{array}{r}20\\ \underline{x90}\\ 0\end{array}$	$\begin{array}{r}20\\ \underline{x90}\\ 1800\end{array}$

Encourage children to formulate the generalization in their own words.

For example, "When multiplying by 10 or multiples of ten record the zero in the ones place and multiply by the number of tens.

One Factor 100

1. Present orally

	Record only	
Since 50 twos = 100 Then 100 twos = ?	$\begin{array}{r}2\\ \underline{x100}\end{array}$	$\begin{array}{r}100\\ \underline{x\ 2}\end{array}$
Since 50 sevens = 350 Then 100 sevens = ?	$\begin{array}{r}7\\ \underline{x100}\end{array}$	$\begin{array}{r}100\\ \underline{x\ 7}\end{array}$
Since 50 fours = 200 Then 100 fours = ?	$\begin{array}{r}4\\ \underline{x100}\end{array}$	$\begin{array}{r}100\\ \underline{x\ 4}\end{array}$

Question children about the product when 100 is a factor.
(See development used when 10 is a factor)

Children should compare the numerals representing the factors with the position of the numerals in the products.

Encourage children to derive the generalization in their own words. For example,

> When we multiply, and one factor is 100 the numerals in the other factor appear again in the product two places to the left. There are two zeros - one in units place and the other in Tens place.

2. Provide practice. Use both vertical and horizontal forms.

$100 \times 4 = n$
$100 \times 9 = n$

$5 \times 100 = n$
$8 \times 100 = n$

$$\begin{array}{r} 4 \\ \underline{\times 100} \end{array} \qquad \begin{array}{r} 100 \\ \underline{\times \ 8} \end{array} \qquad \begin{array}{r} 6 \\ \underline{\times 100} \end{array} \quad \text{etc.}$$

One Factor: Multiple of 100

1. Children arrive at unknown products by adding known products.

Adding in Sequence	Other Patterns
100 x 4 = n	
200 x 4 = n + 400 = ☐	200 x 4 = n
300 x 4 = n + 400 = ☐	400 x 4 = n + n = ☐
400 x 4 = n + 400 = ☐	600 x 4 = n + 800 = ☐
through	800 x 4 = n + 800 = ☐
900 x 4 = n + 400 = ☐	

2. Record as follows:

$$\begin{array}{r} 4 \\ \underline{\times 100} \end{array} \qquad \begin{array}{r} 4 \\ \underline{\times 200} \end{array} \qquad \begin{array}{r} 4 \\ \underline{\times 300} \end{array} \qquad \begin{array}{r} 4 \\ \underline{\times 400} \end{array}$$

$$\begin{array}{r} 100 \\ \underline{\times \ 4} \end{array} \qquad \begin{array}{r} 200 \\ \underline{\times \ 4} \end{array} \qquad \begin{array}{r} 300 \\ \underline{\times \ 4} \end{array} \qquad \begin{array}{r} 400 \\ \underline{\times \ 4} \end{array}$$

Children observe and discuss products.
They express the generalization for multiplying with multiples of 100 in their own words.

For example: When multiplying by 100 or multiples of 100, record the zeros in the ones and tens places, then multiply by the number of hundreds. (2, 3, 4, etc.)

OPERATIONS

$$\begin{tikzcd} + \arrow[r] \arrow[d, leftrightarrow] & \times \arrow[d, leftrightarrow] \\ - \arrow[r] & \div \end{tikzcd}$$

UNIT 39 - DIVISION OF WHOLE NUMBERS

Objectives: To extend division of whole numbers: Vertical algorithm; Quotients through 99; Then through 999; Divisors through 9.

TEACHING SUGGESTIONS

1. Drill multiplication and division facts in specific patterns.

   The following are suggestions for various types of drill.

   a. Find the missing factor.

   Since $5 \times 7 = 35$
   $\square \times 7 = 42$

   Since $9 \times 7 = 63$
   $\square \times 7 = 70$

   Since $15 \times 7 = 105$
   $\square \times 7 = 112$

   Since $100 \times 7 = 700$
   $\square \times 7 = 707$

   Since $25 \times 7 = 175$
   $\square \times 7 = 182$

   Since $3000 \times 7 = 21000$
   $\overset{\square}{7\overline{)21007}}$

   b. Using the Distributive Property of Division with respect to Addition to arrive at unknown quotients from known quotients.

   The following exercises are for dividing by eights.
   Adapt for other facts.

   Supply correct quotients.

   $8\overline{)192}$ as $8\overline{)160 + 32}$ $\qquad$ $8\overline{)192}$ as $8\overline{)96 + 96}$

   $8\overline{)2000}$ as $8\overline{)1600 + 400}$ $\qquad$ $8\overline{)2000}$ as $8\overline{)1000 + 1000}$

   Have children complete the following exercises:

   $128 \div 4 = n$
   as: $(120 + 8) \div 4 = n$
   $(\square \div 4) + (\Delta \div 4) = n$

   $637 \div 7 = n$
   as: $(630 + 7) \div 7 = n$
   $(\square \div 7) + (\Delta \div 7) = n$
   $\square + \Delta = n$

c. Using the Distributive Property of Division with respect to Subtraction.

Have children complete the following:

$57 \div 3 = n$

as: $(60 - 3) \div 3 = n$

$(60 \div 3) - (3 \div 3) = n$

$\square - \Delta = n$

$3\overline{)87}$ as $3\overline{)90} - 3\overline{)3}$

$3\overline{)117}$ as $3\overline{)120} - 3\overline{)3}$

d. Doubling and adding eights. Adapt for other facts.

Necessary background: Adding eight and multiples of eight.

Drill Exercises

$8 + 8 = n$ $\quad 16 + 16 = n$ $\quad 32 + 32 = n$ $\quad 64 + 64 = n$ etc.
$24 + 24 = n$ $\quad 48 + 48 = n$ $\quad 96 + 96 = n$ $\quad 192 + 192 = n$ etc.

Complete the following:

$4 \times 8 = n$
$8 \times 8 = n$
$16 \times 8 = n$
$32 \times 8 = n$

$10 \times 8 = n$
$20 \times 8 = n$
$25 \times 8 = n$
$25\frac{1}{2} \times 8 = n$

$40 \times 8 = n$
$80 \times 8 = n$
$89 \times 8 = n$

$\square \times 8 = 16$ or $8\overline{)16}$ $\quad \square \times 8 = 24$ or $8\overline{)24}$
$\square \times 8 = 32$ or $8\overline{)32}$ $\quad \square \times 8 = 48$ or $8\overline{)48}$
$\square \times 8 = 64$ or $8\overline{)64}$ $\quad \square \times 8 = 56$ or $8\overline{)56}$

e. Subtracting eights from multiples of eight.

When multiplication and division facts are drilled through subtraction, children must know the subtraction facts and extensions involved.

Necessary background: 80 - 8, 160 - 8, 400 - 8,
800 - 8, 800 - 80

Complete the following:

$10 \times 8 = n$ $\quad 20 \times 8 = n$ $\quad 50 \times 8 = n$ $\quad 100 \times 8 = n$
$9 \times 8 = n$ $\quad 19 \times 8 = n$ $\quad 49 \times 8 = n$ $\quad 90 \times 8 = n$

* f. Dividing one factor by 2

$10 \times 8 = n$	$50 \times 8 = n$	$100 \times 14 = n$ [1400]
$5 \times 8 = n$	$25 \times 8 = n$	$50 \times 14 = n$ [700]
$2\frac{1}{2} \times 8 = n$	$12\frac{1}{2} \times 8 = n$	$25 \times 14 = n$ [350]
$1\frac{1}{4} \times 8 = n$	$6\frac{1}{4} \times 8 = n$	$12\frac{1}{2} \times 14 = n$ [175]

Have children observe, then state generalization:

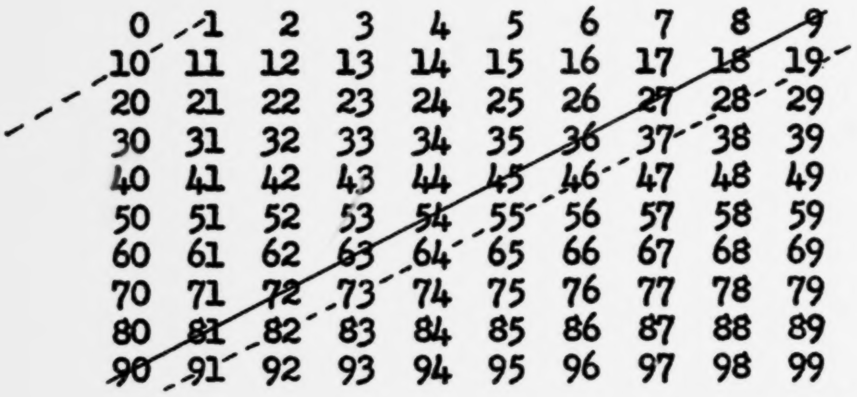
"When one factor is divided by 2 and the other factor is not changed, what happens to the product?"

2. Drills for Division Without and With Remainders

a. Using a number chart

Present a chart showing numbers from 0 to 99. Duplicate a quantity of these charts for later use with other "tables".

Have children draw a line through all the multiples of 9.
(9, 18, 27, 36, etc)



0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Children draw line segments, colored differently, to show which numbers have a remainder of 1 when divided by 9.

Children draw other lines, colored differently, to show a remainder of 2, of 3, etc. They note the patterns that emerge.

Present other number charts similar to the one above, one at a time. Follow the same procedure for multiples of other numbers.

Children name the set of numbers that have a remainder of 0, when divided by 9.
[0, 9, 18, 27, 36, . . .]

Children name the set of numbers that have a <u>remainder of 1</u>, when divided by nine. They note the relationship to 9 and multiples of 9.

[1, 10, 19, 28, 37, . . .]

Children name the set of numbers that have a <u>remainder of 2</u>, when divided by nine. They note the relationship to 9 and multiples of 9.

[2, 11, 20, 29, 38, . . .]

Follow the same procedure for dividing by other numbers.

b. Reinforce properties that apply to division

Have children mark the following true or false.
If false make it true.

$$\square \div \Delta = \Delta \div \square$$
$$64 \div 1 = 64$$
$$84 \div 3 = (60 \div 3) + (24 \div 3)$$

c. If a dividend is a multiple of the divisor, is there a remainder? When does a division exercise have a remainder?

3. Algorithms: Divisors through 9; Quotients through 99

Gradually increase the difficulty by including more difficult quotients. For example:

$3\overline{)257}$ $6\overline{)452}$ $9\overline{)834}$

Encourage children to shorten the recording by using larger, therefore, fewer multiples of ten.

From:

```
    75
  6)452 |
    120 | 20
    332 |
    240 | 40
     92 |
     60 | 10
     32 |
     30 |  5
      2 | 75
```

Encourage

```
    75
  6)452 |
    300 | 50
    152 |
    120 | 20
     32 |
     30 |  5
      2 | 75
```

Advance to

```
    75
  6)452 |
    420 | 70
     32 |
     30 |  5
      2 | 75
```

Solution: 75 R 2

4. Algorithms: Quotients through 999; Divisors through 9

a. Skill in multiplying by 100 and multiples of 100 is basic to the development of division situations which involve 3-place quotients. To help children acquire this skill, the following development is suggested:
Drill: Using Relationships

Present one pattern at a time
Have children record products only
Record children's method for arriving at products

Multiplying in Sequence	Doubling
100 x 4 = n	100 x 4 = n
200 x 4 = n	200 x 4 = n
300 x 4 = n	400 x 4 = n
400 x 4 = n	800 x 4 =(1600 + 1600 read as 16 hundred + 16 hundred)
through	
900 x 4 = n	

Other Sequences

100 x 4 = n	100 x 4 = n	100 x 4 = n
200 x 4 = n	300 x 4 = n	500 x 4 = n
400 x 4 = n	900 x 4 = (12 hundred three times)	600 x 4 = n
600 x 4 = n		700 x 4 = n
800 x 4 = n		etc.

b. Suggested Development

Present a problem situation (quotient between 100 and 200) such as:

The P.T.A. contributed 986 cupcakes for the school Cake Sale. They are to be packed in cellophane bags with 8 cupcakes in each bag. How many bags will the school have to sell?

Ask the children to:

State the action involved
State the process to be used
Record the symbols to be used to solve the problem
[☐ x 8 = 986; $8\overline{)986}$; 986 ÷ 8 = n]
Tell what the symbols say. [How many eights in 986?]

Ask some children how many eights to begin with
Discuss with children and record several ways of solving

Various methods of solving follow:

```
    123               123                123
 8)986             8)986 |            8)986 |
   160   20          320 |   40         400 |  50
   826               666 |              586 |
   320   40          640 |   80         400 |  50
   506                26 |              186 |
   320   40           24 |____3         160 |  20
   186                 2 | 123           26 |
   160   20                              24 |___3
    26                                    2 | 123
    24____3
     2  123
```

Solution: 123 bags and 2 extra cupcakes.

Children will continue to arrive at solutions in a variety of ways depending upon their level of ability.

Using 100 as the first partial quotient.

Discuss the various solutions above.
Encourage children to begin with a still greater number of eights.
When 100 eights are suggested, present the following:

```
    123
 8)986 |
   800 |  100
   186 |
   160 |   20
    26 |
    24 |____3
     2 |  123
```

Compare with solutions above.
Discuss the shortened form.

Check by using the Distributive Property for Multiplication with respect to addition.

```
100 x 8 = 800        984
 20 x 8 = 160       + 2   (extra cupcakes)
  3 x 8 =  24        986
          984
```

Provide for practice. Select divisions in which 100 is the first partial quotient, e.g.,

$7\overline{)864}$ $3\overline{)528}$ $6\overline{)852}$ $4\overline{)611}$ etc.

5. Estimating: Estimating should precede all computation

Provide practice in estimating the size of the quotient when the dividend is in the hundreds.

Will the quotient be in the tens or in the hundreds?
For example:

$7\overline{)317}$ and $7\overline{)864}$

Children decide upon the number of groups with which to begin, by comparing the size of the dividend with the number of groups selected. Children think as follows:

For $7\overline{)317}$ Will there be as many as 10 sevens?
(10 x 7 = 70) too small
Will there be as many as 100 sevens?
(100 x 7 = 700) too large

Continue to compute using few or many multiples of 10 depending upon their ability.

For $7\overline{)864}$ Will there be as many as 10 sevens?
(10 x 7 = 70) too small
Will there be as many as 100 sevens?
(100 x 7 = 700) Yes.

Continue the computation beginning with 100 as the first recorded quotient.

6. More Difficult Computations

a. Present a problem such as:

Children contributed 716 used books for the White Elephant Sale held by the P.T.A. Their mothers tied them into bundles of 3 each. How many bundles did they tie?

Children may use hundreds repeatedly in the partial quotients. Begin with 100 threes. 416 is recorded as a partial dividend. Then ask what large group of threes might be removed next?

```
   238
3)716 |
  300 | 100
  416 |
  300 | 100
  116 |
   90 |  30
   26 |
   24 |   8
    2 | 238
```

Complete the division.

Solution: 238 bundles and 2 extra books

b. Present similar division situations

Discuss the correct placement of the quotients after completing the computations. See examples below.

```
   234
4)936 |
  400 | 100
  536 |
  400 | 100
  136 |
  120 |  30
   16 |
   16 |   4
    0 | 234
```

```
   242
6)1457 |
   600 | 100
   857 |
   600 | 100
   257 |
   240 |  40
    17 |
    12 |   2
     5 | 242
```

```
   323
5)1619 |
   500 | 100
  1119 |
   500 | 100
   619 |
   500 | 100
   119 |
   100 |  20
    19 |
    15 |   3
     4 | 323
```

d. Children who have acquired skill in multiplying by multiples of 100 and who understand the meaning of the numbers they will use in division, may be ready to shorten the recording.

For example, children think as follows:

```
    242
6)1457 |
  1200 | 200
   257 |
   240 |  40
    17 |
    12 |   2
     5 | 242
```

Will there be as many as 10 sixes?
    Yes, but 60 is too small.
Will there be as many as 100 sixes?
    (100 x 6 = 600) Yes
Will there be as many as 200 sixes?
    (200 x 6 = 1200) Yes
Will there be as many as 300 sixes?
    (300 x 6 = 1800) No, too large

Estimate: There will be over 200 groups. Children record 200 and continue the computation.

EVALUATION and / or PRACTICE

SUGGESTED EXERCISES

1. Find quotients

$6\overline{)805}$ $7\overline{)869}$ $3\overline{)642}$

$4\overline{)1373}$ $5\overline{)2228}$

$6\overline{)3927}$ $7\overline{)3269}$ $3\overline{)1718}$

$4\overline{)3878}$ $9\overline{)5149}$

2. Additional exercises and problems may be found in text books.

SETS; NUMBER; NUMERATION

UNIT 40 - CONCEPTS OF EQUIVALENT FRACTIONS EXTENDED; APPLYING THE MULTIPLICATIVE PROPERTY OF "1".

NOTE TO TEACHER

"Equal fractions", by definition have equal numerators and equal denominators.

e.g. $\frac{1}{3} = \frac{1}{3}$ $\qquad$ $\frac{3}{6} = \frac{2+1}{4+2}$

"Equivalent fractions" are fractions (numerals) that name the same fractional number.

e.g. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

In this document "equal fractions" are rarely considered.

The Identity Property for Multiplication states that if a number is multiplied by 1 (The Identity Element for Multiplication), the value of the number is not changed. The Identity Property applies to fractional numbers as well as whole numbers.

$$4 \times 1 = 4; \quad \frac{2}{3} \times 1 = \frac{2}{3}$$

1 may be renamed as a fraction, thus:

$$\frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{4}{6} \qquad \frac{a}{b} = \frac{a}{b} \times \frac{c}{c} = \frac{ac}{bc} \quad b \neq 0, c \neq 0$$

This illustrates the Fundamental Principle of Fractions:

MULTIPLYING NUMERATOR AND DENOMINATOR OF A FRACTION BY THE SAME NUMBER, NOT ZERO, DOES NOT CHANGE THE VALUE OF THE FRACTION.

Objective: To help children understand that changing a fraction to higher terms or to its simplest form involves the Multiplicative Property of "1".

TEACHING SUGGESTIONS

Renaming Fractions: Higher Terms

1. Have children rename $\frac{1}{6}$ as twelfths. $\left[\frac{1}{6} = \frac{2}{12}\right]$

   Ask children:

   The denominator 12 is how many times as large as the denominator 6? [twice as large]

   The numerator 2 is how many times as large as the numerator 1?

   By what number did you multiply both numerator and denominator to change $\frac{1}{6}$ to $\frac{2}{12}$? [2]

2. Have children rename $\frac{1}{4}$ as twelfths. $\left[\frac{1}{4} = \frac{3}{12}\right]$

   Compare both numerators and denominators

   3 is how many times as large as 1? [3 times]
   12 is how many times as large as 4? [3 times]
   By what number do you multiply both terms of $\frac{1}{4}$ to change it to $\frac{3}{12}$?

   Have children multiply both numerator and denominator of $\frac{1}{4}$ by 3.

   $$\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$

3. Have children rename other fractions:

   $\frac{1}{3}$ as sixths; $\frac{1}{3}$ as ninths; $\frac{2}{3}$ as sixths; $\frac{2}{3}$ as ninths; etc.

By what number did you multiply both numerator and denominator in each case to arrive at the equivalent fraction?

What happens to the value of a fraction when the numerator and denominator are multiplied by the same number, except zero?
(Fundamental Principle of Fractions)

Renaming Fractions: Simple Form

Development

1. Have children:

   Write $\frac{2}{6}$ as thirds; $\frac{3}{9}$ as thirds; $\frac{4}{12}$ as thirds.

   Compare the numerators of $\frac{2}{6}$ and $\frac{1}{3}$. Compare the denominators.

   Compare the numerators of $\frac{3}{9}$ and $\frac{1}{3}$. Compare the denominators.

   Tell how they arrive at $\frac{1}{3}$ from $\frac{2}{6}$; from $\frac{3}{9}$; from $\frac{4}{12}$.

2. Simplify the following and tell by which factor they divided both numerator and denominator.

   Change: $\frac{4}{8}$, $\frac{6}{12}$, $\frac{3}{6}$ to halves Change: $\frac{1}{2}$, $\frac{2}{8}$, $\frac{3}{12}$ fourths

   Change: $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$ to thirds Change: $\frac{4}{12}$, $\frac{6}{12}$, $\frac{8}{12}$ to sixths

   Ask children to:

   Reexamine each fractional numeral and its equivalent.
   Discuss what happened to the size of the numerator and denominator after the fraction was changed to its equivalent.
   Discuss what happened to the value of each fraction.
   Tell what you did to the numerator and denominator of each fraction to change to simpler form?
   [Divided numerator and denominator by the same number]

3. Have children change the following to simpler form:

   $\frac{6}{9} = \frac{\square}{n}$ $\left[\frac{2}{3}\right]$ $\frac{12}{16} = \frac{\square}{n}$ $\left[\frac{6}{8}\right]$ $\frac{12}{16} = \frac{\square}{n}$ $\left[\frac{3}{4}\right]$

Ask children:

By what number did you divide the numerator and the denominator of $\frac{6}{9}$ to arrive at $\frac{2}{3}$? [3] of $\frac{12}{16}$ to arrive at $\frac{6}{8}$? [2] of $\frac{12}{16}$ to arrive at $\frac{3}{4}$? [4]

Write the equations as children tell their thinking.

$$\frac{6 \div 3}{9 \div 3} = \frac{2}{3}\ ; \qquad \frac{12 \div 2}{16 \div 2} = \frac{6}{8}\ ; \qquad \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

4. Introduce term "common factor".

5. Discuss:
   a. Simplest form of fractions, those having no common factor other than 1 in the numerator and denominator.
   b. The value of the fraction in relation to the size of the parts into which a whole object has been divided.

6. Children state a generalization in their own words for changing a fraction to its simplest form.

*7. Have children observe the fractions $\frac{2}{3}$, $\frac{7}{6}$

For $\frac{2}{3}$, What is the greatest common factor of the numerator and the denominator? [1]

Is $\frac{2}{3}$ in simplest form?

Consider $\frac{7}{6}$. Can we say that $\frac{7}{6}$ is in simplest form as a fraction?

[Yes] Explain.

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Complete the following statements and tell the common factor used.

$$\frac{1}{3} = \frac{\square}{6}\,; \quad \frac{1}{3} = \frac{\square}{9}\,; \quad \frac{1}{3} = \frac{\square}{12}\,; \quad \frac{3}{4} = \frac{\square}{8}\,; \quad \frac{3}{4} = \frac{\square}{12}$$

2. Supply the correct numeral to complete the equations.

$$\frac{1}{6} = \frac{2 \times \square}{2 \times \triangle} \qquad \frac{1}{6} = \frac{3 \times \square}{3 \times \triangle} \qquad \frac{1}{6} = \frac{4 \times \square}{4 \times \triangle}$$

3. Since $\frac{2}{3} = \frac{6}{9}$, then $\frac{2 \times \square}{3 \times \square} = \frac{6}{9}$ Since $\frac{3}{4} = \frac{9}{12}$, then $\frac{3 \times \square}{4 \times \square} = \frac{9}{12}$

Since $\frac{1}{2} = \frac{4}{8}$, then $\frac{1 \times \square}{2 \times \square} = \frac{4}{8}$ since $\frac{5}{6} = \frac{10}{12}$, then $\frac{5 \times \square}{6 \times \square} = \frac{10}{12}$

4. Complete the following by inserting the correct factor:

$$\frac{2 \div \square}{12 \div \square} = \frac{1}{6} \qquad \frac{3 \div \square}{12 \div \square} = \frac{1}{4} \qquad \frac{4 \div \square}{12 \div \square} = \frac{1}{3}$$

$$\frac{8 \div \square}{12 \div \square} = \frac{2}{3} \qquad \frac{9 \div \square}{12 \div \square} = \frac{3}{4} \qquad \frac{10 \div \square}{12 \div \square} = \frac{5}{6}$$

5. Change the following to simplest form:

$$\frac{2}{6}\,; \quad \frac{4}{6}\,; \quad \frac{3}{9}\,; \quad \frac{6}{9}\,; \quad \frac{4}{10}\,; \quad \frac{6}{10}\,; \quad \frac{8}{10}$$

6. What principle makes it possible to change a fraction to simplest form? Explain, using the problems above. **For example:**

$$\frac{4}{8} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

$$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

$$\frac{4}{8} = \frac{1}{2}$$

In each case numerator and denominator have been divided by the same number

OPERATIONS

UNIT 41 - MULTIPLICATION OF FRACTIONAL NUMBERS: COMMON FORM

NOTE TO TEACHER

Properties of multiplication that apply to whole numbers also apply to fractional numbers. The properties of Distributivity, Commutativity, and Associativity apply.

Distributivity:

$$4 \times 3\tfrac{1}{2} =$$

$$4 \times \ 3 + \frac{1}{2} = (4 \times 3) + \left(4 \times \frac{1}{2}\right)$$

$$\boxed{a \times \left(b + \frac{c}{d}\right) = (a \times b) + \left(a \times \frac{c}{d}\right)}$$

Commutativity:

$$\frac{2}{3} \times \frac{1}{2} = \frac{1}{2} \times \frac{2}{3}$$

$$\boxed{\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}}$$

Associativity:

$$\left(\frac{3}{4} \times \frac{1}{2}\right) \times \frac{2}{3} = \frac{3}{4} \times \left(\frac{1}{2} \times \frac{2}{3}\right)$$

$$\boxed{\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)}$$

The Identity Element for multiplication is 1:

$$1 \times \frac{2}{3} = \frac{2}{3} \times 1 = \frac{2}{3}$$

$$\boxed{1 \times \frac{a}{b} = \frac{a}{b}}$$

The Fundamental Property of Fractions

$$\boxed{\frac{a}{b} \times \frac{c}{c} = \frac{ac}{bc} = \frac{a}{b} \qquad (c \neq 0)}$$

One of the interpretations of multiplication of whole numbers is its relationship to repeated addition. (4 x 3 as 3+3+3+3, read as 4 threes or 4 times 3). Similarly one of the interpretations of multiplication of fractions by a whole number is its relationship to repeated addition.

4 times $\frac{2}{3}$ may be interpreted as: $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$.

Objectives: To help children develop an understanding of multiplying fractions by whole numbers.

To formulate a generalization for multiplication of fractions and applying that generalization.

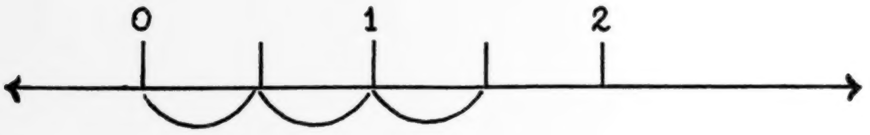
TEACHING SUGGESTIONS

Use problem-solving situations. Children may arrive at solutions "mentally" before using algorithms.

Children should estimate answers and give reasons for their estimates. They should solve these multiplication exercises by using diagrams and repeated addition. Children should be led to realize that repeated addition of fractions can be performed through multiplication by a whole number just as repeated addition of whole numbers can be done through multiplication by a whole number.

1. Suggested problem: I need $\frac{1}{2}$ yd. of oilcloth for each of 3 shelves. How much oilcloth do I need for all the shelves?

Estimate: Less than 2 yards.



or

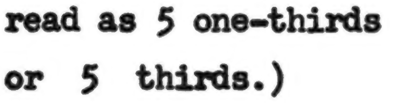
$$\begin{array}{r} \frac{1}{2} \\ \frac{1}{2} \\ \underline{\frac{1}{2}} \\ \frac{3}{2} = 1\frac{1}{2} \end{array}$$

First recorded as: $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$

2. Suggested problem: John wanted to make armbands for the 5 members of the Safety Squad. He required $\frac{1}{3}$ yard of ribbon for each band. How much ribbon did he need?

Children may record the solution of the problem in a variety of ways, including by the use of the number line.

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$



$$\begin{array}{r} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \underline{\frac{1}{3}} \\ \frac{5}{3} = 1\frac{2}{3} \end{array}$$

Note: Multiplication algorithm should not be used at this time to solve the example; it is only being used to record the solution.

$5 \times \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$ ($5 \times \frac{1}{3}$ is read as 5 one-thirds or 5 thirds.)

Children may use number line to solve the following:

2 sets of $\frac{1}{6} = n$ 4 sets of $\frac{1}{8} = n$

6 sets of $\frac{1}{4} = n$ 5 sets of $\frac{1}{3} = n$

etc.

3. Suggested problem: A cake recipe calls for $\frac{3}{4}$ of a cup of milk. How much milk do we need to make 5 cakes for our class party.

Estimate: Less than 5 cups; probably less than 4 cups. Why.

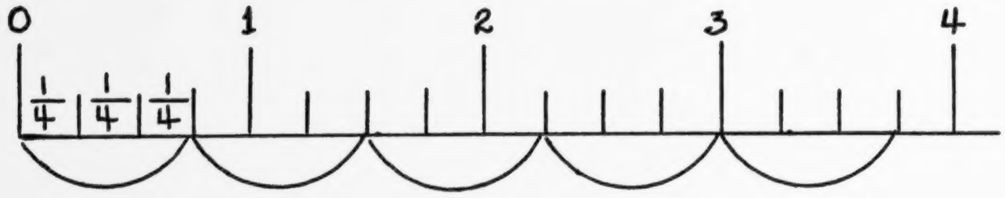
Mental Computation

If each cake needed 1 cup of milk, we would need 5 cups of milk. But each cake needs 1 fourth less. Therefore, 5 cakes need 5 fourths less or 3 and 3 fourths cups.

5 one-fourths are 5 fourths.
5 three-fourths are 15 fourths.

Written Computation

Children may use diagrams and solve by addition.



First recorded as $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{15}{4} = 3\,\frac{3}{4}$

$$\begin{array}{l} \frac{3}{4} \\ \frac{3}{4} \\ \frac{3}{4} \\ \frac{3}{4} \\ \underline{\frac{3}{4}} \\ \frac{15}{4} = 3\,\frac{3}{4} \end{array}$$

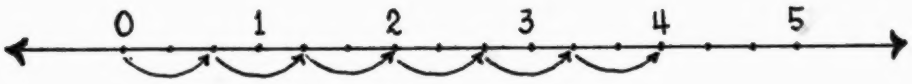
Then recorded as $5 \times \frac{3}{4} = 3\,\frac{3}{4}$.

Check with estimate.

4. Suggested problem:

$\frac{2}{3}$ of a yard of ribbon was needed for bandoliers for each of the 6 members of the Honor Guard. How much ribbon had to be bought?

$$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{12}{3}=4$$



$$6 \times \frac{2}{3}=\frac{12}{3}=4 \qquad \left(6 \times \frac{2}{3} \text{ is read as 6 two-thirds}\right)$$

Emphasize the recording of the problem as a mathematical sentence involving multiplication.

5. Suggestions for developing a generalization:

Present the following exercises one at a time.
Have children solve each by adding. Then have them examine the two factors and the product.

$2 \times \frac{1}{3} = n \quad \left[\frac{2}{3}\right]$

$2 \times \frac{3}{4} = n \quad \left[\frac{6}{4}\right]$

$3 \times \frac{2}{5} = n \quad \left[\frac{6}{5}\right]$

$6 \times \frac{1}{4} = n \quad \left[\frac{6}{4}\right]$

Ask children: Can you discover a rule for multiplying a fraction by a whole number?

Have children state the generalization in their own words. (Multiplying the numerator of the fraction by the whole number gives the numerator of the product. The denominator remains the same.)

Then solve the following multiplications and verify solutions in various ways.

$n = 6 \times \frac{2}{9} \quad \left[\frac{12}{9} = 1\frac{3}{9} = 1\frac{1}{3}\right]$

$n = 9 \times \frac{2}{5} \quad \left[\frac{18}{5} = 3\frac{3}{5}\right]$

$n = 10 \times \frac{1}{4} \quad \left[\frac{10}{4} = 2\frac{1}{2}\right]$

$n = 8 \times \frac{3}{8} \quad \left[\frac{24}{8} = 3\right]$

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Record each of the following as a multiplication and multiply.

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ $\left[4 \times \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \right]$

$\frac{2}{7} + \frac{2}{7} + \frac{2}{7}$ $\left[3 \times \frac{2}{7} = \frac{6}{7} \right]$

Since $4 \times \frac{1}{7} = \frac{4}{7}$

$4 \times \frac{2}{7} = \frac{\square}{7}$ Why? $\left[\frac{2}{7} \text{ is twice } \frac{1}{7} \right]$

Since $4 \times \frac{1}{7} = \frac{4}{7}$

$4 \times \frac{3}{7} = \frac{\square}{7}$ Why?

3. The Connecticut River rose $\frac{1}{3}$ of a foot each hour for 10 hours. How much did the river rise?

4. Susan walked $\frac{3}{5}$ of a mile to the library and discovered that she had forgotten her library card. She walked home to get it and walked back to the library. How far had she walked by the time she reached home with her books?

OPERATIONS

UNIT 42 - FRACTIONAL PARTS OF NUMBERS

Objective: To help children interpret finding a fractional part of a number as multiplication

TEACHING SUGGESTIONS

Finding a Fractional Part of a Number

1. Reinforce the meaning of fractional parts of a group as dividing a group into equal parts (division).

   Have children use pennies, discs, measurement materials if necessary.

   Finding answers in a variety of ways (mental computation) should be emphasized.

   Problems that involve sharing things will be useful for this unit. For example, sharing 10 cookies among 5 children.

2. Reinforce finding fractional parts of numbers through 99.

   Suggested Exercises Using $\frac{1}{2}$, $\frac{1}{4}$, etc. as an Operator.

   Drill:

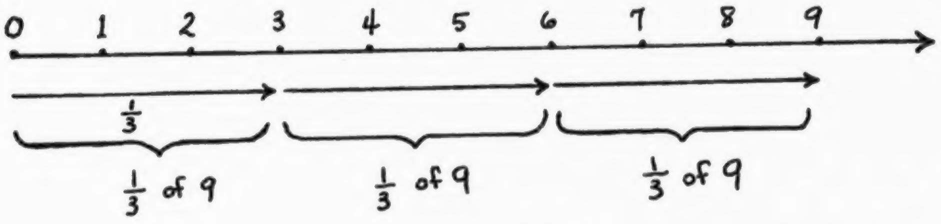
   $\frac{1}{2}$ of numbers through 20, through 100.

   $\frac{1}{4}$ of numbers that are divisible in half, then half again.

   $\frac{1}{8}$ of numbers that are divisible by 8, e.g., 16, 24, 32, 40, etc. (multiples of 8)

   $\frac{1}{3}$ of numbers that are divisible by 3, e.g, 12, 36, 45, etc. (multiples of 3)

Use number rays. For example for $\frac{1}{3}$ of 9 = n



3. Extend development of $\frac{1}{2}$ as operator to find:

$\frac{1}{2}$ of whole hundreds:

$\frac{1}{2}$ of 200 = n
$\frac{1}{2}$ of 400 = n
$\frac{1}{2}$ of 600 = n
$\frac{1}{2}$ of 800 = n

$\frac{1}{2}$ of 100 = n
$\frac{1}{2}$ of 300 = n
$\frac{1}{2}$ of 500 = n
$\frac{1}{2}$ of 700 = n
$\frac{1}{2}$ of 900 = n

$\frac{1}{2}$ of whole thousands:

$\frac{1}{2}$ of 2000 = n
$\frac{1}{2}$ of 4000 = n
$\frac{1}{2}$ of 6000 = n
$\frac{1}{2}$ of 8000 = n

$\frac{1}{2}$ of 1000 = n
$\frac{1}{2}$ of 3000 = n
$\frac{1}{2}$ of 5000 = n
$\frac{1}{2}$ of 7000 = n etc.

4. Finding one fourth of numbers:

a. By dividing in half, then half again

Suggested problem: Find $\frac{1}{4}$ of 600

$\frac{1}{2}$ of 600 = 300; then $\frac{1}{2}$ of 300 = 150

therefore $\frac{1}{4}$ of 600 = 150

or

600 divided into 2 equal parts = 300
300 divided into 2 equal parts = 150

therefore $\frac{1}{4}$ of 600 = 150

$\frac{1}{4}$ of whole hundreds: $\left(\frac{1}{2} \text{ of } \frac{1}{2}\right)$

$\frac{1}{4}$ of 800 = n
$\frac{1}{4}$ of 400 = n
$\frac{1}{4}$ of 600 = n

$\frac{1}{4}$ of 100 = n
$\frac{1}{4}$ of 300 = n
$\frac{1}{4}$ of 500 = n
$\frac{1}{4}$ of 700 = n
$\frac{1}{4}$ of 900 = n

b. By applying the Distributive Property of Multiplication with respect to Addition. For example:

Since 284 = 200 + 80 + 4

$$\frac{1}{4} \text{ of } 284 = \left(\frac{1}{4} \text{ of } 200\right) + \left(\frac{1}{4} \text{ of } 80\right) + \left(\frac{1}{4} \text{ of } 4\right)$$

or

$\frac{1}{4}$ of 284 may be thought through as:

$$\begin{array}{l} \frac{1}{4} \text{ of } 200 = 50 \\ \frac{1}{4} \text{ of } 80 = 20 \\ \frac{1}{4} \text{ of } 4 = 1 \\ \hline \frac{1}{4} \text{ of } 284 = 71 \end{array}$$

$\frac{1}{4}$ of 9000 = n

$$\begin{array}{l} \frac{1}{4} \text{ of } 8000 = 2000 \\ \frac{1}{4} \text{ of } 1000 = 250 \\ \hline \frac{1}{4} \text{ of } 9000 = 2250 \end{array}$$

5. One eighth as an operator

a. Suggested exercise: Find $\frac{1}{8}$ of 300

$\frac{1}{2}$ of 300 = 150

$\frac{1}{2}$ of 150 = 75

Therefore $\frac{1}{4}$ of 300 = 75

$\frac{1}{8} = \frac{1}{2}$ of $\frac{1}{4}$

Therefore $\frac{1}{8}$ of 300 = $\frac{1}{2}$ of 75 = 37 $\frac{1}{2}$

b. One eighth of even hundreds: no fractional remainders

$\frac{1}{8}$ of 1600 = n

$\frac{1}{8}$ of 400 = n

$\frac{1}{8}$ of 200 = n

One eighth of whole hundreds: odd hundreds: fractional remainders:

$\frac{1}{8}$ of 100 = n

$\frac{1}{8}$ of 300 = n

$\frac{1}{8}$ of 500 = n

$\frac{1}{8}$ of 700 = n

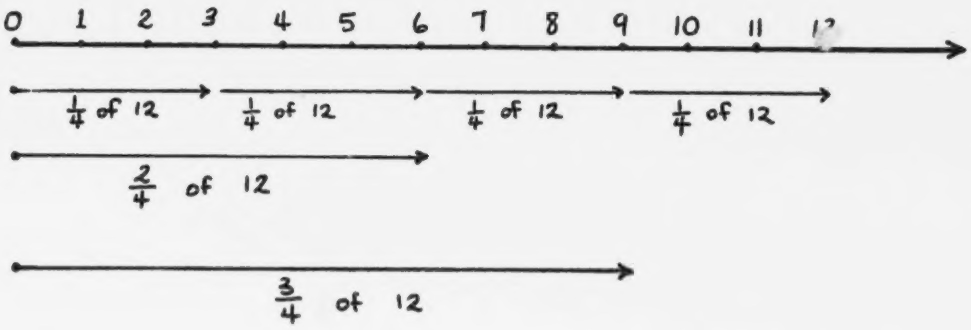
6. Children may find $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$ of numbers in the same way.
For example:

$\frac{1}{3}$ of 7200 = n

$$\begin{array}{l} \frac{1}{3} \text{ of } 6000 = 2000 \\ \frac{1}{3} \text{ of } 1200 = 400 \\ \hline \frac{1}{3} \text{ of } 7200 = 2400 \end{array}$$

7. Finding fractional parts of a number where the numerator of the fraction is greater than 1.

Introduce with a number ray. For example $\frac{3}{4}$ of 12 = n



Discuss with children:

$\frac{2}{4}$ is how many times as great as $\frac{1}{4}$?

$\frac{2}{4}$ of 12 is how many times as great as $\frac{1}{4}$ of 12?

$\frac{3}{4}$ is how many times as great as $\frac{1}{4}$?

$\frac{3}{4}$ of 12 is how many times as great as $\frac{1}{4}$ of 12 ? etc.

Since $\frac{1}{4}$ of 12 = 3, then $\frac{3}{4}$ of 12 = $\square$

Suggested problem: (Related to measures) How many inches are there in $\frac{2}{3}$ of a yard? $\frac{2}{3}$ of 36 = n

$\frac{1}{3}$ of $36 = 12$

$\frac{2}{3}$ is twice $\frac{1}{3}$

Therefore $\frac{2}{3}$ of $36 = 2 \times 12$

$\frac{2}{3}$ of $36 = 24$

Have children solve for "n".

$\frac{1}{4}$ of $36 = 9$, then $\frac{3}{4}$ of $36 = n$

$\frac{1}{8}$ of $56 = 7$, then $\frac{7}{8}$ of $56 = n$

$\frac{1}{6}$ of $72 = 12$, then $\frac{5}{6}$ of $72 = n \times 12$ etc.

8. Suggested exercises for practice

a. $\frac{1}{4}$ of a number $= \frac{1}{2}$ of $\frac{1}{\square}$ of a number.

$\frac{1}{8}$ of a number $= \frac{1}{2}$ of $\frac{1}{\square}$ of a number.

$\frac{1}{8}$ of a number $= \frac{1}{4}$ of $\frac{1}{\square}$ of a number.

$\frac{1}{8}$ of a number $= \frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{\square}$ of a number.

$\frac{1}{6}$ of a number $= \frac{1}{2}$ of $\frac{1}{\square}$ of a number.

$\frac{1}{6}$ of a number $= \frac{1}{3}$ of $\frac{1}{\square}$ of a number.

$\frac{1}{9}$ of a number $= \frac{1}{3}$ of $\frac{1}{\square}$ of a number.

$\frac{1}{10}$ of a number $= \frac{1}{2}$ of $\frac{1}{\square}$ of a number.

$\frac{1}{10}$ of a number $= \frac{1}{5}$ of $\frac{1}{\square}$ of a number.

b. Complete the following equations to make true statements.

$\frac{1}{2}$ of $100 = n$ $\quad$ $\frac{1}{4}$ of $100 = n$ $\quad$ $\frac{1}{2}$ of $\frac{1}{2}$ of 100

$\frac{1}{2}$ of $200 = n$ etc. $\quad$ $\frac{1}{4}$ of $200 = n$ etc.

$\frac{1}{2}$ of $1000 = n$ $\quad$ $\frac{1}{4}$ of $1000 = n$

$\frac{1}{2}$ of $2000 = n$ etc. $\quad$ $\frac{1}{4}$ of $2000 = n$ etc.

$\frac{1}{8}$ of 100 = n $\quad \frac{1}{2}$ of $\frac{1}{4}$ of 100 or

$\frac{1}{4}$ of $\frac{1}{2}$ of 100

$\frac{1}{8}$ of 1000 = n

$\frac{1}{8}$ of 2000 = n etc.

c. Since $\frac{1}{2}$ of 56 = 28
$\frac{1}{4}$ of 56 = $\frac{1}{2}$ of ☐
and $\frac{1}{4}$ of 56 = n

Since $\frac{1}{3}$ of 156 = 52
$\frac{1}{6}$ of 156 = $\frac{1}{2}$ of ☐
and $\frac{1}{6}$ of 156 = n

Since $\frac{1}{2}$ of 128 = 64
$\frac{1}{4}$ of 128 = $\frac{1}{2}$ of ☐
and $\frac{1}{4}$ of 128 = n

Since $\frac{1}{2}$ of 156 = 78
$\frac{1}{6}$ of 156 = $\frac{1}{3}$ of ☐
and $\frac{1}{6}$ of 156 = n

d. Find $\frac{1}{4}$ of 300 $\quad \frac{1}{8}$ of 400 $\quad \frac{1}{2}$ of 96 $\quad \frac{5}{6}$ of 60 $\quad \frac{1}{4}$ of 328

e. If $\frac{1}{2}$ of 600 is 300, $\frac{1}{4}$ of 600 is _____; and $\frac{1}{8}$ of 600 is _____.

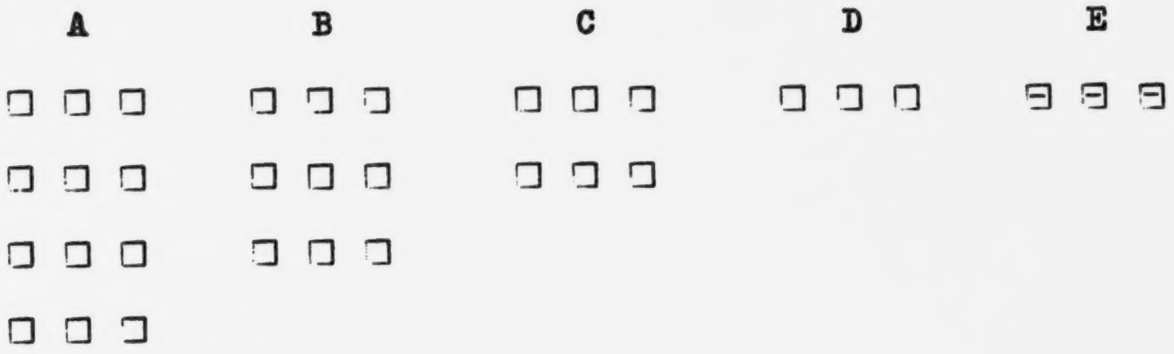
f. John saves $\frac{1}{3}$ of all he earns. One day he earned 96 cents. How much did he save?

g. There are 128 children in Grade 5. $\frac{1}{4}$ of their dental slips have been brought in. How many slips have been brought in?

h. 648 program covers are to be made by the children in Fifth Grade. If Mrs. Jones' class makes $\frac{1}{6}$ of the covers, how many covers will her class make?

Interpretation of Finding a Fractional Part of a Number as Multiplication

1. Have children consider the following diagrams:



How many sets of threes are there in A? [4 sets of the threes]
How many sets of threes are there in B? in C? in D? in E?

As children respond, teacher records horizontally:

4 sets of threes = 12 4 threes = 12 $4 \times 3 = 12$
2 sets of threes = 6 2 threes = 6 $2 \times 3 = 6$
1 set of three = 3 1 three = 3 $1 \times 3 = 3$

2. Ask children to observe the pattern, then to find:

$\frac{1}{2}$ set of threes = $\frac{3}{2}$ $\frac{1}{2}$ three = $\frac{3}{2}$ $\frac{1}{2} \times 3 = \frac{3}{2}$

Have children compare the various ways of expressing the above.

Is 4 x 3 another way of writing 4 of the threes?
Give another way of recording $\frac{1}{2}$ of 3. $\left[\frac{1}{2} \times 3\right]$

3. Arriving at generalizations

Children should record the following using the "x" sign and solve:

$\frac{1}{4}$ of 3 = $\square$ $\frac{1}{5}$ of 2 = $\square$ $\frac{1}{3}$ of 9 = $\square$

Children then examine $\frac{1}{4} \times 3 = \square$, $\frac{1}{5} \times 2 = \square$, etc. and discuss methods of solution. They should be guided to express the generalization in their own words.

4. Extend the understanding to multiplying a whole number by a fraction whose numerator is greater than 1.

Record $\frac{2}{3}$ of 12 as $\frac{2}{3} \times 12$; Record $\frac{3}{4}$ of 5 as $\frac{3}{4} \times 5$;

Record $\frac{5}{8}$ of 72 as $\frac{5}{8} \times 72$.

5. Selecting "of" or "x"

Children should understand that some times it is more practical to think in terms of "of", and at other times to think in terms of "x".

e.g. $\frac{3}{4}$ of 20 as $\frac{1}{4}$ of $20 = 5$ or $\frac{3}{4} \times 20 = \frac{60}{4} = 15$

therefore $\frac{3}{4}$ of $20 = 15$

Which way do you prefer in each of the following? Why?

$\frac{2}{3}$ of 12 or $\frac{2}{3} \times 12$ $\frac{3}{8}$ of 16 or $\frac{3}{8} \times 16$
$\frac{2}{5}$ of 48 or $\frac{2}{5} \times 48$ $\frac{5}{6}$ of 45 or $\frac{5}{6} \times 45$
$\frac{7}{10}$ of 120 or $\frac{7}{10} \times 120$ $\frac{4}{7}$ of 134 or $\frac{4}{7} \times 134$

Another Algorithm for Finding Fractional Parts of Numbers

1. Reinforce renaming a fraction as a whole number using the Identity Element, 1.

For example: $\frac{12}{3} = \frac{12 \div 3}{3 \div 3} = \frac{12 \div 3}{1} = 3\overline{)12} = 4$

Suggested exercises:

$$\frac{24}{8} = \frac{24 \div \square}{8 \div \square} = 24 \div \square = \square\overline{)24} = n$$

$$\frac{150}{25} = \frac{150 \div \square}{25 \div \square} = 150 \div \square = \square\overline{)150} = n$$

$$\frac{2800}{7} = \frac{2800 \div \square}{7 \div \square} = 2800 \div \square = \square\overline{)2800} = n$$

2. Evaluate and /or reinforce finding one half, one third, one fourth, etc. of a number. (Using one half, etc. as the operator on the number.)

   Suggested exercises

   When we divide a number into 2 equal parts, we are finding one ______ of it.

   When we divide a number into 3 equal parts, we are finding one ______ of it.

   When we divide a number into 5 equal parts, we are finding one ______ of it.

   When we divide a number into 7 equal parts, we are finding one ______ of it.

   To find $\frac{1}{7}$ of 294 divide 294 into n equal parts.

3. Discuss as you develop the following:

   Problem: There are 168 children in the Fifth Grade. $\frac{1}{4}$ of them are in the Glee Club. How many children are in the Glee Club?

$$\frac{1}{4} \text{ of } 168 = n$$

$$\begin{aligned} \frac{1}{4} \times 168 &= \frac{168}{4} \\ &= \frac{168 \div 4}{4 \div 4} \\ &= \frac{168 \div 4}{1} \\ &= 4\overline{)168} \\ &= 42 \end{aligned}$$

   Solution: n = 42

   42 = 168 divided into 4 equal parts

   Problem: The community issued 2200 tickets for the dance festival. Grade 5 sold $\frac{1}{8}$ of the tickets. How many tickets did Grade 5 sell?

   $\frac{1}{8}$ of 2200 = n   n = 2200 divided into 8 equal parts.

   Answer: Grade 5 sold 275 tickets.

✶ **Problem:** Grade 6 sold $\frac{3}{8}$ of the 2200 tickets that the Community Association issued for the dance festival. How many tickets did Grade 6 sell?

Refer to the problem suggested for finding $\frac{1}{8}$ of 2200 and discuss:

How you would then find $\frac{3}{8}$ of 2200.

Since $\frac{1}{8}$ of 2200 is $8\overline{)2200}$ or 275, then $\frac{3}{8}$ of 2200 is n x 275

$$\left(\begin{array}{r} 275 \\ \underline{\times\ 3} \\ 825 \end{array}\right)$$

Grade 6 sold 825 tickets.

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Find: a. $\frac{5}{6}$ of 60 $\frac{1}{4}$ of 328

   b. $\frac{1}{4}$ divided into 3 parts

   c. 9 divided into thirds = n

2. Mark the example that gives the larger product:

   $3 \times \frac{3}{4}$ or 3×3; 5×5 or $5 \times \frac{5}{6}$

3. Mother filled $\frac{3}{4}$ of a book of trading stamps. If the book can hold 1200 stamps, how many stamps has she collected?

4. Complete the following:

   $\frac{1}{3} \times 12 = \square$ $12 \times \frac{1}{3} = \square$ $\frac{4}{5} \times 15 = \square$ $15 \times \frac{4}{5} = \square$

5. Verbal problems involving finding fractional parts of numbers may be found in textbooks.

GEOMETRY AND MEASUREMENT

UNIT 43 - MEASUREMENT: WEIGHT

Objectives: To help children find number of ounces in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ pound

To help children find weight that cannot be measured directly.

To introduce concept of Ton

To help children develop tables of relationships

To provide practice in changing to larger and to smaller units.

TEACHING SUGGESTIONS

1. Reinforce number of ounces in 1 pound.

2. Extend development to include number of ounces in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ pound.

The following are suggested experience situations involving weights:

Estimating quantities and cost of refreshments needed for a party.

Sharing quantities and costs among pupils.
Problem situation: Buying candy for the party.
Chicken corn is sold in 4 ounce bags or by the pound.
How shall we buy it? Why?

Keeping individual records to show weight in pounds and fractional parts of pounds. Then, noting gain or loss in weight.

Discussing required postage in relation to weight of mail. Referring to printed table of Post Office for information about rates of 1st class mail, 2nd class mail, air mail, etc. Developing problems based on this information.

Listing items bought by the ounce of $\frac{1}{4}$ pound

Discussing reasons for purchasing small quantities of spices, dried mushrooms, grated cheese, shelled nuts.

Considering size of family when planning menus.

Comparing prices with weight to determine best value.

3. Developing ways of finding weight that cannot be measured directly.

   For a baby or pet: weigh an adult holding baby or pet. Weigh adult. Deduct weight of adult.

   For contents of a jar or pan: weigh empty jar, then filled jar. Deduct weight of empty jar.

4. Suggested practice exercises

   a. A cake recipe requires $1\frac{1}{4}$ pounds of butter. Mother wishes to make double the amount. She will need ____ pounds or ______ ounces.

   b. Mark the following True or False.
   If false, correct the statement.

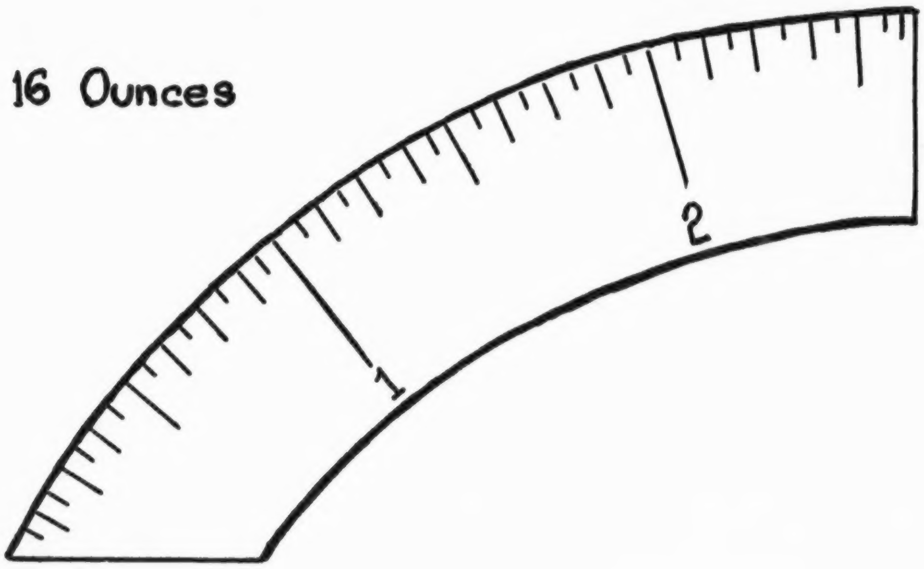
   $2\frac{3}{4}$ lb. $=$ 44 oz.
   8 oz. $= \frac{1}{4}$ lb.
   $1\frac{1}{8}$ oz. $>$ 18 oz.

   c. Complete the following:

   $2\frac{1}{2}$ lb. $= \square$ oz.
   16 oz. $= \frac{\square}{8}$ lb.
   The number of ounces in $\frac{1}{4}$ lb. is _____.

5. Collect pictures of dials or portions of dials of weighing devices. Have children explain how the device is used and read. Include a hanging spring scale, bathroom scale and equal arm balances.

   Construct a model of the dial on a meat scale. Divide each pound interval to show ounce divisions.



6. Introduce unit of Ton (2000 lbs.)

   Suggested Experiences

Family Car	Discuss weight in relation to license fee.
Trucks	Capacity is measured in tons. Have children look at the side of trucks and report findings to class.
Elevators	Notices in elevators state limit of weight in terms of ton. Estimate the average weight per person before they find the number of people who can safely use an elevator.
Bridges	Limits of weight are postea at the terminals.

   Reinforce equivalents

   Reinforce equivalents between ounces and pounds; pounds and tons: Experience situations involving change to smaller units.

Purchasing packaged and canned goods; comparing quantities to aid in determining the more economical purchases.

7. Develop tables of relationships.

Tables of relationships may be prepared by the children.

Fractional Parts of Pounds Expressed in Ounces	Fractional Parts of the Ton Expressed in Pounds
1 lb. = □ oz.	1 ton = □ lb.
$\frac{1}{2}$ lb. = □ oz.	$\frac{1}{2}$ ton = □ lb.
$\frac{1}{4}$ lb. = □ oz.	$\frac{1}{4}$ ton = □ lb.
$\frac{1}{8}$ lb. = □ oz.	
$\frac{3}{4}$ lb. = □ oz.	

8. Provide practice in changing to larger and smaller units.

20 oz. = 1 lb. □ oz. or 1 $\frac{1}{4}$ lb.
3000 lb. = 1 ton □ lb. or 1 $\frac{1}{2}$ tons
1 $\frac{3}{4}$ lb. = 16 oz. and □ oz. = □ oz.
2 $\frac{1}{4}$ ton = 4000 lb. and □ lb. = □ lb.

9. Discuss letters and numbers on the side of trucks in New York State and their meaning.

NYUW - New York Unladen Weight
NYML - New York Maximum Load
NYMGW - New York Maximum Gross Weight

Devise problems to find the net weight of a load on a truck.

EVALUATION and / or PRACTICE
SUGGESTED PROBLEMS

1. Candy is sometimes sold in 8 oz. bags. The storekeeper bought 25 lb. can of candy. How many 8 oz. bags can be filled?

2. The cost of sending a 1 ounce letter or package by First Class Mail is 5 cents for 1 ounce. John mailed a small package which weighed $\frac{1}{4}$ lbs. How much did he pay for stamps?

3. A 2 lb. box is half full of sugar. How many ounces of sugar are in the box?

4. Pick up one dozen new pencils. About how much do you think they weigh?

   1 lb. $\frac{1}{4}$ lb. 3 oz. 8 oz.

   How can you check the estimate?

5. The clerk in the Post Office weighed a package for Mrs. White. She sent it by First Class Mail and had to pay 40 cents. Rate: 5 cents per oz.

   a. Did the package weigh more or less than 1 lb.?
   b. How many ounces did the package weigh?

6. A box of mustard is labeled, Contents, 2 oz. The mustard weighs what part of a pound?

7. A bale of cotton weighs $\frac{1}{4}$ ton. A truck carried 48 bales to a ship.

   a. How many tons were on the truck?
   b. How many pounds did each bale weigh?

8. Two ounces of meat are put into each sandwich at the "Sandwich Shop".

   a. How many sandwiches can be made from 1 pound of meat?
   b. How many sandwiches can be made from 1 pound 6 ounces?
   c. How many pounds are needed to make 2 dozen sandwiches?

9. Is it safe for an empty truck weighing 2500 pounds to cross a bridge when the sign reads, "Limit 3 Tons"? Yes? No? Why?

10. A truck marked "Capacity 3 Tons" can carry a load weighing . . . . pounds.

11. It is safe for an elevator in a Department Store to carry a load of a $\frac{1}{2}$ ton. How many children, each weighing about 50 pounds, can ride in it at one time?

12. Additional problems may be found in textbooks.

GEOMETRY AND MEASUREMENT

UNIT 44 - MEASUREMENT: TIME

Objectives: To extend understanding of periods of calendar time
To introduce concept of seconds in a minute
To organize tables to show relationships
To introduce concept of time zones

TEACHING SUGGESTIONS

Clock Time

1. Discuss day's activities done by the clock.

2. Introduce need for seconds as a unit of measure

   Timing athletic events
   Taking pictures, developing pictures
   Testing pulse reading
   Television programs

3. Introduce concept of seconds in a minute.
   Use a watch and a stop watch.

4. Organize tables of time in various ways. For example;

   60 sec. = 1 _____
   60 min. = 1 _____
   24 hr. = 1 _____
   7 days = 1 _____
   52 weeks = 1 _____

5. Reinforce the relationships between units of time and fractional parts of these units, and develop other tables. For example;

   60 min. = _____ hr.
   30 min. = _____ hr.
   15 min. = _____ hr.
   45 min. = _____ hr. etc.

6. Discuss the meaning of an 8 hour day; a 40 hour week; overtime; etc.

*7. Have children explore: **(Optional)**

Naval Observatory Time

Timekeeping through the ages:
History of telling time by sandglass, time candle,
position of the sun, shadows, water clocks, sundial,
Twenty-four Hour Clock, Ship's Bells

8. Suggested exercises and problems

a. Underline the unit of measure which you would use for each of the following:

Sue wants to time the baking of a cake

minutes seconds days

Measuring the time for the 100 yard dash

seconds hours minutes and seconds

b. Insert the correct symbol, > or < :

45 min. _____ $\frac{1}{4}$ hr.

$\frac{1}{3}$ min. _____ 15 sec.

$\frac{1}{4}$ hr. _____ 25 min.

c. Arrange these measures from longest time to shortest time:

a minute 70 seconds an hour a day a half minute
a second a half hour 20 hours 25 hours

d. Fill in the blanks:

$2\frac{1}{2}$ hr. = _____ min.

$\frac{1}{4}$ hr. = _____ min.

3 min. = _____ sec.

120 sec. = _____ min.

$\frac{1}{2}$ day = _____ hr.

30 sec. = _____ min.

45 min. = _____ hr.

80 min. = _____ hr.

* e. A ship sailed from New York at 2 P.M. on Monday and docked in London 5 days later at 12 Noon. How many hours did the trip take?

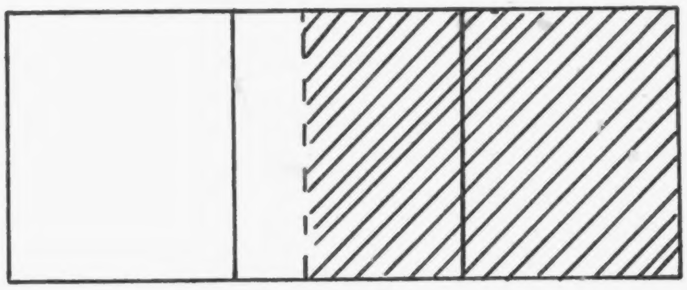
f. Mark the following statements true (T) or false (F).

There are 50 seconds in a minute.
75 minutes = 1 hr. 15 min. = 1 $\frac{1}{2}$ hr.
10 minutes = $\frac{1}{6}$ hr.
6:45 P.M. to 9:15 P.M. is 2 $\frac{1}{2}$ hrs.

g. Draw a line from the time recorded in Column A to the words which have the same meaning in Column B.

Column A	Column B
9:45	10 minutes after 11
6:50	10 minutes to 3
2:50	half past 3
7:15	50 minutes after 6
11:10	a quarter past 7
3:30	a quarter to 10

h. This whole bar is drawn to show 3 hours.



Which of the following does the shaded part of the bar show?

about 45 min. about 1 hour and 45 min.
about 1 hour and 15 min.

Calendar Time

Concept of Leap Year

1. Reinforce number of days in a year.

   Tell children it takes the earth 365 days, 5 hours, 48 minutes, and 49 seconds to make its way around the sun so the length of one year is not exactly 365 days. (This is called a Solar Year.)

   Ask children:

   > **Since the year is considered to consist of 365 days what adjustment is made for the 5 hours, 48 minutes and 49 seconds of the Solar Year?**
   >
   > Approximately what part of a day is this additional time?
   >
   > [$\frac{1}{4}$ of a day]
   >
   > How many years will it take until enough time has accumulated to make another day?
   >
   > Can you tell how our calendar is adjusted to take care of this time?

   Tell children that correction is made on our calendar every fourth year by adding one day to the month of February. This fourth year is called Leap Year and has 366 days.

*2. Children may explore: **(Optional)**

   Various types of calendars

   World, Gregorian, Hebrew, Chinese, Mayan

3. Have children investigate meaning of: decade, score, century, half century, centennial year, sesquicentennial year, generation.

   Relate these terms to time periods in Social Studies, Art, Music, etc. For example, How long ago did the Westward Movement occur?

   Strengthen meaning of terms by constructing various Time Lines:
   - Individual Time Line of child's life. Include ideas of decade.
   - Extend Time Line by decades to a century.
   - Divide centuries into half centuries and decades.
   - Refer to Social Studies textbooks for suggested situations for Time Lines.

4. Suggested problems for practice:

   a. Arrange these measures from shortest time to longest time:

      17 days 14 years a season 2 months
      3 weeks 2 decades a century

   b. What did Abraham Lincoln mean when he said, "Four score and seven years ago" ?

   c. In what century:

      Do we live?
      Did the American Revolution take place?
      Did Columbus discover New World?
      Did Julius Caesar live?

   d. The first Thanksgiving Day took place in 1621.
      In what century was that?
      How many years ago did that happen?

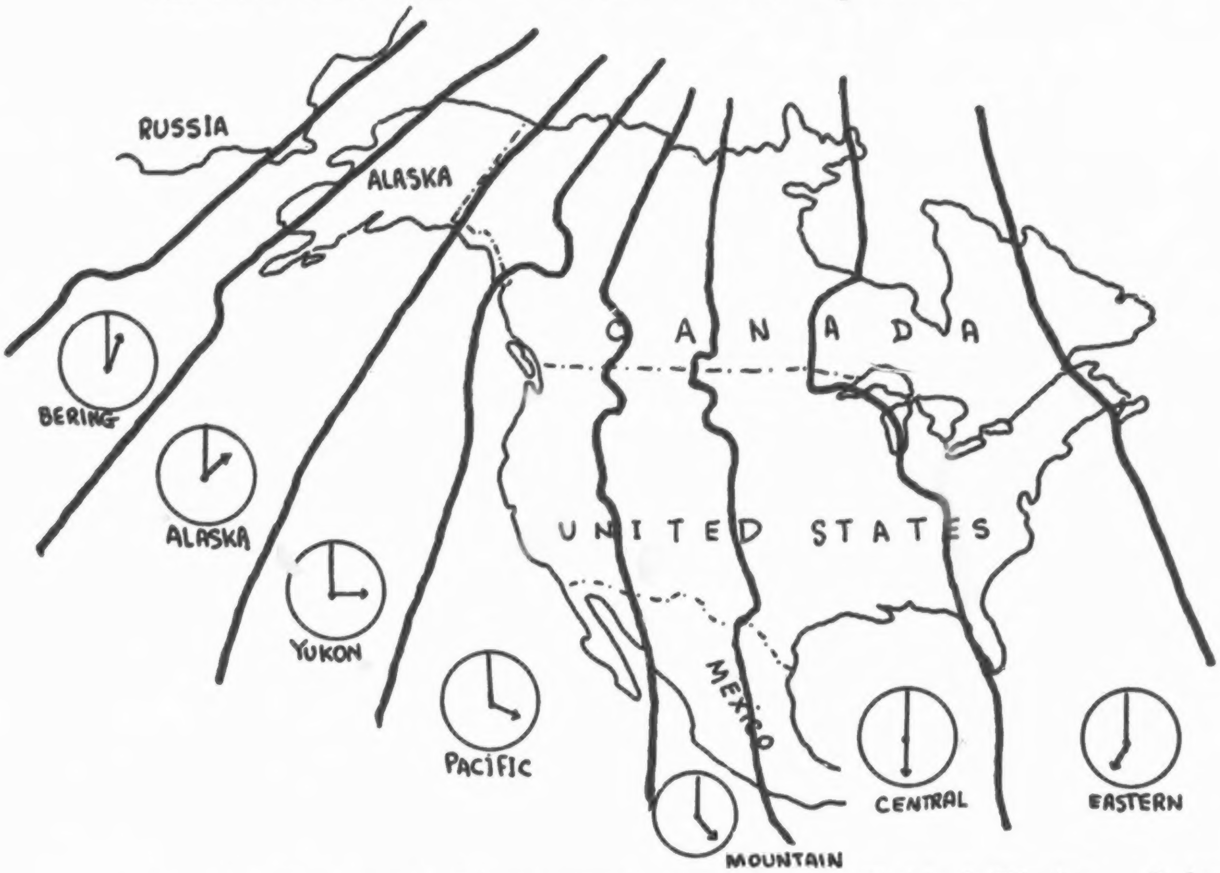
      The Star Spangled Banner was written in 1814.
      In what century was it written?

      Betsy Ross made our flag in 1777.
      What century was that?
      Which half of the century?

   e. How many scores of years are there before the next century?
      On what date will the twenty-first century begin?
      In what century did the year 34 A.D. occur?

Time Zones in the United States

Discuss time zones as shown in the map below.


RUSSIA
ALASKA
CANADA
UNITED STATES
MEXICO
BERING
ALASKA
YUKON
PACIFIC
MOUNTAIN
CENTRAL
EASTERN

1. In how many time zones are the 50 United States of America? [7]

2. As you travel from New York to San Francisco do you move your watch ahead or back as you enter each time zone? Why?

3. A Television program shown at 7:00 P.M. in Denver will be seen in New York at what time?

[5:00 P.M.]

4. When it is noon in the State of Hawaii, what time is it in the State of Pennsylvania?

[5:00 P.M.]

5. Consult newspaper on any given day for time of sunrise in New York to discuss the following:

   When it is sunrise in New York, what time is it in Alaska?

6. Refer to the rap above to find whether the United States shares time zones with any other countries.

SETS; NUMBER; NUMERATION

UNIT 45 - NUMERATION: ROMAN SYSTEM

NOTE TO TEACHER

Continued study of the Roman System of Numeration will reinforce an understanding of the Decimal System of Numeration, will emphasize the efficiency of the Decimal System of Numeration and will reinforce the distinction between number and numeral.

Objective: To compare Roman and Hindu-Arabic Systems of Numeration

TEACHING SUGGESTIONS

1. Test children's ability to translate Roman numerals into Arabic numerals and the reverse, for numbers through 500.

2. Introduce M as the Roman numeral representing 1000. Have the children derive the Roman numeral for 900 by using the subtractive principle: CM = 100 less than 1000.

3. Reinforce the principles used to combine the seven symbols:
   I, V, X, L, D, C, M

   The Rule of Addition: VI, LX, DC, etc.
   The Rule of Subtraction: IV, XL, CD, etc.
   The Rule of Repetition: III, XXX, CCC, etc. (In later Roman Notation, a symbol is repeated no more than 3 times)

   Symbols which cannot be repeated: V because 2 fives = X;
   L because 2 fifties = C;
   D because 2 five hundreds = M.

Have children explain why we say the Roman System of Numeration involves "Repetition"; "Addition"; "Subtraction". Do any of these principles apply to The Hindu-Arabic System?

4. Children may construct a chart to compare numbers represented by Roman numerals and Hindu-Arabic numerals.

ONES:	1	2	3	4	5	6	7	8	9
	I	II	III	IV	V	VI	VII	VIII	IX
TENS:	10	20	30	40	50	60	70	80	90
	X	XX	XXX	XL	L	LX	LXX	LXXX	XC
HUNDREDS:	100	200	300	400	500	600	700	800	900
	C	CC	CCC	CD	D	DC	DCC	DCCC	CM
THOUSANDS:	1000								
	M								

5. Children may construct a chart to make comparisons between systems of numeration. For example,

Hindu Arabic	Roman
Number of Symbols: 10	Number of Symbols: 7
1, 2, 3, 4, 5, 6, 7, 8, 9, 0	I V X L C D M
Symbol "0" indicates a zero number of ones, tens, hundreds, etc.	No symbol for "zero" number of I's, V's, X's, etc.
Notation	Notation
System - simple	System - complicated
7 70 700 Two digits needed to write all three numerals	VII LXX DCC Six letters needed to write all three numerals
888 1 digit needed	DCCCLXXXVIII 12 letters needed

(chart continued)

Base: 10	Base: 10
10 = 10 x 1 100 = 10 x 10 1000 = 10 x 10 x 10	X = 10 x 1 C = 10 x 10 M = 10 x 10 x 10 For convenience, symbols for midway points are used. I. . . V . . . X I. . . L . . . C I. . . D . . . M
Principles	**Principles**
Addition Place Value	Repetition Addition Subtraction

Have children refer to the chart to:

> Compare the number of symbols used in the Roman System (7) with the number of symbols used in the Decimal System. (10)
> Tell how the use of the symbol for zero makes the Decimal System more efficient.
> Think out relative values (ratios) of the basic symbols in the Roman System;
> 2 to 1, 5 to 1, etc.
>
> M = (2 x D) D = (5 x C) C = (2 x L) L = (5 x X)
> X = (2 x V) V = (5 x 1) I = 1
>
> Compare the principles of the Roman System with those of the Hindu-Arabic System. They discuss the advantages of our system of numeration.

6. Children should make the following comparisons:

Hindu Arabic System of Numeration	Roman System of Numeration
Relative Values	**Relative Values**
The relative value represented by a particular digit is 10 times the value of the same digit to its right.	The relative value represented by the basic letters are 5 to 1, 2 to 1, 5 to 1, 2 to 1, 5 to 1, and 2 to 1.

(Relative values continued)

Thous.	Hun.	Tens	Ones
1	1	1	1

V = 5 times I
X = 2 times V
L = 5 times X
C = 2 times L
D = 5 times C
M = 2 times D

Computation is Simple

	Tens	Ones
	3	7
+	5	[illegible]
	9	[illegible]

Computation is Complicated

Fifties	Tens	Fives	Ones	
	XXX	V	II	
L		V	III	
L	XXX	VV	IIIII	=

LXXXXV = XCV

2 3
x 3
6 9

Fifties	Tens	Fives	Ones	
	XX		III	23
			III	x 3
	XX		III	(1 x 23)
	XX		III	(1 x 23)
	XX		III	(1 x 23)
	XXXXXX		IIIIIIIII	
L	X		IX	

Ask children:

a. In the Base 10 System upon what does the value of each digit in the numeral 444 depend?

b. How would 444 be written in the Roman System?

c. What is the sum of the values of each basic symbol in LXVI ?

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Write the following as Roman Numerals: 39; 44; 279; 500; 177.

2. Write the following as Arabic Numerals:

LXXII; XLV; LIII; MC; MMCD.

3. Write the Roman Numerals counting by 100, from 100 to 1000.

4. Read the following:

   CCC [3 hundred] MCD [1400] MCMLXIV [1964]

5. Tell what rule was applied to combine each of the following:

   XV CM DIII

6. Show that:

   XIX + XXXI = L [19 + 31 = 50] M - CD = DC [1000 - 400 = 600]
   C - XL = LX [100 - 40 = 60] MDCC + CCC = MM [1700 + 300 = 2000]

7. From each set of numerals select the numeral that does not belong. Explain your reasoning.

25 + 50	75	LXXV	95 - 15	[95 - 15]
600	DC	CD	500 + 100	[CD]
400 + 100 + 6	DXXVI	526	2 x 263	[400+100+6]

8. In the numerals XLIII and LXIII the same symbols are used. Why do they not name the same number?

9. In adding CMLII and CDX, which of the following is correct? Why

   CM + CD + LXII , MDCCLXII

* 10. What historical event do you associate with each of these Roman Numerals?

   MDCCLXXVI [1776] MCDXCII [1492] MDCCCXLIX [1849]
   MDCCCLXIII [1863]

* 11. Try to multiply XLVII by XXXVII. Then multiply 47 by 37. Compare the two products.

* 12. Why do you think the Romans did not include a symbol for zero with their numerals?

SETS; NUMBER; NUMERATION

UNIT 46 - NUMERATION: EXTENDING UNDERSTANDING OF DECIMAL (BASE 10) SYSTEM

Objectives: To develop Place Value through Billion
To extend ability to use expanded notation

TEACHING SUGGESTIONS

1. Evaluate children's understanding of the Decimal System of Numeration. Suggested questions follow:

   Draw a line under the numeral that applies to the word symbol.

   Three million five thousand twenty-eight
   3,005,280   3,005028   3,500,028   None of these

   Draw a line under the "6" which represents six million.
   6,260,000   56,351   7,627,000   326,803   None of these

   How many sets of 10 objects can be combined to make a set of 2160 objects?
   6   21   160   216

   How many sets of 100 objects yield 7000 objects?
   7   70   700   7000

   How many thousands does it take to make a million?
   1 thousand   10 hundred   1 hundred   10 thousand

   The circled 5 is how many times as great as the underlined 5?

   $\textcircled{5}5,\underline{5}55$

   500   10,000   1,000   100   [100]

   Which of these means
   10,000:   100 thousand   100 hundreds   100 tens   None of these
   [100 hundreds]

100,000: 1000 thousands 1000 tens 100 hundreds None of these
[none]
1,000,000: 1000 tens 1000 hundreds 1000 thousands None of these
[1,000 thousands]

2. Reinforce place value through millions.

Record the numeral 1,111,111

Have children record the same numeral on a place value chart and indicate periods.

Millions			Thousands			Units		
		Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
		1	1	1	1	1	1	1

Discuss:

The role of "ten" in our system of numeration indicating values as shown below.

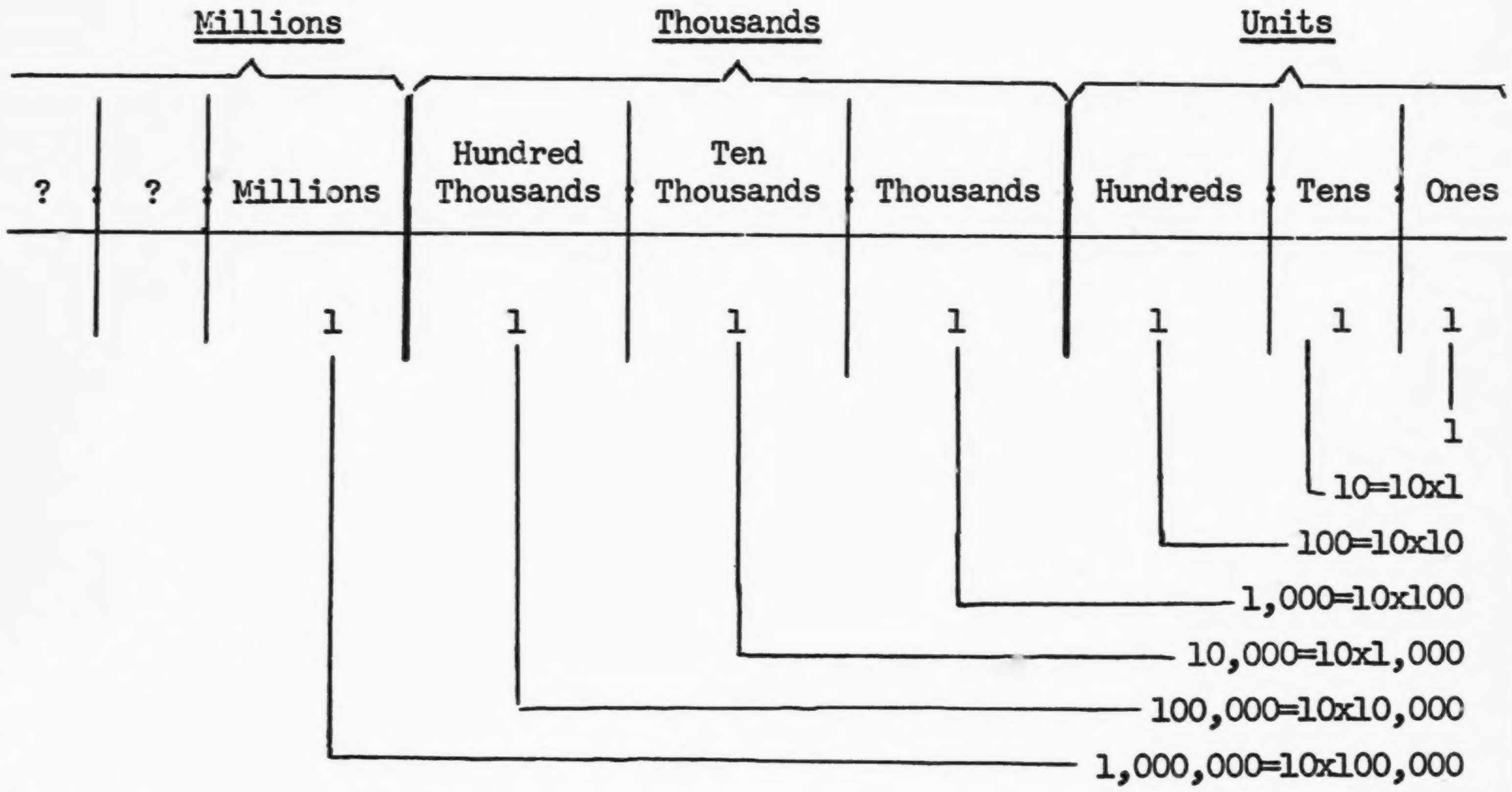
(10 sets of 10 for 100; 10 sets of 100 for 1000; etc.)

The number of digits; Why ten digits are sufficient.

The Place Value represented by each digit (ten times as great as the same digit one place to the right; $\frac{1}{10}$ as great as the same digit one place to the left)

3. Develop Place Value through billion.

A Place Value chart such as the following may be presented.


Millions
Thousands
Units
?
?
Millions
Hundred Thousands
Ten Thousands
Thousands
Hundreds
Tens
Ones
1
1
1
1
1
1
1
1
10=10x1
100=10x10
1,000=10x100
10,000=10x1,000
100,000=10x10,000
1,000,000=10x100,000

Ask children to:

Record the digit 1 to the left of millions place.
Suggest a label for the unmarked place and reasons for your choice.
Evaluate each suggestion in terms of the 1 and 10 relationship.
[10 sets of 1,000,000 are 10,000,000]
Label the unmarked place Ten Millions.
Extend the chart to show 10,000,000= 10 x 1,000,000
Extend the chart to show hundred million, to billion, etc.
For example, 10 x 10,000,000 = 100,000,000

* Discuss one billion in England vs. one billion in America.
(In England one billion = one million million)

4. Repeat development above substituting the digit 2 in each place.

Have children compare the 2 in the tens place with the 2 in the ones place.

What does the 2 in the tens place mean? [2 tens or 20]
Twenty is how many times as large as 2? [10 times]
Can you name 20 in another way? [2 sets of 10; 2 x 10]

Have children compare the 2 in the hundreds place with the 2 in the tens place and continue as with tens.

Children should continue the comparisons, moving one place at a time to the left.

5. Extend understanding of expanded notation.

Children complete the following:

$$77{,}777 = 70{,}000 + \square + 700 + \Delta + 7$$

$$= (7 \times n) + (7 \times 1{,}000) + (7 \times 100) + (7 \times 10) + (7 \times 1)$$

$$5{,}243 = \square + \Delta + 40 + 3$$

$$= (5 \times n) + (2 \times 100) + (4 \times n) + (3 \times 1)$$

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Separate these numerals into periods.

6769954 30003548 230000300 30023021

2. Write the following as numerals using digits:

One hundred million, fifty thousand four
Three million, five thousand ten
Thirty million, two hundred six thousand, fifty
Forty thousand forty

3. Write each of the following in two ways to show the meaning of place value.

999,999 65,327 28,043

20,000 + 8,000 + 40 + 3
(2 x 10,000) + (8 x 1000) + (4 x 10) + (3 x 1)

✱ 4. How many complete thousands, hundreds, tens, ones are in the following:

3,214,268

3,214 thousands
32,142 hundreds
321,426 tens
3,214,268 ones

547

No thousands
5 hundreds
54 tens
547 ones

OPERATIONS

UNIT 47 - SET OF WHOLE NUMBERS: ADDITION AND SUBTRACTION

Objectives: To maintain computational skill in Addition and Subtraction
To provide practice in applying Properties of Addition and Subtraction

TEACHING SUGGESTIONS

1. Provide practice in:

Applying the Associative Property of Addition for all whole numbers: $(a+b) + c = a + (b+c)$

Have children complete the equations below. Regroup the second addend "crossing a ten". Children should understand that in each case "8" is renamed as (5+3) first. Then by the associativity of addition:

$$5 + 8 = (5+\square) + 3 = n \quad [\text{here } 5 + (5+3) = (5+5) + 3]$$
$$25 + 8 = (25 + \square) + 3 = n$$
$$235 + 8 = (235 + 5) + \square = n$$
$$3225 + 8 = (3225 + \square) + 3 = n$$

Using Inverse Operations

"Adding n" and "subtracting n" are inverse operations because one undoes what the other does. For example:

$$12 + 4 = 16 \text{ vs. } 16 - 4 = 12$$

$$13 - 5 = 8 \text{ vs. } 8 + 5 = 13$$

Children relate adding to subtracting.

$$9 + 4 = 13 \qquad 19 + 4 = 23$$
$$13 - \square = 9 \qquad \square - 4 = 19$$

Then extend to adding to and to subtracting from numbers in the hundreds, thousands.

$119 + 4 = n$ $\qquad$ $6758 + 7 = n$

$\square - 4 = 119$ $\qquad$ $\square - 7 = 6758$

Applying the Commutative Property of Addition: $a + b = b + a$ for all whole numbers a, b.

Present:

Higher decade addition, e.g. $6 + 39 = 39 + \square = n$;
$7 + 48 = 48 + \square = n$; etc.

Adding to numbers in the hundreds, e.g. $3 + 138 = 138 + \square = n$
$5 + 246 = 246 + \square = n$

Adding to numbers in the thousands, e.g. $2 + 1119 = 1119 + \square = n$
$5 + 2238 = 2238 + \square = n$

Children should summarize:

The order of the addends may be interchanged without changing the sum. Reversing the addends sometimes helps to make an addition easier: e.g.

$7 + 5$ is easier than $5 + 7$

$1119 + 2$ is easier than $2 + 1119$

2. Evaluate understanding of the application of mathematical principles.
Complete the following statements:
Since $748 + 76 + 393 = 1217$, $\qquad$ $76 + \square + 748 = 1217$

For $960 + 750 = n$; underline the expressions below that can replace n.

$(960 + 700) + 50$ $\qquad$ $(9 + 60\ ) +)\ 7 + 50)$
$(900 + 700) + (60 + 50)$ $\qquad$ $(9 + 7) + (60 + 50)$

Change each false statement to make it a true statement

$76 + 0 = 0 + 76$ $\qquad$ $97 - 58 = 58 - 97$

$127 - 9 = 128 - 10$ $\qquad$ $230 + 70 = 70 + 230$

Determine an efficient method for evaluating each of the following. Explain.

$2654 + 2999 + 3999 = n$ [$2654 + 3000 + 4000 - 2$]

$6542 - 4998 = n$ [$6542 - 5000 + 2$]

$1218 + 1219 = n$ [$1218 + 1218 + 1$]

$3050 - 1526 = n$ [$3050 - 1525 - 1$]

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. If $T = 625$, $R = 245$ and $V = 875$, find the value of n.

   $T + V = n$ [1500] $V - T = n$ [250] $T + R + V = n$ [1745]

   $T + n = V$ [250] $T - n = R$ [380]

2. Arrange the following in increasing order beginning with the sum or remainder of least value.

   Solve each example to check your work.

Set A	Set B
3475 - 240	13,509 + 417
3475 - 218	13,509 + 421
3475 - 256	13,509 + 468
3475 - 231	13,509 + 442
3475 - 264	13,509 + 439

3. Replace each frame with a numeral and solve for n.

   $375 + 75 = 375 + \square + 50 = n$ $215 - 45 = 215 - \square - 30 = n$

   $185 + 45 = 185 + \square + 30 = n$ $450 - 75 = 450 - \square - 25 = n$

   $1265 + 55 = 1265 + \square + 20 = n$ $118 - 36 = 118 - \square - 18 = n$

4. Write an equation to find the perimeter of a rectangle whose width is 36" and whose length is 78".

5. Write the equation to find the temperature at 10 P.M. if it dropped 14° from the 8 A.M. temperature of 23°.

* 6. Suppose A + B + C = D **(Optional)**

   a. If B was doubled, how much would D increase?

   b. If each addend was doubled, how much would the sum be increased?

   c. If A was increased by 6 and C was decreased by 12, what would be the effect on D.

7. Select suitable verbal problems from textbooks and from other curriculum areas.

OPERATIONS

UNIT 48 - SET OF WHOLE NUMBERS: MULTIPLICATION; PROPERTIES APPLIED; HORIZONTAL AND VERTICAL FORMAT

Objectives: To extend multiplication of whole numbers: products in the tens of thousands; one factor through 9.

To give practice in multiplying with dollars and cents.

To introduce multiplication with both factors through 99.

To extend development involving one factor through 99, the other factor in the hundreds.

TEACHING SUGGESTIONS

1. Continue to provide practice. Refer to units 36, 37, 38 and textbooks for: equations, patterns, procedures in computation, (horizontal and vertical format).

2. Make sure that children understand:

   Renaming numbers of larger value, e.g.

   $24,246 = 20,000 + 4,000 + 200 + 40 + 6$

   Place value concepts

   Regrouping for exchange

Products In the Tens of Thousands: One Factor Through 9, Vertical Format

1. Present a problem situation: An airplane pilot makes 3 trips each week to a city 4,143 miles away. How far does he travel each week?

2. Children should:

   Estimate product
   Record the estimate
   Compute
   Compare the results of the computation with the estimate
   Check the work by addition and / or the distributive property

For example, for the problem above:

Estimate: $n > 12{,}000$ miles; $n > 12{,}300$ miles

Computation:

$$\begin{array}{r} 4143 \\ \underline{x\ \ \ 3} \\ 12{,}429 \end{array}$$

Check: (The standard algorithm for multiplication depends upon the application of the Distributive Property of Multiplication with respect to Addition).

$$\begin{aligned} 3 \times 4143 &= (3 \times 4000) + (3 \times 100) + (3 \times 43) \\ &= 12{,}000 + 300 + 129 \\ &= 12{,}300 + 129 \\ &= 12{,}429 \end{aligned}$$

Children should compare product with estimate.

3. Suggested practice exercises.

$$\begin{array}{r} 3212 \\ \underline{x\ \ \ 8} \end{array} \qquad \begin{array}{r} 1402 \\ \underline{x\ \ \ 9} \end{array} \qquad \begin{array}{r} 6058 \\ \underline{x\ \ \ 4} \end{array} \qquad \text{etc.}$$

Dollars and Cents

Horizontal Format

1. Suggested Multiplication Exercises: Products through $999.99

Ask children to solve the following using the Distributive Property of Multiplication with respect to Addition:
For example:

$$\begin{aligned} 2 \times \$75.16 &= 2 \times (\$75 + .16) \\ &= \square + \$.32 = n \end{aligned}$$

$2 \times \$\ 55.34 = \square + \$\ .68 = n$
$2 \times \$332.45 = \square + \$\ .90 = n$
$2 \times \$427.06 = \square + \$\ .12 = n$
$2 \times \$280.25 = \square + \$\ .50 = n$
$2 \times \$265.90 = \square + \$1.80 = n$
etc.

$2 \times \$\ 84.26 = n + \$\ .52 = n$
$7 \times \$\ 21.02 = n + \$\ .14 = n$
$3 \times \$323.30 = n + \$\ .90 = n$
$6 \times \$104.10 = n + \$\ .60 = n$
$4 \times \$215.40 = n + \$1.60 = n$
etc.

Vertical Format

2. Introduce exercises which involve products through $999.99. For example:

a. $\begin{array}{r} \$86.87 \\ \underline{\times\ 2} \end{array}$ Estimate: $n > \$160$ or $n > \$172$

Compute:

$$\begin{array}{r} \$86.87 \\ \underline{\times\ 2} \\ \$173.74 \end{array}$$

Children note the regrouping of tens of dollars as hundreds and tens.

Check:

$$\begin{array}{r} \$\ 86.87 \\ \underline{86.87} \\ \$173.74 \end{array} \quad \text{or} \quad \begin{array}{rl} 2 \times \$86. & = \$172.00 \\ 2 \times \$\ \ .87 & = \underline{\$\ \ 1.74} \\ & \ \ \ \$173.74 \end{array}$$

b. $\begin{array}{r} \$72.14 \\ \underline{\times\ 5} \end{array}$ Estimate: More than $350 or More than $360

Compute:

$$\begin{array}{r} \$\ 72.14 \\ \underline{\times\ 5} \\ \$360.70 \end{array}$$

Children note that 36 tens of dollars is regrouped as 3 one hundred dollars and 6 tens of dollars.

Check: By addition or by applying the Distributive Property of Multiplication with respect to Addition.

c. $\begin{array}{r} \$235.61 \\ \underline{\times\ 3} \end{array}$ Estimate: Between $600 and $715

Compute:

$$\begin{array}{r} \$235.61 \\ \underline{\times\ 3} \\ \$706.83 \end{array}$$

Children note the value and position of each digit of the numeral in the product.

Check: By addition or by applying the Distributive Property of Multiplication with respect to Addition.

Both Factors Through 99

Horizontal Format

1. Make sure children are able to multiply by 10 and multiples of 10.

2. Reinforce additions needed to derive unknown products from known products. For example:

Adding Groups in Sequence

$10 \times 34 = n$ (read as 10 thirty-fours)
$20 \times 34 = n + 340 = ?$
$30 \times 34 = n + 340 = ?$
$40 \times 34 = n + 340 = ?$
through
$90 \times 34 = n + 340 = ?$

Doubling Groups

$10 \times 34 = ?$
$20 \times 34 = \square + \square = n$
$40 \times 34 = \square + \square = n$
$80 \times 34 = \square + \square = n$

$30 \times 34 = \square$
$60 \times 34 = \square$

Doubling and Adding Groups

$10 \times 34 = \square$
$20 \times 34 = \square + \square = ?$
$30 \times 34 = \square + 340 = ?$

$40 \times 34 = \square + \square = ?$
$80 \times 34 = \square + \square = ?$
$90 \times 34 = \square + 340 = ?$

$30 \times 34 = ?$
$60 \times 34 = ?$
$70 \times 34 = \square + 340 = n$

$30 \times 34 = 1020$
$60 \times 34 = \square + \square$
$90 \times 34 = \square + 1024 = n$

3. Suggested development

a. For $11 \times 39 = n$

A child may say:
10 thirty nines = 390
1 thirty nine = 39
390 and 30 are 420 and
9 more are 429

Teacher could record:
$10 \times 39 = 390$
$1 \times 39 = 39$
$390 + 30 + 9 = 429$

b. For 21×34

A child may say:
10 thirty fours = 340
20 thirty fours = 680
1 thirty four = 34
680 and 34 = 714

Teacher could record:
$10 \times 34 = 340$
$20 \times 34 = 680$
$1 \times 34 = 34$
$680 + 34 = 714$

c. Have children complete exercises like the following:

$20 \times 36 = n + n = n$
$40 \times 36 = n + n = n$
$41 \times 36 = n + 36 = ?$ Why?

$? \times 23 = 230$ $\quad ? \times 12 = 120$
$? \times 20 = 400$ $\quad ? \times 30 = 900$ etc.

$23 \times 16 = n + 48 = ?$
$21 \times 83 = n + 83 = ?$
$27 \times 32 = n + 224 = ?$

d. Have children change each false sentence to make it a true sentence.

$12 \times 104 = 1040$
$14 \times 216 = 2160 + 800$
$20 \times 379 = 3790 + 3790$
$9 \times 246 = 2460 - 10$
$16 \times 138 = 6 \times 138$ plus 6×138
$32 \times 17 = 17 \times 23$

Vertical Format

Reinforce children's understanding that:

> The standard algorithm for multiplication depends upon the application of the Distributive Property of Multiplication with respect to Addition.

1. Present problem:

> Copies of the school newspaper are being sent to 14 classes this morning. If each class receives 32 copies, how many newspapers will be distributed?

2. Have children suggest various ways of solving the multiplication.

3. Introduce the conventional form. Suggest that children compute beginning with the ones.

$$\begin{array}{r} 32 \\ \underline{\times 14} \end{array} \text{ newspapers}$$

Estimate: More than 320;
or
More than 440

Record:
$$\begin{array}{r} 32 \\ \underline{\times\ 4} \\ 128 \end{array} \qquad \begin{array}{r} 32 \\ \underline{\times\ 10} \\ 320 \end{array} \qquad \begin{array}{r} 128 \\ \underline{+\ 320} \\ 448 \end{array}$$

$$\begin{array}{rl} 32 & \text{newspapers (refer to problem and compute again)} \\ \underline{x14} & \\ 128 & (4 \times 32) \\ \underline{320} & (10 \times 32) \\ 448 & \text{newspapers } (14 \times 32) \end{array}$$

4. Extend development to include products in the thousands.
For example: For the problem shown below we think of 60 as 60 + 2.
Therefore $36 \times 62 = 36 \times (60 + 2) = (36 \times 60) + (36 \times 2)$

When we compute using the conventional vertical algorithm we apply the commutative property and begin with the ones. Then
$36 \times 62 = (36 \times 2) + (36 \times 60)$

$$\begin{array}{rl} 36 & \\ \underline{x62} & \\ 72 & (2 \times 36) \\ \underline{2160} & (60 \times 36) \\ 2232 & (62 \times 36) = (2 \times 36) + (60 \times 36) \end{array}$$

Children may check by using the Commutative Property: $36 \times 62 = 62 \times 36$

$$\begin{array}{r} 62 \\ \underline{x36} \end{array}$$

5. Extend development to include one factor in the hundreds.

$$\begin{array}{rl} 136 & \\ \underline{x\ 72} & \\ 272 & (2 \times 136) \\ \underline{9520} & (70 \times 136) \\ 9792 & (72 \times 136) = (2 \times 136) + (70 \times 136) \end{array}$$

Ask children to explain how the Distributive Property is being used here.

Multiplication exercises may be presented in various ways.

$56 \times 807 = n$
Multiply 807 by 56
Multiply 56 and 807
Find the product of 56 and 807
If one factor is 56, and the other factor is 807, what is the product?

$$\begin{array}{r} 807 \\ \underline{x56} \end{array}$$

Exercises may be read in a variety of ways.

Problem	May Be Read as:
$62 \times 458 = n$	62 four hundred fifty eights
	62 times 458
$\begin{array}{r} 458 \\ \underline{\times\ 62} \end{array}$	458 taken 62 times
	458 multiplied by 62

Numerals may be read in various ways. Children decide on the most convenient way for a specific purpose.

7800 may be read as: seventy eight hundred or
7 thousand 8 hundred.

Products may be estimated in a variety of ways.

For: $\begin{array}{r} 132 \\ \underline{\times\ 27} \end{array}$

Children may think:

$10 \times 130 = 1300$
$20 \times 130 = 2600$ or
$30 \times 130 = 3900$

$10 \times 132 = 1320$
$20 \times 132 = 2640$
$30 \times 132 = 3960$

Children should record estimate only not the thinking involved:

$27 \times 132 > 2600$ or $27 \times 132 > 2640$

6. Extend development to include products in the tens of thousands.

Suggested problem: Mr. Smith donated 596 library books to each of 62 schools in our city. How many books did Mr. Smith donate?

$62 \times 596 = n$

Children should estimate the product first. Estimates will vary according to the ability of the child.

Estimates:

Since $10 \times 500 = 5000$
then $60 \times 500 = 6$ times as much
$60 \times 500 = 30{,}000$
$62 \times 596 > 30{,}000$
or

Since $60 \times 600 = 36{,}000$
then 62×600 is $36{,}000 + 1200$
62×596 is about $37{,}200$
Children record: n is about 37,200

or
Since $10 \times 600 = 6000$ (596 is almost 6000)
then 60×600 is 6 times as much
$60 \times 600 = 36{,}000$
62×596 is about $36{,}000$

Children should record estimate, compute, then compare products with estimates.

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. 62×596

$$\begin{array}{r} 596 \\ \underline{\times\ 60} \end{array} \qquad \begin{array}{r} 596 \\ \underline{\times\ \ 2} \end{array} \qquad \begin{array}{r} 596 \\ \underline{\times\ 62} \end{array}$$

2.

$$\begin{array}{rcl} & & 596 \\ & & \underline{\times\ 62} \\ \square & = & 60 \times 596 \\ \underline{\Delta} & \underline{=} & \underline{\ \ 2 \times 596} \\ n & = & 62 \times 596 \end{array} \qquad\qquad \begin{array}{rcl} 596 & & \\ \underline{\times\ 62} & & \\ 1192 & = & \square \times 596 \\ \underline{35760} & \underline{=} & \underline{\square \times 596} \\ 36952 & = & \square \times \Delta \end{array}$$

3. Present other multiplication problems. Include zeros in one factor.

$$\begin{array}{r} 368 \\ \underline{\times\ 79} \end{array} \qquad \begin{array}{r} 607 \\ \underline{\times\ 58} \end{array} \qquad \begin{array}{r} 740 \\ \underline{\times\ 86} \end{array} \qquad \begin{array}{r} \$3.64 \\ \underline{\times\ \ \ 58} \end{array} \qquad \begin{array}{r} \$2.98 \\ \underline{\times\ \ \ 95} \end{array}$$

Check by multiplying with the tens digit first.

4. In how many different ways can you find the product of each of the following:

50×12; $6 \times 4 \times 25$; 16×125; $A \times B \times C$

5. Additional practice exercises may be found in textbooks.

OPERATIONS

UNIT 49 - SET OF WHOLE NUMBERS: DIVISION; QUOTIENTS IN THOUSANDS; INTRODUCE DIVISORS GREATER THAN NINE

Objectives: To maintain skill in division of whole numbers: divisors through 9, quotients through 999.

To extend ability to divide: quotients in the thousands, divisors through 9.

To develop dividing by numbers greater than 9.

TEACHING SUGGESTIONS

Dividing By Numbers Through 9

1. **Test** children's ability to multiply numbers through 9 by:
   Ten and Multiples of Ten (10 x 8, 40 x 8, 70 x 8)
   Hundreds and Multiples of One Hundred (100 x 7, 300 x 7, 800 x 7)
   Thousand and Multiples of One Thousand (1000 x 9, 2000 x 9, 1200 x 9)

2. **Test** children's ability to estimate quotients.
   Present problems with simpler dividends to less mature children.

   Suggested problem: A gift of 2864 library books was donated to our city schools. If 9 books were sent to each school, how many schools received the books? (How many nines are in 2864?)

   Estimate: Children may think:
   $100 \times 9 = 900$
   $200 \times 9 = 1800$
   $300 \times 9 = 2700$
   $400 \times 9 = 3600$

   Children record $n > 300$, $n < 400$
   n is between 300 and 400

3. **Suggested Problem:** How many sets of sixes in a collection of 5871 things?

Estimate: $n < 1000$; $n > 900$; n is between 900 and 1000.

Computation:

$$\begin{array}{r|r} 978 & \\ 6\overline{)5871} & \\ \underline{5400} & 900 \\ 471 & \\ \underline{420} & 70 \\ 51 & \\ \underline{48} & \underline{8} \\ 3 & 978 \end{array}$$

Verify the answer by using the Distributive Property of Multiplication with respect to Addition.

$$\begin{array}{rlr} 900 & \text{sixes} = & 5400 \\ 70 & \text{sixes} = & 420 \\ \underline{8} & \text{sixes} = & \underline{48} \\ 978 & & 5868 \end{array}$$

Answer: 978 sets of six and 3 left over.

978 sixes = 5868;
5868 + 3 = 5871

Children compare solution with estimate.

4. Introduce problems with quotients in the thousands. Children should continue to arrive at answers in a variety of ways depending upon their level of ability. For example:

$$\begin{array}{r|r} 1405 & \\ 3\overline{)4216} & \\ \underline{900} & 300 \\ 3316 & \\ \underline{1800} & 600 \\ 1516 & \\ \underline{1500} & 500 \\ 16 & \\ \underline{15} & \underline{5} \\ 1 & 1405 \end{array}$$

$$\begin{array}{r|r} 1405 & \\ 3\overline{)4216} & \\ \underline{1500} & 500 \\ 2716 & \\ \underline{1500} & 500 \\ 1216 & \\ \underline{1200} & 400 \\ 16 & \\ \underline{15} & \underline{5} \\ 1 & 1405 \end{array}$$

Using 1000 in the partial quotient

$$\begin{array}{r|r} 1405 & \\ 3\overline{)4216} & \\ \underline{3000} & 1000 \\ 1216 & \\ \underline{1200} & 400 \\ 16 & \\ \underline{15} & \underline{5} \\ 1 & 1405 \end{array}$$

Using 1000 and multiples of 100 in the partial product

$$\begin{array}{r|r} 2553 & \\ 3\overline{)7659} & \\ \underline{3000} & 1000 \\ 4659 & \\ \underline{3000} & 1000 \\ 1659 & \\ \underline{1500} & 500 \\ 159 & \\ \underline{150} & 50 \\ 9 & \\ \underline{9} & \underline{3} \\ & 2553 \end{array}$$

Shortening the Computation

$$\begin{array}{r|r} 2553 & \\ 3\overline{)7659} & \\ \underline{6000} & 2000 \\ 1659 & \\ \underline{1500} & 500 \\ 159 & \\ \underline{150} & 50 \\ 9 & \\ \underline{9} & \underline{3} \\ & 2553 \end{array}$$

5. Present other exercises.

$7\overline{)489}$ $9\overline{)1623}$ $6\overline{)23614}$ $8504 \div 8$

How many nines are there in 28,642? $n \times 8 = 1728$

Dividing By Numbers Greater Than 9

1. Test ability to multiply numbers through 99 by tens; by hundreds. For example:

$20 \times 20 = n$
$40 \times 20 = n$
$80 \times 20 = n$

$30 \times 20 = n$
$60 \times 20 = n$
$90 \times 20 = n$

$20 \times 45 = n$
$30 \times 45 = n$
$40 \times 45 = n$

$100 \times 10 = n$
$200 \times 10 = n$
$300 \times 10 = n$

$200 \times 40 = n$
$400 \times 40 = n$
$500 \times 40 = n$

$200 \times 34 = n$
$400 \times 34 = n$
$800 \times 34 = n$

2. Emphasize estimating quotients first and as a check.

Exercise: How many twenty-threes in 851?

$23\overline{)851}$ $851 \div 23 = n$ $n \times 23 = 851$

By Doubling Numbers

Since $10 \times 23 = 230$
$20 \times 23 = 460$
and $40 \times 23 = 920$

Since the dividend is 851; the quotient < 40.

By Doubling and Adding Numbers

Since $10 \times 23 = 230$
$20 \times 23 = 460$
and $30 \times 23 = 690$
and $40 \times 23 = 920$

$30 \times 23 = 690$. Since the dividend is 851; the quotient > 30.

Children should record: Estimate: $n > 30$, $n < 40$, n is between 30 and 40.

Then check solution with estimate and verify the quotient.

3. Development

Problem: Tom's job is to distribute the school newspaper to the children on the fourth floor. He has 238 copies. If each class receives 32 newspapers, how many classes will get copies from Tom?

Children should interpret problem: (dividing the papers into stacks or groups of 32).

They can then record symbols: $32\overline{)238}$

Then interpret the symbols: How many thirty-twos are there in 238?

Ask children to solve the problem beginning with as many thirty-twos as they wish.

The first solution should be recorded on the chalkboard as the teacher and the children work together.

Ask the children to solve the same problem in other ways. Various solutions follow:

```
      7
32)238 |
    64 | 2
   174 |
   128 | 4
    46 |
    32 | 1
    14 | 7
```

```
      7
32)238 |
    96 | 3
   142 |
    96 | 3
    46 |
    32 | 1
    14 | 7
```

```
      7
32)238 |
   160 | 5
    78 |
    64 | 2
    14 | 7
```

Solution: 7 classes
14 extra newspapers

Compare the various solutions.

Have children check the quotient by:

Multiplying the quotient by the divisor and adding the remainder.

```
  32 newspapers
x  7
 224 newspapers
```

```
 224 newspapers
+ 14 newspapers
 238 newspapers
```

4. During the initial presentation of division exercises different methods should be encouraged and discussed.

```
     12
23)289 |
    69 | 3
   220 |
   138 | 6
    82 |
    69 | 3
    13 | 12
```

```
     12
23)289 |
   115 | 5
   174 |
   115 | 5
    59 |
    46 | 2
    13 | 12
```

```
     12
23)289 |
    92 | 4
   197 |
   184 | 8
    13 | 12
```

```
     12
23)289 |
   230 | 10
    59 |
    46 | 2
    13 | 12
```

Solution: 12 groups, 13 left over

The advantage of beginning with 10 as the first partial quotient should be noted.

$$\begin{array}{r|r} \multicolumn{1}{r}{\underline{12}} & \\ 31)\overline{392} & \\ \underline{310} & 10 \\ 82 & \\ \underline{62} & \underline{2} \\ 20 & 12 \end{array} \qquad \begin{array}{r|r} \multicolumn{1}{r}{\underline{14}} & \\ 17)\overline{249} & \\ \underline{170} & 10 \\ 79 & \\ \underline{68} & \underline{4} \\ 11 & 14 \end{array} \qquad \begin{array}{r|r} \multicolumn{1}{r}{\underline{17}} & \\ 24)\overline{412} & \\ \underline{240} & 10 \\ 172 & \\ \underline{120} & 5 \\ 52 & \\ \underline{48} & \underline{2} \\ 4 & 17 \end{array}$$

Children can check by using: The Distributive Property of Multiplication with respect to Addition.

$$\begin{array}{r|r} \multicolumn{1}{r}{\underline{12}} & \\ 31)\overline{392} & \\ \underline{310} & 10 \\ 82 & \\ \underline{62} & \underline{2} \\ 20 & 12 \end{array} \qquad \begin{array}{l} 10 \times 31 = 310 \\ \ \ 2 \times 31 = \ \ 62 \end{array}$$

$$\begin{array}{r} 310 \\ \underline{+\ 62} \\ 372 \end{array} \qquad \begin{array}{r} 372 \\ \underline{+\ 20} \\ 392 \end{array} \qquad \text{or} \qquad \begin{array}{r} 31 \\ \underline{\times\ 12} \\ 62 \\ \underline{310} \\ 372 \end{array} \qquad \begin{array}{r} 372 \\ \underline{+\ 20} \\ 392 \end{array}$$

Various algorithms for $23)\overline{719}$ follow:

$$\begin{array}{r|r} \multicolumn{1}{r}{\underline{31}} & \\ 23)\overline{719} & \\ \underline{230} & 10 \\ 489 & \\ \underline{230} & 10 \\ 259 & \\ \underline{230} & 10 \\ 29 & \\ \underline{23} & \underline{1} \\ 6 & 31 \end{array} \qquad \begin{array}{r|r} \multicolumn{1}{r}{\underline{31}} & \\ 23)\overline{719} & \\ \underline{460} & 20 \\ 259 & \\ \underline{230} & 10 \\ 29 & \\ \underline{23} & \underline{1} \\ 6 & 31 \end{array} \qquad \begin{array}{r|r} \multicolumn{1}{r}{\underline{31}} & \\ 23)\overline{719} & \\ \underline{690} & 30 \\ 29 & \\ \underline{23} & \underline{1} \\ 6 & 31 \end{array}$$

Solution: 31 groups and 6 left over.

Check: Use Distributive Property

Various algorithms, for 31)1679 follow:

$$\begin{array}{r|r} \multicolumn{1}{r}{54} & \\ 31\overline{)1679} & \\ \underline{620} & 20 \\ 1059 & \\ \underline{620} & 20 \\ 439 & \\ \underline{310} & 10 \\ 129 & \\ \underline{124} & \underline{4} \\ 5 & 54 \end{array}$$

$$\begin{array}{r|r} \multicolumn{1}{r}{54} & \\ 31\overline{)1679} & \\ \underline{1240} & 40 \\ 439 & \\ \underline{310} & 10 \\ 129 & \\ \underline{124} & \underline{4} \\ 5 & 54 \end{array}$$

$$\begin{array}{r|r} \multicolumn{1}{r}{54} & \\ 31\overline{)1679} & \\ \underline{1550} & 50 \\ 129 & \\ \underline{124} & \underline{4} \\ 5 & 54 \end{array}$$

5. Provide practice. Relate to problem situations.

   a. $14\overline{)\ 86}$ $12\overline{)\ 95}$ $21\overline{)\ 87}$ $41\overline{)\ 91}$ $24\overline{)\ 85}$ etc.

   $23\overline{)137}$ $31\overline{)192}$ $25\overline{)217}$ $15\overline{)137}$ $24\overline{)189}$ etc.

   $21\overline{)2364}$ $41\overline{)1765}$ $52\overline{)2137}$ $33\overline{)2638}$ etc.

   b. If the perimeter of a square garden plot is 576 ft., how long is each side?

   c. If an elevator may carry at most 23 people at a time, how many trips must it make to carry down 169 people?

   d. If a grocer has egg cartons that hold a dozen eggs, how many will he need to hold 418 eggs?

6. Additional practice exercises and verbal problems may be found in textbooks.

SETS; NUMBER; NUMERATION

UNIT 50 - SET OF FRACTIONAL NUMBERS: SIXTEENTHS; LOCATING OTHER FRACTIONS ON THE NUMBER LINE

Objectives: To develop concept of sixteenths
To help children make comparisons among fractions
To help children locate $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. of fractions on number line

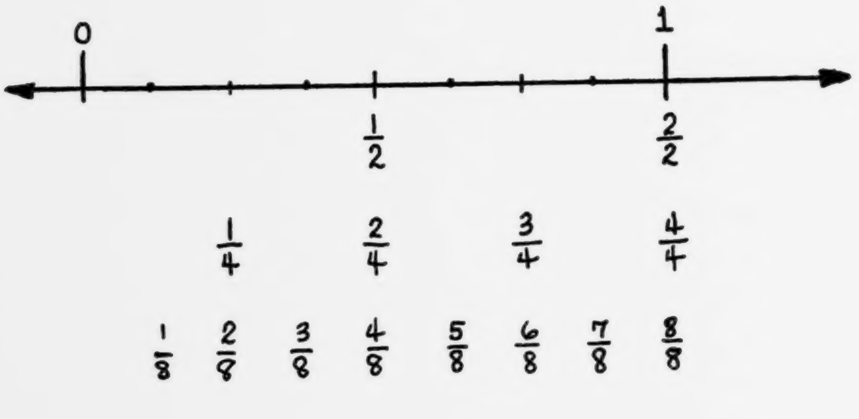
TEACHING SUGGESTIONS

Locating Sixteenths on the Number Line By Successive Bisection

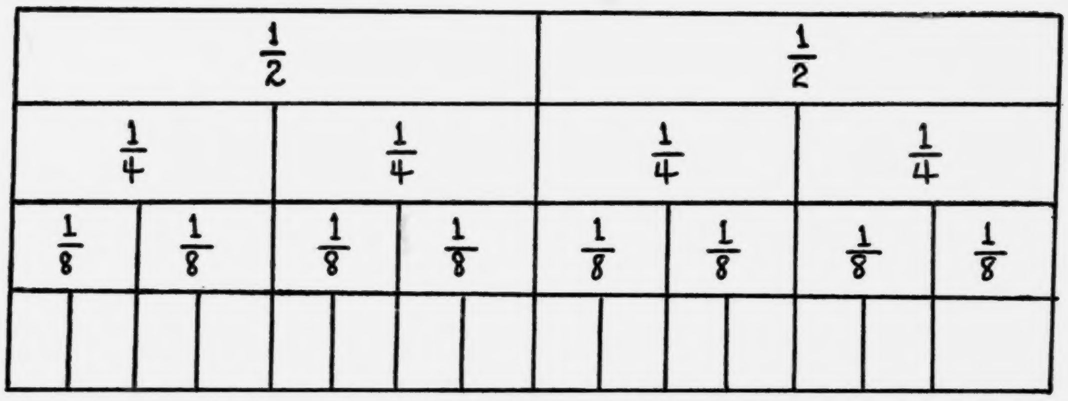
Using One-Half as an Operator

1. **Reinforce finding eighths. Use a number line.**

   Children should draw part of a number line showing 1 unit of length. They divide this segment into 2 equal parts and indicate the midpoint $\left(\frac{1}{2}\right)$, into 4 equal parts and label the endpoint of each part; into 8 equal parts and label each endpoint.



2. Have children develop a chart like the one below by dividing each eighth interval into two equal parts.



Ask children:

What name can we give each unlabeled part? $\left[\frac{1}{16}\right]$ Why?

How many sixteenths are in $\frac{1}{8}$? [2] in $\frac{2}{8}$? . . . . . $\frac{8}{8}$?

How many sixteenths are in $\frac{1}{4}$? [4] in $\frac{1}{2}$? in 1 ?

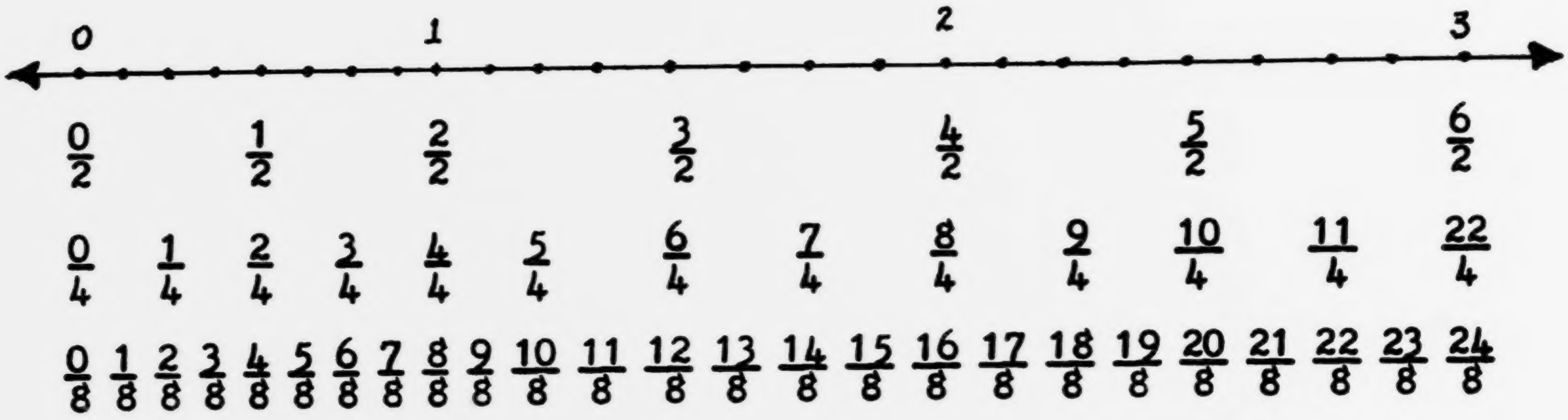
3. Have children use this chart, a number ray, and the symbols > and < to complete the sentences below.

$\frac{1}{2}$ ___ $\frac{1}{4}$ [>] $\frac{3}{8}$ ___ $\frac{1}{4}$ [>] $\frac{3}{4}$ ___ $\frac{5}{8}$ [>]

$\frac{1}{8}$ ___ $\frac{1}{4}$ [<] $\frac{1}{2}$ ___ $\frac{3}{8}$ [>] $\frac{3}{8}$ ___ $\frac{2}{4}$ [<]

$\frac{3}{16}$ ___ $\frac{3}{8}$ [<] $\frac{7}{8}$ ___ $\frac{9}{16}$ [>] $\frac{5}{16}$ ___ $\frac{3}{4}$ [<]

Children refer to the number line below to help answer the questions suggested.


0
1
2
3
0/2
1/2
2/2
3/2
4/2
5/2
6/2
0/4
1/4
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3/4
4/4
5/4
6/4
7/4
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9/4
10/4
11/4
22/4
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1/8
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7/8
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11/8
12/8
13/8
14/8
15/8
16/8
17/8
18/8
19/8
20/8
21/8
22/8
23/8
24/8

Which number is smaller, $\frac{17}{8}$ or $\frac{16}{8}$? $\left[\frac{16}{8}\right]$

Which is farther to the left on the number line, $\frac{16}{8}$ or $\frac{17}{8}$? $\left[\frac{16}{8}\right]$

Which number is smaller $\frac{10}{16}$ or $\frac{12}{8}$? $\left[\frac{10}{16}\right]$

Which is farther to the left on the number line? $\left[\frac{10}{16}\right]$

Which number is smaller, $\frac{17}{8}$ or $\frac{15}{8}$? $\left[\frac{15}{8}\right]$

Which is farther to the left on the number line? $\left[\frac{15}{8}\right]$

Which number is smaller $\frac{16}{16}$ or $\frac{4}{2}$? $\left[\frac{16}{16}\right]$

Which is farther to the left on the number line ?

$\frac{9}{16}$ or $\frac{1}{2}$? ; $\frac{0}{8}$ or $\frac{0}{16}$?

Relate to music: a quarter note is given half the time of a half note; an eighth note is given half the time of a quarter note; etc.

4. Compare a number line with a ruler graduated in sixteenths. Have children show with rulers why $\frac{1}{8}$ may be interpreted as

$\frac{2}{16}$; $\frac{1}{4}$ as $\frac{4}{16}$; etc.

Locating Other Fractions Using One-Half As The Operator

1. Ask children to draw another number line. Divide a unit length into 2 equal parts. Label each part.

   If each sixteenth were divided into 2 equal parts, how many equal parts would there be in 1 interval of length? [32]

   What is each part called? $\left[\frac{1}{32}\right]$

   Children note that taking $\frac{1}{2}$ of a number creates a new number one half the size. Discuss further subdivisions into 2 equal parts (sixty-fourths, one hundred twenty-eighths, etc.)

   What happens to the value of the fraction as the denominator increases? decreases? is doubled? is halved? Why?

2. Have children continue to use number lines and rulers where possible to locate other fractions.

3. Discuss :

   a. The meaning of:

   $\frac{1}{6}$ as $\frac{1}{2}$ of $\frac{1}{3}$ $\quad$ $\frac{1}{12}$ as $\frac{1}{2}$ of $\frac{1}{6}$ $\quad$ $\frac{1}{24}$ as $\frac{1}{2}$ of $\frac{1}{12}$

   $\frac{1}{14}$ as $\frac{1}{2}$ of $\frac{1}{7}$ $\quad$ $\frac{1}{28}$ as $\frac{1}{2}$ of $\frac{1}{14}$ $\quad$ $\frac{1}{56}$ as $\frac{1}{2}$ of $\frac{1}{28}$ etc.

   b. The coordinate point halfway between:

   0 and 1 $\quad$ 0 and $\frac{1}{2}$ $\quad$ 0 and $\frac{1}{4}$ $\quad$ 0 and $\frac{1}{16}$

   etc.

   1 and $1\frac{1}{8}$ $\left[\frac{1}{16}\right]$ $\quad$ 1 and $1\frac{1}{16}$ $\left[\frac{1}{32}\right]$ $\quad$ 1 and $\frac{1}{32}$ $\left[\frac{1}{64}\right]$

Can we continue to divide any segment into 2 equal parts?
What is the limit of such subdivisions?

Locating Other Fractions Using One-Third, One-Fourth, etc. As Operators

Suggested exercises: Use number line or other aids when desirable.

1. $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{\square}$ $\qquad$ $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{\square}$ $\qquad$ $\frac{1}{3}$ of $\frac{1}{10} = \frac{1}{\square}$

   What part of $\frac{1}{6}$ is $\frac{1}{18}$? $\qquad$ $\frac{1}{6}$ is how many times $\frac{1}{18}$?

2. Show different ways of obtaining $\frac{1}{16}$, $\frac{1}{18}$, $\frac{7}{24}$, etc. as the result of subdividing a unit interval.

3. Complete the following:

   Since $\frac{1}{2}$ of $\frac{1}{12} = \frac{1}{24}$ $\qquad$ Since $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$
   then $\frac{1}{2}$ of $\frac{2}{12} = \frac{\square}{24}$ $\qquad$ then $\frac{1}{3}$ of $\frac{2}{5} = \frac{\square}{15}$
   and $\frac{1}{2}$ of $\frac{5}{12} = \frac{\square}{24}$ $\qquad$ and $\frac{1}{3}$ of $\frac{4}{5} = \frac{\square}{15}$

   Since $\frac{1}{4}$ of $\frac{1}{10} = \frac{1}{40}$
   then $\frac{1}{4}$ of $\frac{2}{10} = \frac{\square}{40}$
   and $\frac{1}{4}$ of $\frac{7}{10} = \frac{\square}{40}$

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Tell whether we are considering a number or a numeral when we:

   Add fractions [number]
   Use the decimal form [numeral]
   Change fractions to simpler form [numeral]
   Compare fractions [number]

2. If the interval on a ruler between zero and one is divided into 8 equal parts, what is each equal part called?

3. Draw part of a number line. Label the points that correspond to the numbers 0, 1, 2, 3. Consider the distance between 0 and 1 as the unit of length. Divide each unit of length into 3 equal parts. Label these points.

4. Label the points on the number line that represent the numbers:

$$1\frac{2}{3}, \quad 2\frac{1}{3}, \quad \frac{5}{6}, \quad \frac{7}{6}, \quad \frac{5}{12}$$

5. List 3 fractional numerals that name 0; 1; 2; etc.

6. Rename $\frac{6}{2}$ in 4 different ways.

7. Compare $\frac{3}{5}$ with $\frac{5}{3}$. How do you know which of them is less than 1? more than 1?

8. In the fraction $\frac{12}{12}$, give an interpretation to 12 in the numerator, the 12 in the denominator.

9. Using a ruler with one-fourth inch markings, how can we measure $1\frac{3}{8}$ inches?

10. There is a marker on the Thruway for every tenth of a mile. How many markers are there in $\frac{3}{5}$ of a mile? in $1\frac{3}{10}$ miles? in 100 miles? in n miles?

11. Which of the following are names for whole numbers?

$$\frac{10}{5}, \quad \frac{7}{8}, \quad \frac{0}{3}, \quad \frac{13}{12}, \quad \frac{12}{4}, \quad \frac{6}{1}, \quad \frac{8}{9}$$

12. Insert the correct symbols >, =, < in the placeholder to compare the following:

$\frac{2}{3}$ of a yd. □ $\frac{2}{5}$ of a yd. $\frac{4}{12}$ of an hr. □ $\frac{2}{6}$ of an hr.

$\frac{1}{8}$ of a gal. □ $\frac{1}{2}$ of a qt.

13. Rewrite the following set in order starting with the smallest.

$\frac{9}{8}, \frac{3}{8}, \frac{7}{8}, \frac{5}{8}, \frac{11}{8}$ $\frac{3}{12}, \frac{3}{4}, \frac{3}{5}, \frac{3}{8}, \frac{3}{2}$

$\frac{3}{9}, \frac{1}{2}, \frac{3}{12}, \frac{2}{10}, \frac{1}{6}$

14. Mark each of the following statements as either True or False. Draw a number line for each pair to show the correct relationship.

$\frac{1}{2} > \frac{2}{3}$ $\frac{2}{3} < \frac{3}{4}$ $\frac{17}{8} < \frac{2}{1}$ $\frac{0}{12} < \frac{0}{6}$

15. What is the smallest number of parts into which a unit can be divided? The largest?

16. Show on the number line whether one-fourth or one-half is closer to one-third.

17. As the number of equal parts into which a unit is divided is increased, how does the size of the part change?

GEOMETRY AND MEASUREMENT

UNIT 51 - MEASUREMENT: LENGTH; SCALE DRAWING

Objectives: To extend concepts of length to fractional parts of an inch.
To emphasize approximate measurements.
To help children interpret drawings made to scale.

TEACHING SUGGESTIONS

Fractional Parts of an Inch

1. Have children estimate inches, inch, and half inch using handspan and fingers to show approximate size.

2. Provide each child with a rectangle (paper which can be easily be folded - 1" by $1\frac{1}{2}$ ").

   Fold the paper inch into halves. Discuss halves and compare with 1/2 inch markings on the standard ruler. Provide practice in estimating 1/2 inch (margins, space between lines, space between letters when printing, etc.) Always verify estimates.

3. Provide each child with a strip of durable paper (oaktag), 12 inches by 1 inch, for the construction of his own ruler.

   Reinforce concept of inch by having children use their inch rectangle to mark off the inches on their own oaktag ruler.

   Verify with a standard ruler.

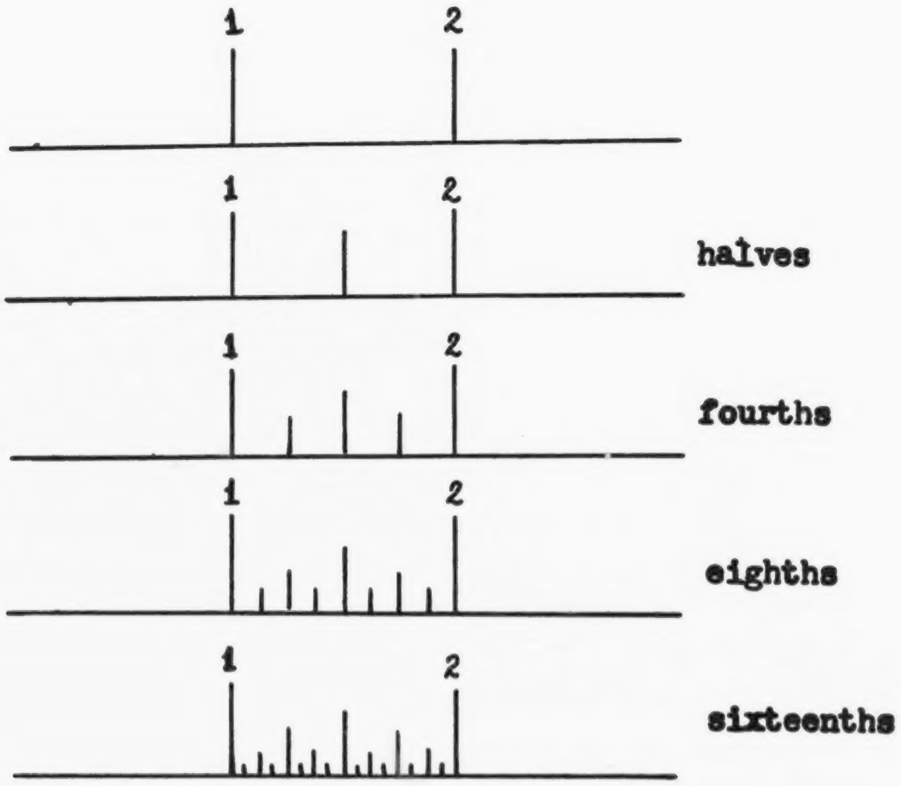
   Discuss with children a convenient place to write the numerals 1 - - - 12 on the oaktag rulers. Compare with standard rulers.

   Have children mark off lengths of $\frac{1}{2}$ inch on their rulers by using the inch rectangle folded in half.

In a similar way have children complete rulers. They can mark off fourths and eighths by relating fourths to halves (1/2 of 1/2), and eighths to fourths (1/2 of 1/4).

4. Reinforce equivalents for halves, fourths, eighths and sixteenths.

Suggestion:



5. Develop the ability to measure the following lengths: 1 inch, 2 inches, 3 inches, etc. starting with any inch marking on the ruler.

Develop the ability to measure the following lengths: 1, 2, or 3 inches starting from any half-inch marking on the ruler; from any marking.

Develop the ability to measure the following lengths: half-inch segments from inch markings, 1/4 inch segments from 1 inch markings.

6. Provide practice in:

Locating various lengths on a ruler.
Drawing various lengths, e.g., 4 inches, $2\frac{1}{4}$ inches, $3\frac{1}{2}$ inches, $5\frac{7}{8}$ inches, etc. using a ruler.

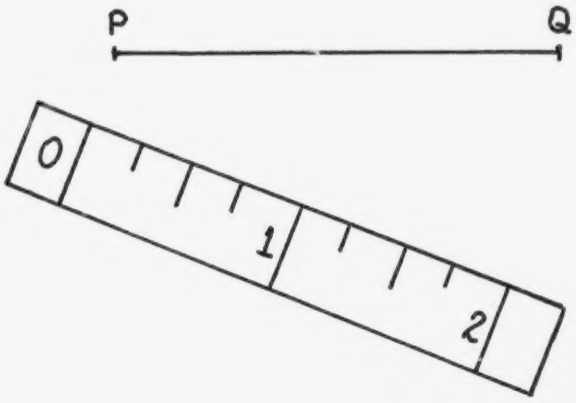
Estimating various lengths, then checking estimates. Counting using fractional parts of the inch, e.g., begin at $3\frac{7}{8}$ inches, count forward by $\frac{3}{4}$ inches; begin at $\frac{3}{16}$ of an inch count forward by $\frac{1}{2}$ inch; etc.

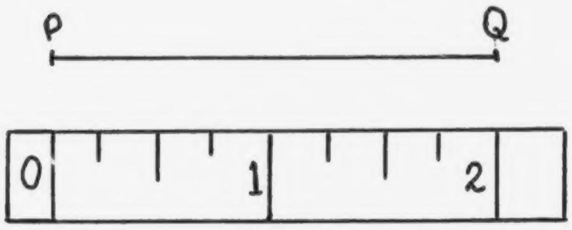
7. Ask children:

How many one-fourths of an inch are there in 1 inch: in 3 inches; in $3\frac{1}{4}$ inches?

Draw a line segment $\frac{3}{4}$ of an inch in length. How many $\frac{1}{8}$ inches in this line segment? How many $\frac{1}{4}$ inches?

Which of these drawings shows the correct way to place a ruler to measure a segment? Why?



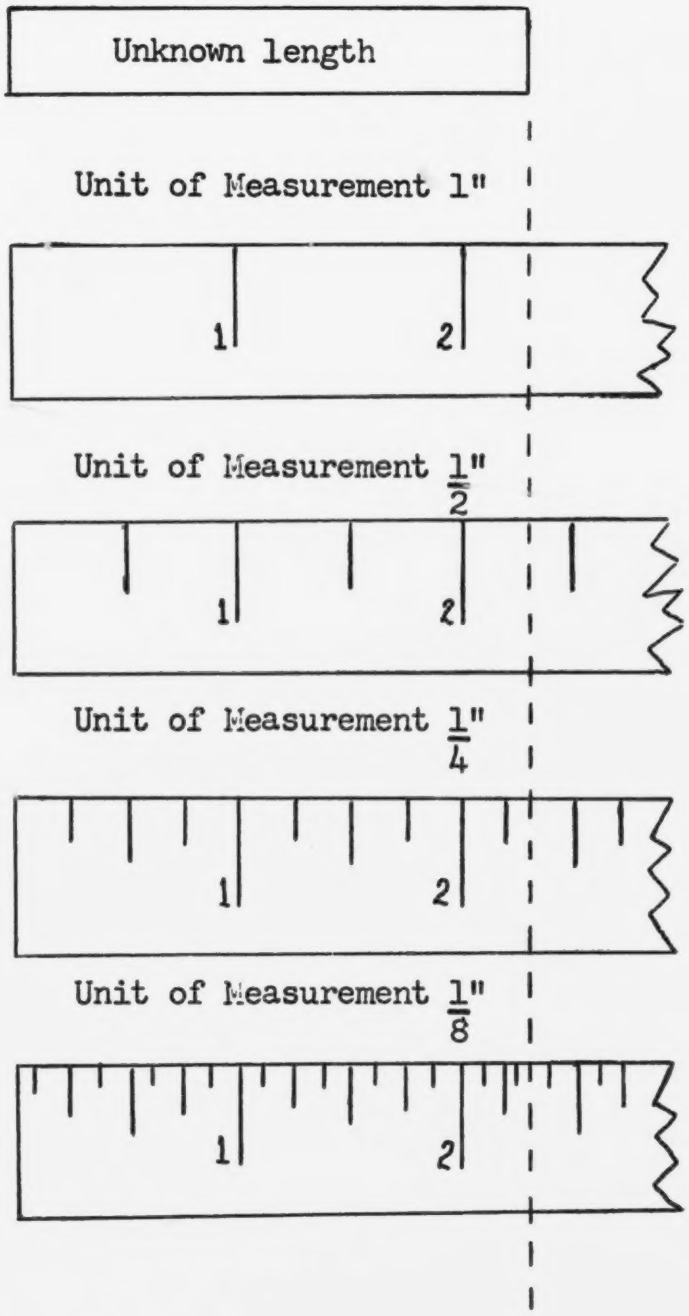


Approximate Measurements

Have children:

Examine rulers scaled in various ways: $\frac{1}{4}$ in., $\frac{1}{8}$ in., $\frac{1}{16}$ in.

Measure an unknown length to the nearest 1", $\frac{1}{2}$", $\frac{1}{4}$", $\frac{1}{8}$" as shown:


Unknown length
Unit of Measurement 1"
1
2
Unit of Measurement $\frac{1}{2}$"
1
2
Unit of Measurement $\frac{1}{4}$"
1
2
Unit of Measurement $\frac{1}{8}$"
1
2

Measurement is 2" to nearest 1"

Measurement is $\frac{5}{2}$" ($2\frac{1}{2}$")
to nearest $\frac{1}{2}$"

Measurement is $\frac{9}{4}$" ($2\frac{1}{4}$")
to nearest $\frac{1}{4}$"

Measurement is $\frac{19}{8}$" ($2\frac{3}{8}$")
to nearest $\frac{1}{8}$"

Have children:

Note that the numbers expressing the measurement are exact whereas the measurement is approximate.

Discuss which ruler to select to give the more precise measurement.

Note that the smaller the unit of measure used, the more precise the measurement will be to the desired length.

Measure various items, e.g., books, papers, length and width of desk, etc. with rulers of different scales.

Draw a line segment $2\frac{3}{16}$ inches using rulers of different scales.

PRACTICE EXERCISES

1. Complete each of the following and verify with a ruler.

$\frac{1}{2}$ of $\frac{1}{4}$" = ... "

$\frac{1}{4}$" + $\frac{1}{4}$" + ..." = $\frac{5}{8}$"

$\frac{1}{8}$" + $\frac{1}{8}$" + $\frac{1}{8}$" + ..." = $\frac{1}{2}$"

2. Which is nearer to the 1" mark on your ruler? $\frac{5}{8}$" or $\frac{3}{4}$" ?

3. If you want to draw a segment $3\frac{3}{8}$ in. long and start at the 2" mark on your ruler, where would you stop?

Interpretation of Drawings Made to Scale

1. Examine a map of New York State.

   Discuss the mileage from New York City to Albany (about 150 miles) with the length shown on the map.

Interpret the meaning of the legend shown on the map.

Note that:

A map is a drawing made to scale.
A scale drawing is a map or diagram of a place or object that is too large or too small to draw in actual size.
A scale drawing has the same shape as the real thing it represents.

2. Present a list of items:

a baseball field	a flag	classroom
a snowflake	the school garden	Map of Mexico

Discuss for each of the above:

Should the scale drawing be larger or smaller than the actual object? Why?

What must be considered before making the drawing?

3. Draw a line segment one inch long on the chalkboard.

Tell children:

If this line segment 1 inch long, represents 1 mile, we say that "the segment is drawn to a scale of 1 inch to 1 mile".

We write: "Scale: 1 inch represents 1 mile"

or

1" = 1 mi.

What does the equal sign mean in this case?

4. Have children write two statements for each of the following:

A line segment is drawn to a scale of 1 inch to 1 foot.
A segment is drawn to a scale of $\frac{1}{2}$ in. to 10 feet.

5. Have children draw a line segment to represent $2\frac{1}{2}$ miles if:

a. 1 inch represents 1 mile

b. 1 inch represents $\frac{1}{2}$ mile

c. 1 inch represents 2 miles

SETS; NUMBER; NUMERATION

UNIT 52 - NUMBER SYSTEM: "CLOCK" ARITHMETIC

NOTE TO TEACHER

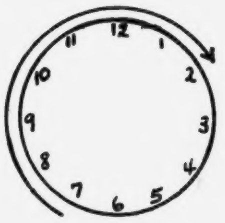
In "Clock" Arithmetic we shall be concerned with a number system that involves only a finite set of numbers, whereas in our usual arithmetic we have been dealing with an infinite set of numbers. In "Clock" Arithmetic, using a 12-hour clock, the finite set of numbers is:

{1, 2, 3, 4, 5, 6. 7, 8, 9, 10, 11, 12}

"Clock" Addition

Let us define "addition of two clock numbers" as follows:

"3 + 2" shall mean
2 hours after 3 o'clock
Therefore, 7 + 7 = 2;
6 + 6 = 12; etc.



"Clock" addition is a binary operation in which we operate on two numbers, a and b, to arrive at a third number, c.

Some Properties of "Clock" Addition

In studying "Clock" Arithmetic, children have an excellent opportunity to discuss and discover the properties that apply to addition and subtraction of the numbers of a finite set.

Zero It should be observed that, in "Clock" Arithmetic, using a 12-hour clock, the number 12 plays the same role that zero does in our usual arithmetic. 12 is the Identity Element in "Clock" addition:
$3 + 12 = 3$; $8 + 12 = 8$; $a + 12 = a$ for all a

Closure Since the sum of any two numbers in this finite set is an element in the same set, addition is said to be closed for "Clock" Arithmetic where any number a, plus any number b, results in a number within that finite set.

Commutative Property The Commutative Property holds true in "Clock" Arithmetic. For example: $2 + 7 = 7 + 2$ (Seven hours after 2 o'clock is 9 o'clock as is 2 hours after 7 o'clock.)

Associative Property for Addition The Associative Property holds true in "Clock" Arithmetic. For example:

$$(2 + 5) + 3 = 2 + (5 + 3)$$
$$7 + 3 = 2 + 8$$

5 hours after 2 o'clock is 7 o'clock and 3 hours after 7 o'clock brings us to 10 o'clock which is the same as 8 hours after 2 o'clock.

Clock Subtraction Subtraction for "Clock" Arithmetic is defined as follows:

$3 - 2$ shall mean 2 hours before or earlier than 3 o'clock

Therefore, $7 - 7 = 12$;
$6 - 8 = 10$; etc.

Properties as They Apply To Subtraction

Commutativity for Subtraction does not apply. For example:

$$5 - 2 \neq 2 - 5$$

Two hours before 5 o'clock (3 o'clock) is not the same as 5 hours before 2 o'clock (9 o'clock). It is important that children understand that a property must apply in all cases in order that it be considered a property. There are special instances in subtraction with "Clock" Arithmetic where the Commutative Property does apply.

For example, using the 12-hour clock observe that:

$$10 - 4 = 4 - 10$$
$$9 - 3 = 3 - 9$$
$$8 - 2 = 2 - 8; \text{ etc.}$$

(Note that the result each time is 6.)
However, since the Commutative Property does not apply in every case, it is not a property of Subtraction in "Clock" Arithmetic.

Associativity for Subtraction does not apply. For example:

$$(8 - 3) - 2 \neq 8 - (3 - 2)$$

3 o'clock is not the same as 7 o'clock

Closure for Subtraction
This finite set of numbers {1, 2, 3, 4 ... 12} is closed with respect to subtraction since the difference of any two numbers is an element of this set. For example:

$$8 - 3 = 5; \quad 4 - 6 = 10$$

Note that in this respect "Clock" Arithmetic differs from our ordinary Arithmetic in which the set of whole numbers is not closed for subtraction.

OBJECTIVES:

To introduce or reinforce "Clock" Arithmetic.
To observe Properties of Addition and Subtraction in "Clock" Arithmetic.
To observe that an arithmetic based on the number of the days of the week (7) is also a system based on a finite set of numbers.
To give children an opportunity to explore, to discover.

TEACHING SUGGESTIONS

Addition and Subtraction: Using a 12-Hour Clock

1. Reinforce Addition and Subtraction in "Clock" Arithmetic using a 12-hour clock.
   Present the set of numbers: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
   Represent this set on a clock face.
   Reinforce meaning of clockwise, counter clockwise.
   Children should explore:
      Meanings for 3 + 2 on the clock face (2 hours after 3 o'clock)
                   5 + 7 (7 hours after 5 o'clock), etc.
      Meanings for 12 - 3 on the clock face (3 hours before 12)
                   3 - 4 (4 hours before 3 o'clock)
                   6 - 12 (12 hours before 6 o'clock); etc.

Ask children:

What is the meaning of $7 + 7 = 2$? $6 + 8 = 2$?
$8 + 5 = 1$? $3 - 9 = 6$?

Emphasize that we are dealing with only the numbers represented on the clock face and we call this "Clock" Arithmetic.

2. Suggested practice exercises:
Children should use a clock face if necessary.

a) If the time is now 4 o'clock, 6 hours later it will be __ o'clock.
If the time is now 7 o'clock 12 hours later it will be __ o'clock.
If the time is now 3 o'clock 10 hours later it will be __ o'clock.

b) In "Clock" Arithmetic
$4 + 6 = \square$ $8 + 9 = \square$
$7 + 7 = \square$ $3 + 10 = \square$

c) If the time is now 4 o'clock, 6 hours earlier it was __ o'clock.
If the time is now 7 o'clock 12 hours earlier it was __ o'clock.
If the time is now 3 o'clock 10 hours earlier it was __ o'clock.

d) In "Clock" Arithmetic
$8 - 6 = \square$ $8 - 10 = \square$
$8 - 8 = \square$ $8 - 12 = \square$

Properties of Addition and Subtraction in "Clock" Arithmetic

1. Commutativity in Addition
Ask children to use a clock face to decide whether or not addition in "Clock" Arithmetic is Commutative. Have them give 3 examples to "prove" their decision.

2. Commutativity in Subtraction
Children should discover whether the Commutative Property holds for subtraction.
Have them use a clock face to discover whether:

$8 - 6 \overset{?}{=} 6 - 8$; (Read as: Does $8 - 6 = 6 - 8$?)
$9 - 2 \overset{?}{=} 2 - 9$;
$12 - 3 \overset{?}{=} 3 - 12$; etc.

The children discover that $8 - 6 \neq 6 - 8$, etc.

Ask children:

Do you think that the Commutative Property applies to Subtraction in "Clock" Arithmetic? [No]
Let us verify our conclusion.

Does $10 - 4 \overset{?}{=} 4 - 10$ [Yes, each equals 6]
$9 - 3 \overset{?}{=} 3 - 9$ [Yes]
$8 - 2 \overset{?}{=} 2 - 8$ [Yes]
$7 - 1 \overset{?}{=} 1 - 7$ [Yes]

Tell children that a property in mathematics can only be called a property if it applies in every case. Since the Commutative Property does not apply in every case, their first conclusion was correct.
The Property of Commutation for Subtraction does not apply to the finite set of numbers on a 12 hour clock.
They compare Commutativity for Addition and Subtraction with the System of Whole Numbers and the System of Numbers in "Clock" Arithmetic.

3. Associativity in Addition
Ask children to use the clock face to discover whether the following are true or false:
$(3 + 4) + 6 = 3 + (4 + 6)$
$(5 + 3) + 2 = 5 + (3 + 2)$
Children note that the Associative Property of Addition applies to "Clock" Arithmetic.

4. Associativity in Subtraction
In the same way have children discover that the Associative Property does not hold true for Subtraction.

5. Identity Element for Addition
Ask children to complete the following:
6 + 12 = □ 7 + 12 = □
8 + 12 = □ n + 12 = □
In "Clock" Arithmetic, which number is the Identity Element? Explain.
In the System of Whole Numbers, which number is the Identity Element?

6. Closure
Help children to discover that in "Clock" Arithmetic the set of numbers is closed with respect to Addition and Subtraction.
8 + 2 = □ 9 - 3 = □
7 = 6 = □ 6 - 8 = □
4 = 9 = □ 3 - 5 = □
Ask children:
Is the sum or difference always in the given set of numbers?
Compare this with Addition and Subtraction of Whole Numbers.

Addition and Subtraction: "Days of the Week" Arithmetic

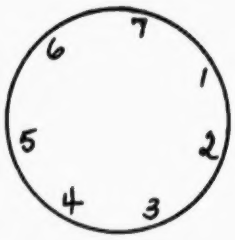
1. Discuss the numerical positions of the days of the week, referring to the calendar. Let Sunday be day 1, etc.

2. Children should refer to one week on the calendar and answer the following:

What is 3 days after Tuesday? [Friday]
What number have we assigned to Friday? [6]

Write a mathematical sentence to describe
that 3 days after Tuesday is Friday [$3 + 3 = 6$]
What is 4 days after Friday? [Tuesday]
Write a mathematical statement to show this [$6 + 4 = 3$]
Write mathematical statements to show what is
4 days after Saturday [$7 + 4 = 4$]
7 days after Sunday [$1 + 7 = 1$]

3. Have children represent the numbers for the days of the week in the same way that the numbers on the clock are represented.



Discuss similarities and differences between "Clock" Arithmetic and "Days of the Week" Arithmetic:

Both have a limited number of symbols.
In one case the set of symbols is $\{1, 2, 3 \ldots 12\}$.
In the other case the set of symbols is $\{1, 2, 3 \ldots 7\}$.
Place Value is not involved.
Zero is not used in either one.

4. Have children explore the following Properties for Addition using $\{1, 2, 3, 4, 5, 6, 7\}$.
   a. Is addition commutative? For example: $7 + 3 \stackrel{?}{=} 3 + 7$, etc.
   b. Is addition associative? For example: $(7 + 3) + 2 \stackrel{?}{=} 7 + (3 + 2)$
   c. What number of the set corresponds to the role of zero? [7]
   d. Is this finite set closed for addition?

5. Have children explore Subtraction using $\{1, 2, 3, 4, 5, 7\}$.
   a. How would you describe subtraction?
   b. Is subtraction commutative? For example:
      $7 - 3 \stackrel{?}{=} 3 - 7$, etc.
   c. Does the Associative Property apply to subtraction? For example:
      $(7 - 3) - 2 \stackrel{?}{=} 7 - (3 - 2)$
   d. Is this finite system closed for subtraction?

6. Have children compare the properties of operation when using "Clock" Arithmetic and when using "Days of the Week" Arithmetic.

SETS; NUMBER; NUMERATION

*UNIT 53 - NUMBER: THE MODULO 12 NUMBER SYSTEM (Optional)

NOTE TO TEACHER

In Clock Arithmetic we considered the following __finite__ set of numbers:

$$C = \{1, 2, 3, .., 10, 11, 12\}$$

In Unit 52 the children were guided to discover that 12 plays the role of a __zero__, in that
$2 + 12 = 2$, $6 + 12 = 6$, $\square + 12 = \square$
for __any__ replacement for Set C.
If we therefore use 0, instead of 12 we have the set:

$$\{0, 1, 2, .., 11\}$$

which we can call a "Modular Number System".

__Objectives:__ To introduce concept of a Modular Number System.

To introduce addition in a Modulo 12 System.

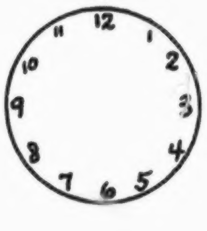
To observe the properties of addition in the Modulo 12 System.

TEACHING SUGGESTIONS

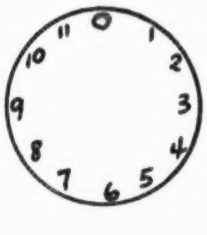
__Concept of A Modular Number System__

1. Teacher may use two figures as shown:

A



B



Ask children what system Figure A represents.
[A system using a 12 hour clock.]

Ask children referring to Figures A and B:

How many symbols are used in Figure A?
List that set of symbols. [{1, 2, 3,... 12}]
How many symbols are used in Figure B?
List that set of symbols. [{0, 1, 2, 3,... 11}]
Compare the 2 sets of symbols. [same number of, but different symbols]
What number in Figure B takes the place of 12 in Figure A? [0]

2. Have children answer the following:

For "Clock Arithmetic" as represented by Figure A:

12 + 2 = ☐ 12 + 6 = ☐ 12 + 9 = ☐

For Figure B:

0 + 2 = ☐ 0 + 11 = ☐ 0 + 7 = ☐

Children should note that 12 and 0 play the same role for both sets of numbers. (Identity Element)

Tell children that:

As we used the set of symbols including only {1, 2, 3,... 12} to add and subtract in "Clock" Arithmetic we are now going to use the set of symbols including only {0, 1, 2, 3,..., 11}.

We call a number system consisting of a limited number of symbols a Modular Number System.

Addition in Modulo 12 Number System

1. Develop with children a table such as the one below for adding two numbers in the Modulo 12 Number System.

+	0	1	2	3	4	5	6	7	8	9	10	11
0	[0]	[1]										
1	[1]											
2												
3												
4												
•												
•												
•												
11												

a. Teacher should reinforce concepts of a finite number system.
Ask:

What symbols are used in clock arithmetic?

[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]

How many symbols are used in clock arithmetic?
What symbols are used in the table above?
How many symbols are used in the table above?
What are the 12 symbols?

b. Have children complete the following and then insert the correct numeral on the chart.

0 + 0 = ☐ 0 + 1 = ☐ 0 + 2 = ☐ etc.

1 + 0 = ☐ 1 + 1 = ☐ 1 + 11 = ☐ etc.

c. Children complete the table.

2. Have children observe that:

In this table only a specific number of symbols are used.
No new numbers are involved. (Closure)
Place value does not apply.

Properties As They Apply to Addition in the Modulo 12 Number System.

1. Review meaning of "Addition - Modulo 12".

For example: 3 + 9 = 0; 11 + 4 = 3; etc.

2. Commutative Property

Ask children:

What is the sum of 9 + 4 as shown in this table? [1]

What is the sum of 4 + 9 as shown in this table? [1]

Is addition commutative when we use this system? [Yes] Why?

Have children give other examples to illustrate that Modulo 12 addition is commutative.

3. Associative Property

Ask children:

What is the sum of (4 + 5) + 6? [3]

What is the sum of 4 + (5 + 6)? [3]

Does (4 + 5) + 6 = 4 + (5 + 6)? [Yes]

Is this addition associative? [Yes, the way the numbers are grouped does not change the sum.]

Have children verify this with (3 + 5) + 2; (7 + 4) + 3; etc.

4. Closure

Continue to use the table to help children discover that the sum of any numbers in this finite system (Modulo 12) is always a number of this finite set.

Present addition exercises. Have children find the sums.

6 + 5 = □ 10 + 4 = □ 8 + 4 = □ , etc.

Note that the sum is always one of the twelve numbers in the system.

Point out that this set of numbers is closed with respect to addition.

5. Compare the Identity Element for Addition in Clock Arithmetic and the Identity Element for Addition in Modulo 12 Arithmetic.

a. Present a series of exercises referring to the clock.

1 + 12 = □
2 + 12 = □
3 + 12 = □
4 + 12 = □

9 + 12 = □
10 + 12 = □
11 + 12 = □
12 + 12 = □

12 + 5 = □
12 + 6 = □
12 + 7 = □
12 + 8 = □

Have children note that 12 is the identity element in clock arithmetic. Any number added to 12, or 12 added to any number results in that number.

Ask children:

What number in the System of Whole Numbers is the Identity Element for Addition? [0]

What number in Clock Arithmetic is the Identity Element for Addition? [12]

b. Refer to the table for Modulo 12.
Present a series of exercises.

1 + 0 = □ ;   3 + 0 = □ ;   9 + 0 = □ ;   0 + 11 = □ ; etc.

Have children note that 0 is the Identity Element for Modulo 12. Any number added to 0, or 0 added to any number results in that number.

*Other Modular Systems

NOTE TO TEACHER

Clock Arithmetic involves 12 numbers because of the way clocks are constructed. We use this as a motivation for Clock Arithmetic and Modulo 12 Arithmetic to point up arithmetic dealing with a finite number system.

Other finite number systems are important too. For example, in Grade 6, the Arithmetic Modulo 2 will take advantage of properties of even and odd numbers.

If time and interest permit, another finite number system that can be introduced for individual exploration by students is Modulo 7 Arithmetic which we can call "Days of the Week" Arithmetic making use of the periodicity of the 7 days of the week.

TEACHING SUGGESTIONS

1. Tell children that we can develop a finite number system with any number of elements greater than 1.

2. Have children explore a finite number system with 7 elements.

   They add in modulo 7.
   They find out whether properties apply to this modulo.
   e.g. For Modulo 7:

   $4 + 5 = \square$, $5 + 4 = \square$ (Commutativity)

   $(4 + 5) + 3 = \square$; $4 + (5 + 3) = \square$ (Associativity)

   $5 + 0 = \square$; $3 + 0 = \square$; $6 + 0 = \square$ (Identity)

   What is the identity element in Modulo 7?

   Children should note that the properties that apply in the set of whole numbers also apply to addition in the finite number systems.

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Solve the following exercises and identify the property.

   $3 + 0 = 0 + \square$ (modulo 12)

   $3 + 9 = \square + 3$ (modulo 12)

   $(7 + \square) + 3 = 7 + (5 + 3)$

*2. Have children explore the Properties of Addition for Modulo 7.

*3. Make an Addition Table for Modulo 7.

*4. The Table for exercise 3 is

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3*	4	5	6	0
2	2	3*	4	5*	6	0	1
3	3	4	5*	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

Have children draw a line from the upper left hand corner to the lower right hand corner.

Children should observe that the section of the table above the line is symmetric to the section of the Table below the line.

Have children show how the commutative property is demonstrated on this grid.

Have children star 2 + 1, then 1 + 2
3 + 2, then 2 + 3 etc.

OPERATIONS

UNIT 54 - SET OF WHOLE NUMBERS: EVEN AND ODD NUMBERS

Objectives: To determine divisibility of a number by two.

To observe the results of adding,subtracting,multiplying even and odd numbers.

TEACHING SUGGESTIONS

1. Reinforce meaning of even and odd numbers.

2. Extend to generalization for divisibility by 2.

   Discuss even numbers. Determine rule for divisibility by 2. A number is divisible by 2 if and only if the number formed by the digit in the units place is divisible by 2.

3. Addition and Subtraction with Even and Odd Numbers.

   a. Write 2 mathematical sentences to show whether an even number or an odd number results when:

      An even number is added to an even number.

      An even number is subtracted from an even number.

      An odd number is added to an even number.

      An odd number is subtracted fron an even number.

      An even number is subtracted from an odd number.

      An odd number is added to an odd number.

      An odd number is subtracted from an odd number.

b. Fill in the blanks with the word odd and even to make a true statement.

The sum of two even numbers is always an ________ number.

The sum of two odd numbers is always an ________ number.

The sum of an odd and even number is always an ______ number.

c. Fill in these tables e.g. since Odd + Odd = Even we fill in an E in the circled box.

+	O	E
O	Ⓔ	
E		

−	O	E
O		
E		

4. Multiplication with Even and Odd numbers.

Have children draw the following charts.

A

X	1	3	5	7
1				
3				
5				
7				

B

X	2	4	6	8
2				
4				
6				
8				

C

X	2	4	6	8
1				
3				
5				
7				

D

X	1	3	5	7
2				
4				
6				
8				

Children note that:

Chart A has 2 sets of odd numbers.
Chart B has 2 sets of even numbers.
Charts C and D have 1 set of even numbers and 1 set of odd numbers.

They complete each chart by recording all products.

Children complete the following statements:
The product of 2 even numbers is always an ________ number.
The product of 2 odd numbers is always an ________number.
The product of an odd and an even number is always an __________ number.

Children should generalize by completing and interpreting an array such as the following:

X	odd	even
odd	[odd]	[even]
even	[even]	[even]

X	O	E
O	O	E
E	E	E

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. If the replacement set for n is the set of odd numbers,write all the values of n to make the following sentences true:

   $n \times 8 < 50$ $n \times 3 < 30$

   $7 \times n < 49$ $n \times 9 < 81$

2. If the replacement set for n is the set of even numbers,write three values of n to make the following sentences true:

   $n \times 6 < 50$ n has 3 as a factor

   $3 \times n$ is an even number n has 7 as a factor

3. Write three illustraions for each sentence below using numerals. Tell whether the product will be odd or even.

   even times odd odd times odd

   even times even odd times even

4. Write five even multiples of 3. Write four even multiples of 9.
   Write five even multiples of 8. Write four odd multiples of 5.
   Write five even multiples of 7. Write four odd multiples of 7.

5. Can a multiple of 8 ever be an odd number? Discuss.

6. Can a multiple of any even number be an odd number?

7. Can a multiple of any odd number be an even number?

8. Can the sum of 4 odd numbers be an odd number?

*9. Have children consider the set of even numbers { 0,2,4,6, ... }

Which of the following properties are true for this set? Give an example of each.

The Commutative Property for Addition.

The Associative Property for Addition.

Closure for Addition.

The Commutative Property for Multiplication.

The Distributive Property for Multiplication.

Closure with respect to Multiplication.

*10. Have children consider the Set of Odd Numbers and observe that:

Closure for Addition does not appply to this set of numbers.

[3 + 5 = 8]

Multiplication is closed within this set.

OPERATIONS

UNIT 55 - MULTIPLICATION OF WHOLE NUMBERS: EXPLORING PATTERNS

OBJECTIVE: To drill multiplication facts and the extension of facts by the use of patterns.

TEACHING SUGGESTIONS

Patterns For Multiplication Facts

The discovery of patterns by children helps them to see relationships and gives them insight into mathematical structure.

Patterns as Shown in the Multiplication Table

1. Have children use rexographed outlines or graph paper to organize a multiplication "table" as shown below:

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6							27
4	0	4								36
5										45
6										54
7										63
8										72
9										81

Include:
The operational symbol "x"; 0 to 9 in the left hand column; 0 to 9 along the top row.

2. Ask children to complete one row at a time as shown above and to observe patterns that show differences between consecutive numbers.

   What are the differences between two consecutive numbers in each vertical column?

   What is the relationship between the differences and the heading?

3. Ask children to examine the product in any one column to note:

   Commutativity: Show how the product of any two factors is not affected by the order of the factors. [8 x 7 = 7 x 8, etc.]

   Zero in Multiplication: n x 0 = 0
   Note the horizontal and vertical rows of zero. Why are there complete lines of zero.

   Property of "1" in Multiplication: n x 1 = n
   Note the numerals in the second rows, and compare these with the heading row. Why are they the same?

   Children should note the numerals in the second column and compare these with the numerals at the left. Why are these the same?

   Pattern Showing Factors:
   Note that 12 appears four times. Why? 30 appears only twice. Why?

   Find the numerals that appear only once. Why?

Patterns for Multiples of 9

1. Have children name the multiples of 9, beginning with 9. Teacher should record: {9, 18, 27 ...}

   How much is added each time?

   How do the digits in the one's place change? Why?

   How do the digits in the ten's place change? Why?

2. Ask children to note the sum of the digits as they add the ones and tens digit in each multiple of 9. For example:

   9: $0 + 9 = 9$
   18: $1 + 8 = 9$
   27: $2 + 7 = 9$

   What is the sum of the digits each time?

3. Write 3 or more equations in the following sequences:

   $1 \times 9 = (1 \times 10) - 1 = n$ Why do we subtract 1?
   $2 \times 9 = (2 \times 10) - (2 \times 1) = n$ Why do we subtract 2?
   $3 \times 9 = (3 \times 10) - 3 = n$ Why do we subtract 3?

   What is the pattern in the successive numbers that are subtracted from 10, 20, 30, etc.?

4. Ask children to find the product of 17×9; 32×9; 163×9. Add the ones, tens, hundreds digits of each product.

   What is the sum?

   What will the sum be if you add the ones, tens, etc. digits of any multiple of 9? [a multiple of 9]

   Generalization: Have children tell how they will know if a number is divisible by 9.

5. Have children make the following chart from 1 to 100.

1	2	3	4	5	6	7	8	⑨	10
11	12	13	14	15	16	17	⑱	19	20
21	22	23	24	25	26	㉗	28	29	30
31	32	33	34	35	㊱	37	38	39	40
41	42	43	44	㊺	46	47	48	49	50
51	52	53	(54)	55	56	57	58	59	60
61	62	(63)	64	65	66	67	68	69	70
71	(72)	73	74	75	76	77	78	79	80
(81)	82	83	84	85	86	87	88	89	(90)
91	92	93	94	95	96	97	98	(99)	100

   Circle all the multiples of 9. Observe the pattern.

   Why does the encircled numeral move 1 column to the left each time?

   Why is "90" an exception?

Compare each multiple of 9 with the next whole-decade number:

9 with 10; 18 with 20; 27 with 30; etc.

(1 x 9 is 1 less than 10; 2 x 9 is 2 less than 20; etc.)

Patterns for Multiples of 8

1. Have children name the multiples of 8, beginning with 8.
   Teacher should record: {8, 16, 24 ...}

   By how much does the units digit decrease each time? Why?

2. Write 3 or more equations in the following sequences:

$$\begin{array}{l}1 \times 8 = (1 \times 10) - 2 \qquad\qquad = 10 - 2 = n \\ 2 \times 8 = (2 \times 10) - (2 \times 2) = 20 - 4 = n \\ 3 \times 8 = (3 \times 10) - (3 \times 2) = 30 - 6 = n\end{array}$$

   [Note the use of the Distributive Property]

   What is the pattern in the successive numbers that are subtracted from 10, 20, 30, etc. to arrive at multiples of 8?

3. Have children make a number chart from 1 to 100 as for multiples of 9. Circle the multiples of 8.
   They should observe the pattern and discuss:

   In what columns are there no multiples of 8? Why?

   In each column what is the difference between the successive multiples?

Patterns for Multiples of Other Numbers

Ask children to make other number charts from 1 to 100.
They block out the multiples of 3; of 7; etc.
Children should try to discover patterns and discuss what they find.

Sequence Showing Patterns

Ask children to find the pattern in the sequences below and write the next two numbers:

1, 2, 4, 8, 16, __, __, [Power of 2]
1, 3, 9, 27, __, __, [Power of 3]
1, 2, 6, 24, __, __, [1, 1 x 2, 1 x 2 x 3, 1 x 2 x 3 x 4, etc.]

SUGGESTED EXERCISES

1. Write a complicated addition fact that can be solved by multiplication.

2. Make a "table" of Multiplication Facts and answer the questions below.

   Why are all the numerals in the second vertical column the same as the numerals at the left side?

   Why are all the numerals in the second vertical row the same as the numerals across the top of the chart?

   The product of any number and 1 is ——? [Identity Element]

   Most of the numerals in the multiplication chart are repeated at least 2 times. Why? (Commutativity)

   When two whole numbers are multiplied, what kind of number will the product always be? [Whole Number] (Property of Closure)

3. Write three numerical sentences to illustrate the statement:

   If you change the order of the factors you do not change the value of the product. (Commutativity)

4. Answer the following:

   Is every multiple of 6 also a multiple of 3? [Yes] Why?

   Is every multiple of 3 also a multiple of 6? [No] Why?

   One factor of 24 is 8. Write the other factor.

   Factor 24 in different ways.

5. Complete each of the following open sentences to make the statements true.

   7 x 4 = 4 x □
   6 x □ = 5 x 6

   3 x 10 = 6 x □
   6 x 4 = 12 x □

   8 x 9 = 36 + □
   5 x 7 = (4 x 7) + □
   7 x 9 = 27 + 27 + □
   9 x 6 = (10 x 6) − □
   9 x 9 = 90 − □

6. Examine each of the following equations and answer the questions below:

$$7 \times 9 = (4 \times 9) + (3 \times 9)$$
$$7 \times 9 = (5 \times 9) + (2 \times 9)$$
$$7 \times 9 = (6 \times 9) + (1 \times 9)$$

How did you rename 7×9 in each equation?

What was the product each time?

How did renaming the 7 affect the product?

How can you find a product that you may not know?

What property is being used? [Distributive]

7. Solve the following open sentences in as many ways as you can:

$$\begin{aligned} 7 \times 9 &= (3 + 4) \times 9 = (\square \times 9) + (\Delta \times 9) = n \\ &= (5 + \Delta) \times 9 = (5 \times 9) + (\Delta \times 9) = n \\ &= (6 + \Delta) \times 9 = (6 \times 9) + (\Delta \times 9) = n \end{aligned}$$

$$\begin{aligned} 16 \times 7 &= (\square + \square) \times 7 = (\square \times 7) + (\square \times 7) = n \\ &= (9 + \Delta) \times 7 = (9 \times 7) + (\Delta \times 7) = n \\ &= (7 + \Delta) \times 7 = (7 \times 7) + (\Delta \times 7) = n \end{aligned}$$

$$\begin{aligned} 6 \times 8 &= (n \times 8) + (n \times 8) \\ &= (4 \times 8) + (n \times 8) \\ &= (5 \times 8) + (n \times 8) \end{aligned}$$

$$\begin{aligned} 14 \times 8 &= (n \times 8) + (n \times 8) \\ &= (9 \times 8) + (n \times 8) \\ &= (8 \times 8) + (n \times 8) \end{aligned}$$

8. Find the largest value for n that will make the sentence true, if the replacement set = the set of whole numbers.

   $n \times 8 < 17$ $\quad$ $n \times 8 < 25$ $\quad$ $n \times 8 < 30$

   $n \times 8 < 34$ $\quad$ $n \times 8 < 50$ $\quad$ $n \times 8 < 60$

   $n \times 8 < 45$ $\quad$ $n \times 8 < 38$ $\quad$ $n \times 8 < 23$

   $n \times 8 < 85$ $\quad$ $n \times 8 < 69$ $\quad$ $n \times 8 < 55$

* 9. Just For Fun (Puzzles generally intrigue children)

Have children:

Choose any one of the following numbers:

6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Multiply the number by 2.

Now add 10.

Take one half of the sum.

Subtract from this, the number originally selected.

What is your answer? [5]

Can you explain why the answer will always be 5.

$$\left[\begin{array}{l} \frac{2n + 10}{2} = n + 5 \\ \\ (n + 5) - n = 5 \end{array}\right]$$

Make the same puzzle using other numbers and following the same pattern of operations.

OPERATIONS

UNIT 56 - MULTIPLICATION OF WHOLE NUMBERS : SQUARES OF NUMBERS

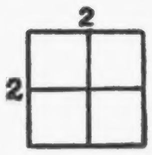
Objective: To develop understanding of squares of whole numbers.

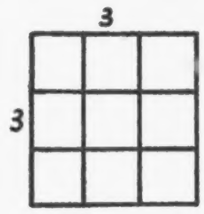
TEACHING SUGGESTIONS

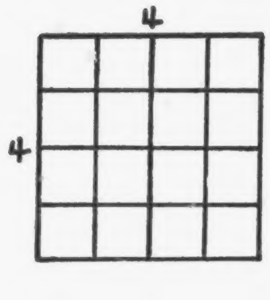
1. Begin by using squares to show patterns.

Have children use unruled paper or graph paper to mark off and count the number of boxes in a:

2 by 2 square	3 by 3 square	4 by 4 square
or	or	or
2 x 2	3 x 3	4 x 4





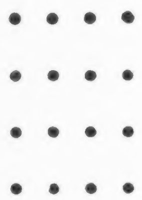


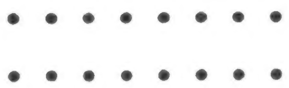
Tell children that another way of referring to 2 twos, 3 threes, 4 fours is 2 squared or 2 square, 3 squared or 3 square, 4 squared or 4 square.

Why are the products 4, 9, 6 referred to as squares of 2, 3, 4?

Have children observe that 16 is also the product of 8 x 2, but it is only because a number is the product of the same factor used two times that it is considered a perfect square.

Have children illustrate that the representation of 4 x 4 as an array results in a perfect square, while the representation of 8 x 2 as an array does not.



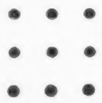


2. Children make diagrams to show the squares of other numbers:

The square of 5; 6 squared; 7 x 7; etc.

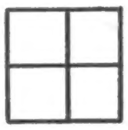
Have children observe that 1, 4, 9, 16 etc. are called "squares" because they represent the number of elements in square arrays.

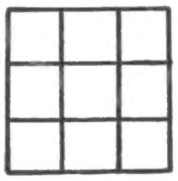
4 may be represented as ⁚ ⁚ ;

9 as  ; etc.

Children also observe that:

A 1 by 1 square contains 1 box

A 2 by 2 square contains 4 boxes 

A 3 by 3 square contains 9 boxes 

etc.

3. Children should draw squares like the ones below and fill in the multiplication facts.

For a 2 x 2 square

x	1	2
1	1	2
2	2	4

For a 4 x 4 square

x	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

For a 5 x 5 square

x	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

Ask children to count the number of boxes in a 2 x 2 square; a 4 x 4 square, etc.
They observe that a 2 by 2 square contains 2 x 2 = 4 boxes; a 4 by 4 square contains 4 x 4 = 16 boxes; etc.

Ask children to draw diagonals of the squares in each array. They should note that all the squared numbers lie along one diagonal. Why?

SUGGESTED EXERCISES

1. Solve and construct diagrams to complete the solutions.

The square of 6 = □ x □ = Δ
The square of 8 = □ x □ = Δ
7 squared = □ x □ = Δ
9 squared = □ x □ = Δ
1 squared = □ x □ = Δ

4 = n squared n = ?
9 = n squared n = ?
16 = n squared n = ?
25 = n squared n = ?
36 = n squared n = ?

2. Which of the following are squares? Of what numbers?

7, 9, 54, 64, 19, 49, 80, 81

3. What are the equal factors that result in the following squares?

25, 36, 64, 4, 9

4. Find the square of each number:

3, 5, 7, 9, 2, 4, 6, 8, 0, 1

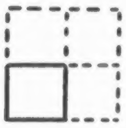
5. 36 is the product of 2 x 18, and of 6 x 6. Which pair of factors can be represented by an array to show that 36 is a square? Why?

*6. Interesting Numbers (Optional)

Start with a 1 unit square. □

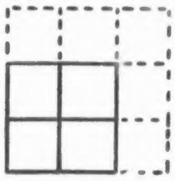
How many unit squares must we add to the 1 unit square to make a 2 by 2 square? [3]

Show this in a diagram.



Show this as a sum. [1 + 3 = 2 squared = 4]

Extend the 2 by 2 square to a 3 by 3 square.



How many units did you add? [5]

Show this as a sum. [1 + 3 + 5 = 3 squared = 9]

Extend the 3 by 3 square to a 4 by 4 square.

What pattern of numbers do you observe?

$$1 + 3 = 2 \text{ squared} = 4$$

$$\underbrace{1 + 3}_{\text{2 squared}} + 5 = 3 \text{ squared} = 9$$

$$\underbrace{1 + 3 + 5}_{\text{3 squared}} + 7 = 4 \text{ squared} = 16$$

What do you observe about the number that is added each time?

Can you continue the pattern for a 5 by 5 square? for a 6 by 6 square?

OPERATIONS

UNIT 57 - DIVISION OF WHOLE NUMBERS: PROPERTIES OF "1" AND ZERO

Objectives: To develop an understanding of the property of "1" in division.

To compare the properties of "1" in all operations.

To help children arrive at generalizations for divisor, dividend, quotient relationshops.

To develop an understanding of the role of zero in division.

TEACHING SUGGESTIONS

Property of "1" in Divisor

Quotient "1"

1. In each of the following exercises have children complete the open sentence.

$[\;] \times 7 = 7$ $\qquad 7\overline{)7}^{\,\square}$ $\qquad [\;] \times 35 = 35$ $\qquad 35\overline{)35}^{\,\square}$

$[\;] \times 8 = 8$ $\qquad 8\overline{)8}^{\,\square}$ $\qquad \square \times n = n$ $\qquad n\overline{)n}^{\,\square}$

$[\;] \times 9 = 9$ $\qquad 9\overline{)9}^{\,\square}$

2. Discuss and state the generalization:

   Any number, except zero, divided by itself equals 1.

   Therefore, $n \div n = 1$ if $n \neq 0$.

Divisor "1"

1. Present open sentences in which "1" is a given factor.

Children solve and discuss each quotient.

$\square \times 1 = 7$ $\quad \begin{array}{r}\square\\ 1\overline{)7}\end{array}$ $\quad \square \times 1 = 35$ $\quad \begin{array}{r}\square\\ 1\overline{)35}\end{array}$

$\square \times 1 = 8$ $\quad \begin{array}{r}\square\\ 1\overline{)8}\end{array}$ $\quad \square \times 1 = n$ $\quad \begin{array}{r}\square\\ 1\overline{)n}\end{array}$

2. Develop, then state the generalization.

Any number, divided by 1 results in the same number.
Therefore, $\frac{n}{1} = n$

Ask children: "Do we have to be careful about zero in this case"? [No]

Comparing Properties of "1" in all Operations for the Set of Whole Numbers

1. Have a chart similar to the one below.
Develop:

Addition	Subtraction	Multiplication	Division
8 + 1 = 9 1 + 8 = 9	8 - 1 = 7 1 - 8 (Subtraction is not always possible in the set of whole numbers)	8 x 1 = 8 1 x 8 = 8	8 ÷ 1 = 8 1 ÷ 8 (Division is not always possible in the set of whole numbers)

2. Discuss and compare properties of "1" with respect to the different operations.

What happens when:

1 is added to 8?	8 is added to 1?
1 is multiplied by 8? (8x1)	8 is multiplied by 1? (1x8)
1 is subtracted from 8?	8 is divided by 1?

3. Have children compare the role of 1 with respect to:

Addition vs. Subtraction
Multiplication vs. Division

Decide when the results are the same as the original number and when they are different.

4. Discuss the role of 1 as an "Identity" element in multiplication.

5. Compare with 0 for addition.

$$\begin{bmatrix} a \times 1 & = & 1 \times a & = & a \\ a + 0 & = & 0 + a & = & a \end{bmatrix}$$

Divisor - Dividend - Quotient Relationship

Present sets of equations. Have children compare dividends, divisors, and quotients and state the relationships for each set, as follows:

1. Doubling the Dividend: Keeping the Divisor Constant

$\begin{array}{r}1\\6\overline{)6}\end{array}$	$6 \div 6 = 1$	$\begin{array}{r}3\\8\overline{)24}\end{array}$	$24 \div 8 = 3$
$\begin{array}{r}\square\\6\overline{)12}\end{array}$	$12 \div 6 = \square$	$\begin{array}{r}\square\\8\overline{)48}\end{array}$	$48 \div 8 = \square$
$\begin{array}{r}\square\\6\overline{)24}\end{array}$	$24 \div 6 = \square$	$\begin{array}{r}\square\\8\overline{)96}\end{array}$	$96 \div 8 = \square$
$\begin{array}{r}\square\\6\overline{)48}\end{array}$	$48 \div 6 = \square$	$\begin{array}{r}\square\\8\overline{)192}\end{array}$	$192 \div 8 = \square$

Develop the generalization:

If the dividend is doubled and the divisor remains the same, the quotient is doubled.

2. Dividing the Dividend by 2: Keeping the Divisor Constant

$\begin{array}{r}\square\\6\overline{)24}\end{array}$	$24 \div 6 = \square$	$\begin{array}{r}\square\\8\overline{)48}\end{array}$	$48 \div 8 = \square$
$\begin{array}{r}\square\\6\overline{)12}\end{array}$ Why?	$12 \div 6 = \square$	$\begin{array}{r}\square\\8\overline{)24}\end{array}$ Why?	$24 \div 8 = \square$
$\begin{array}{r}\square\\6\overline{)6}\end{array}$	$6 \div 6 = \square$	$\begin{array}{r}\square\\8\overline{)12}\end{array}$	$12 \div 8 = \square$
$\begin{array}{r}\square\\6\overline{)3}\end{array}$	$3 \div 6 = \square$	$\begin{array}{r}\square\\8\overline{)6}\end{array}$	$6 \div 8 = \square$

Ask this question:

> If a dividend is divided by 2 and the divisor remains the same, how has the quotient changed?

Have the children state the result as a generalization.

3. Doubling the divisor: Keeping the Dividend Constant

24 ÷ 3 = 8
24 ÷ 6 = □ Why?
24 ÷12= □
24 ÷24= □

68 ÷ 4 = 17
68 ÷ 8 = □ Why?
68 ÷16= □
68 ÷32= □

Ask this question:

> If a divisor is doubled and the dividend remains the same, how has the quotient changed?

4. Dividing the Divisor by 2: Keeping the Dividend Constant

24 ÷ 24 = 1
24 ÷ 12 = □ Why?
24 ÷ 6 = □
24 ÷ 3 = □

120 ÷ 120 = 1
120 ÷ 60 = □ Why?
120 ÷ 30 = □
120 ÷ 15 = □

If a divisor is divided by 2 and the dividend remains the same, how has the quotient changed?

5. Doubling Both Dividend and Divisor

6 ÷ 3 = 2
12 ÷ 6 = □ Why?
24 ÷ 12 = □
48 ÷ 24 = □

18 ÷ 6 = 3
36 ÷ 12 = □
72 ÷ 24 = □
144 ÷ 48 = □

If a divisor is doubled and the quotient remains the same, how has the dividend changed?

If a dividend is doubled and the quotient remains the same, how has the divisor changed?

If both dividend and divisor are doubled, how has the quotient changed?

6. Dividing Both Dividend and Divisor by 2

$64 \div 16 = 4$
$32 \div 8 = \square$ Why?
$16 \div 4 = \square$
$8 \div 2 = \square$

$240 \div 80 = 3$
$120 \div 40 = \square$ Why?
$60 \div 20 = \square$
$30 \div 10 = \square$

$480 \div 120 = 4$
$240 \div 60 = \square$ Why?
$120 \div 30 = \square$
$60 \div 15 = \square$

If a divisor is divided by 2, and the quotient remains the same, how has the dividend changed?

If a dividend is divided by 2, and the quotient remains the same, how has the divisor changed?

If both the dividend and divisor are divided by 2, how has the quotient changed?

Zero in Division

Zero Divided by a number (Zero as the dividend)

1. Present a series of divisions and their related multiplications.

$3 \div 1 = n$ $n \times 1 = 3$
$2 \div 1 = n$ $n \times 1 = 2$
$1 \div 1 = n$ $n \times 1 = 1$
$0 \div 1 = n$ $n \times 1 = 0$

Ask children:

How many ones are there in three? in two? in one?
How many ones are there in zero?

2. Continue with:

$0 \div 2 = n$ $n \times 2 = 0$
$0 \div 3 = n$ $n \times 3 = 0$
$0 \div 4 = n$ $n \times 4 = 0$

How many twos are there in zero?
What was the value of n that made $n \times 1 = 0$ true? $n \times 2 = 0$? $n \times 3 = 0$?

3. Write the following open sentences involving division, as equivalent open sentences involving multiplication and solve for n.

$0 \div 4 = n$ ($n \times 4 = 0$; $n = 0$)
$0 \div 9 = n$
$0 \div 17 = n$
$0 \div 35 = n$

$0 \div 5 = n$
$0 \div 8 = n$
$0 \div 19 = n$
$0 \div 63 = n$

> If the dividend is zero and the divisor is any non-zero number, what will the quotient be?

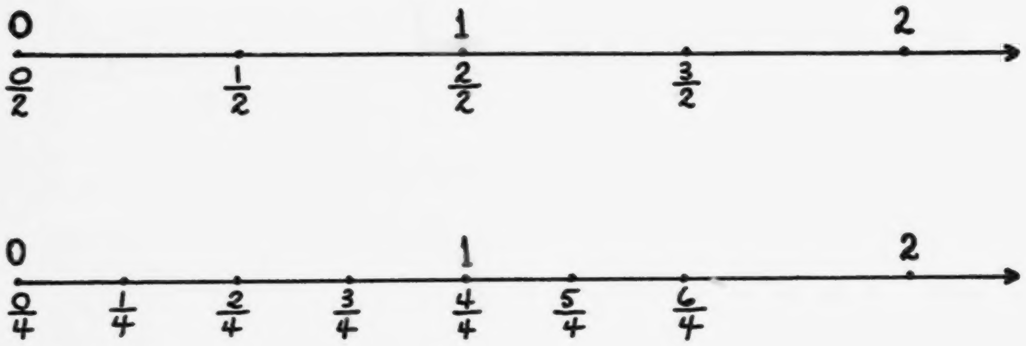
Have children observe that we are considering non-zero divisors. Zero as a divisor is a special case which will be considered later.

Have children state the generalization:

> If zero is divided by any non-zero number, the quotient is zero.

4. Relate to fraction form. Use number rays.

a.



b. Show on a number ray 3 other ways of representing zero as a fraction.

$$\left[\frac{0}{3}, \frac{0}{5}, \frac{0}{8} - - -\right]$$

SETS; NUMBER; NUMERATION

UNIT 58 - SET OF FRACTIONAL NUMBERS: CONCEPTS OF FIFTHS AND TENTHS; COMMON FRACTIONAL FORM

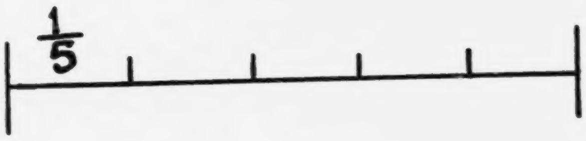
Objectives: To strengthen the concept of fifths and tenths.

To help children understand the relationship of tenths to one; tenths to tenths; tenths to fifths.

TEACHING SUGGESTIONS

Meaning of Fifths

1. Have children draw a line segment indicating one unit. The whole is then divided into fifths (5 equal parts).



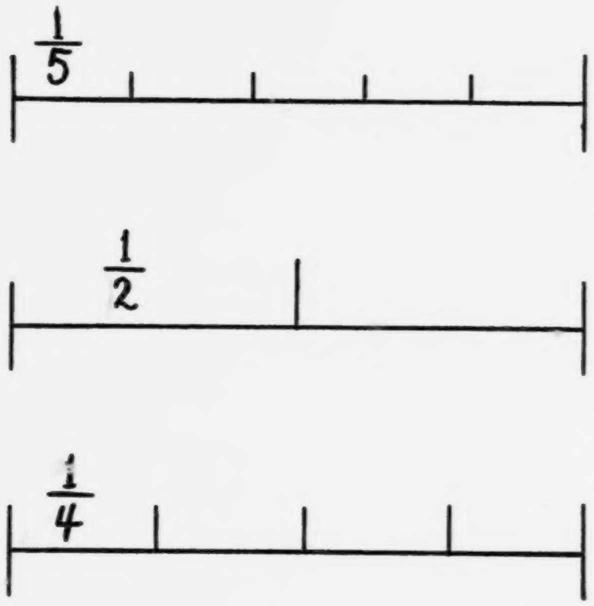
Have children discover that:

Fifths cannot be derived from halves, fourths, eighths, thirds, sixths.

Fifths are so named when a unit has been divided into 5 equal parts.

2. Comparing fifths with: halves, fourths, eighths, thirds, sixths.

   a. Children should draw several lines of equal length.



Which fractional parts are larger than $\frac{1}{5}$? Smaller?

b. Draw 3 lines of equal length.

A On segment A locate $\frac{1}{2}$; $\frac{1}{5}$

B On segment B locate $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$

C On segment C locate $\frac{1}{3}$; $\frac{1}{5}$

Similarly compare: fifths and sixths; fifths and eighths.

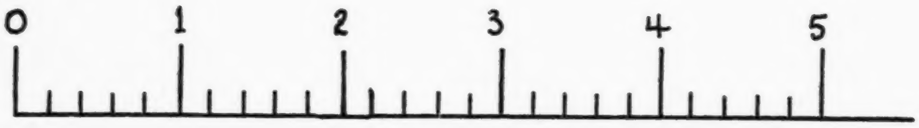
c. Ask children to insert the correct symbol (< , >) to compare each pair of the following fractional parts.

$\frac{1}{5}$ — $\frac{1}{4}$ $\frac{1}{2}$ — $\frac{1}{5}$

$\frac{2}{5}$ — $\frac{1}{4}$ $\frac{3}{5}$ — $\frac{3}{4}$ etc.

3. **Equivalents**

Draw a number line divided into fifths that includes more than 1 unit, thus:



Have children locate the following: $1\frac{1}{5}$, $1\frac{3}{5}$, $4\frac{4}{5}$, etc.

Reinforce concept that these points also represent distances from the zero point.

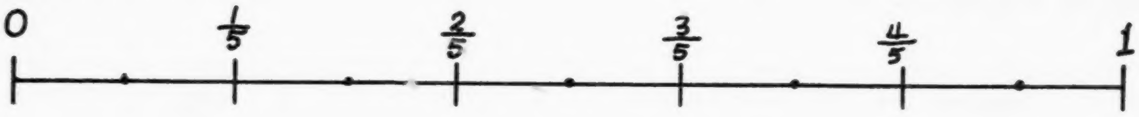
Have children complete the following:

$\frac{5}{5} = \square$ $\frac{6}{5} = \square$ $\frac{7}{5} = \square$ $\frac{12}{5} = \square$ etc.

Concept of Tenths

1. Tenths derived from halves of fifths

Direct children to draw a line segment divided into five equal parts. They then label each fifth, and divide each fifth into 2 equal parts.



What is each part called now?
Label each part.
How much is $\frac{1}{2}$ of $\frac{1}{5}$?
How much is $\frac{1}{2}$ of $\frac{2}{5}$? of $\frac{3}{5}$? of $\frac{4}{5}$? of $\frac{5}{5}$?
Since $\frac{1}{5} = \frac{2}{10}$, $\frac{1}{2}$ of $\frac{1}{5} = \frac{\square}{10}$

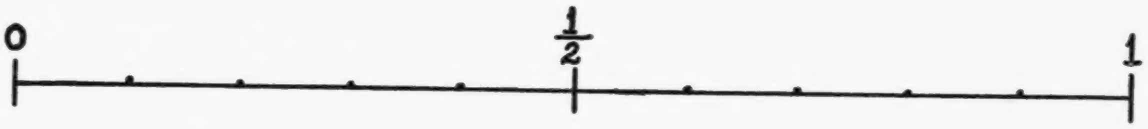
Since $\frac{2}{5} = \frac{4}{10}$, $\frac{1}{2}$ of $\frac{2}{5} = \frac{\square}{10}$

Since $\frac{1}{5} = \frac{2}{10}$, $\frac{1}{10}$ is what part of $\frac{1}{5}$?

$\frac{2}{10}$ is what part of $\frac{2}{5}$?

2. **Tenths derived from fifths of halves.**

Draw a line segment and divide it into 2 equal parts.
Then divide each half into 5 equal parts.



What is each part called now? $\left[\frac{1}{10}\right]$

How much is $\frac{1}{5}$ of $\frac{1}{2}$? Find this length from zero on the line segment.

How much is $\frac{2}{5}$ of $\frac{1}{2}$? Draw an arrow on the line segment to show this.

Have children continue to find fifths of one half in the same way.

Since $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$, $\frac{2}{5}$ of $\frac{1}{2} = \frac{\square}{10}$

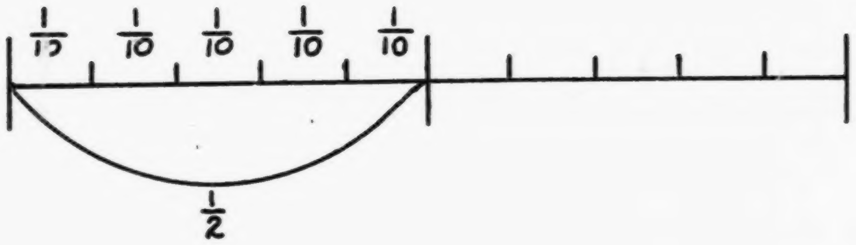
Since $\frac{2}{5}$ of $\frac{1}{2} = \frac{2}{10}$, $\frac{4}{5}$ of $\frac{1}{2} = \frac{\square}{10}$

3. Equivalent Fractions

Use number lines.

Have children complete the following:

Tenths and Halves

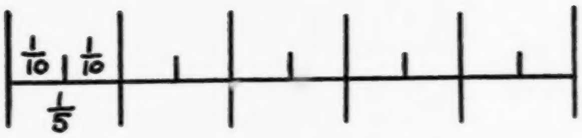


$\frac{5}{10} = \frac{\square}{2}$ $\frac{7}{10} = \frac{1}{2} + \frac{\square}{10}$ $\frac{9}{10} = \frac{1}{2} + \frac{\square}{10}$ $\frac{10}{10} = \square$

$\frac{6}{10} = \frac{1}{2} + \frac{\square}{10}$ $\frac{8}{10} = \frac{1}{2} + \frac{\square}{10}$ $\frac{10}{10} = \frac{1}{2} + \frac{\square}{10}$ $\frac{10}{10} = \frac{\square}{2}$

Since $\frac{5}{10} = \frac{1}{2}$, $\frac{6}{10} = \frac{1}{2} + \frac{\square}{10}$ and $\frac{8}{10} = \frac{1}{2} + \frac{\square}{10}$

Tenths and Fifths



$\frac{2}{10} = \frac{\square}{5}$

$\frac{4}{10} = \frac{\square}{5}$

$\frac{6}{10} = \frac{\square}{5}$

$\frac{8}{10} = \frac{\square}{5}$

$\frac{10}{10} = \frac{\square}{5}$

$\frac{3}{10} = \frac{\square}{5} + \frac{1}{10}$

$\frac{5}{10} = \frac{\square}{5} + \frac{1}{10}$

$\frac{7}{10} = \frac{\square}{5} + \frac{1}{10}$

$\frac{9}{10} = \frac{\square}{5} + \frac{1}{10}$

Compare tenths with: halves, fourths, eighths, thirds, sixths, fifths using line diagrams.

4. Counting

Use line diagrams. Have children record fractions on the number line as they count.

a. Counting forward - groups of $\frac{2}{5}$

$$\frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \ldots \quad \text{then } \frac{2}{5}, \frac{4}{5}, 1\frac{1}{5}, \ldots \left[1\frac{3}{5}, 2, 2\frac{2}{5}\right]$$

$$\frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots \quad \text{then } \frac{3}{5}, 1, 1\frac{2}{5}, \ldots \left[1\frac{4}{5}, 2\frac{1}{5}, 2\frac{3}{5}\right]$$

b. Counting backward - groups of $\frac{3}{5}$

$$\frac{13}{5}, \frac{10}{5}, \frac{7}{5}, \ldots \quad \text{then } \frac{13}{5}, 2, \frac{7}{5}, \ldots \left[\frac{4}{5}, \frac{1}{5}\right]$$

$$\text{then } 2\frac{3}{5}, 2, \ldots \left[1\frac{2}{5}, \frac{4}{5}, \frac{1}{5}\right]$$

c. Counting forward

$$\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \ldots \quad \text{then } \frac{1}{10}, \frac{3}{10}, \frac{1}{2}, \frac{7}{10}, \frac{9}{10}, \ldots$$

$$\frac{1}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10}, \frac{9}{10}, \ldots \quad \text{then } \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \ldots$$

d. Counting backward

$$\frac{11}{10}, \frac{10}{10}, \frac{9}{10}, \frac{8}{10}, \frac{7}{10}, \ldots \quad \text{then } 1\frac{1}{10}, 1, \frac{9}{10}, \frac{4}{5}, \frac{7}{10}, \ldots$$

$$\frac{9}{10}, \frac{7}{10}, \frac{5}{10}, \frac{3}{10}, \frac{1}{10}, \ldots \quad \text{then } \frac{9}{10}, \frac{7}{10}, \frac{1}{2}, \frac{3}{10}, \frac{1}{10}, \ldots$$

e. Additional Suggestions.

Have children change to simplest form as they count.

Count by $\frac{3}{5}$ starting with $1\frac{3}{5}$

Count by $\frac{3}{10}$ starting with $1\frac{1}{2}$

Count backward by $\frac{2}{5}$ starting with 2

Count backward by $\frac{3}{10}$ starting with 2

Find the distance between the points.

$\frac{4}{5}$ and $1\frac{1}{5}$; $1\frac{1}{5}$ and $2\frac{1}{5}$; $3\frac{2}{5}$ and $4\frac{3}{5}$, etc.

5. Relationship of tenths to one; Tenths to tenths

a. Have children insert the correct numeral to complete the equation below. Refer to a line diagram when necessary.

$\frac{2}{10} = \square$ times $\frac{1}{10}$ $\frac{3}{10} = \square$ times $\frac{1}{10}$ $\frac{6}{10} = \square$ times $\frac{2}{10}$ etc.

$\frac{10}{10} = \square$ times $\frac{1}{10}$ $\frac{10}{10} = \square$ times $\frac{2}{10}$ $\frac{10}{10} = \square$ times $\frac{5}{10}$

$1 = \square$ times $\frac{1}{10}$ $1 = \square$ times $\frac{2}{10}$ $1 = \square$ times $\frac{5}{10}$

b. Solve the following problems:

$\frac{1}{10}$ is what part of: $\frac{2}{10}$, $\frac{3}{10}$, — — — $\frac{10}{10}$, 1 ?

$\frac{2}{10}$ is what part of: $\frac{4}{10}$, $\frac{6}{10}$, $\frac{8}{10}$, $\frac{10}{10}$, 1 ?

c. What is the relationship between:

$\frac{1}{10}$ and $\frac{10}{10}$; $\frac{10}{10}$ and $\frac{1}{10}$; $\frac{1}{10}$ and 1; 1 and $\frac{1}{10}$

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Use the symbol > to show which fraction of each pair is greater.

   $\frac{2}{3}$ or $\frac{4}{5}$ $\qquad$ $\frac{5}{10}$ or $\frac{3}{5}$ $\qquad$ $\frac{10}{10}$ or 2

2. Which is smaller? Use the symbol < to show this.

   $\frac{3}{5}$ or $\frac{3}{8}$ $\qquad$ $\frac{2}{5}$ or $\frac{2}{10}$ $\qquad$ $\frac{5}{6}$ or $\frac{5}{10}$

3. Which is greater? Explain.

   $\frac{3}{4}$ or $\frac{3}{5}$ $\qquad$ $\frac{5}{6}$ or $\frac{5}{8}$

4. Count forward. Fill in the missing fractions.

   $\frac{1}{10}, \frac{1}{5}, \_\_, \frac{2}{5}, \_\_, \frac{3}{5}, \_\_$

5. Draw a number line and show that:

   $\frac{3}{5} < \frac{7}{10}$; $\qquad$ $\frac{9}{10} < \frac{3}{2}$; $\qquad$ $\frac{8}{10} = \frac{4}{5}$

6. Name the numbers from the smallest to the greatest.

   $\frac{3}{7}, \frac{3}{10}, \frac{3}{5}, \frac{3}{2}$

7. Complete the following:

   $1\frac{5}{10} = 1\frac{\square}{10}$; $\qquad$ $\frac{17}{10} = 1\frac{\square}{10}$; $\qquad$ $\frac{30}{10} = \square$; $\qquad$ $\frac{19}{10} = 1\frac{9}{\square}$

SETS; NUMBER; NUMERATION

UNIT 59 - SET OF FRACTIONAL NUMBERS: TENTHS; DECIMAL FORM

NOTE TO TEACHER

A decimal is a numeral. It is another form for recording a number written as a fraction whose denominator is 10 or a power of 10. Both $\frac{1}{10}$ and .1 are numerals that name the same idea - the fractional number "one-tenth."

Children should understand the relationship of:

Tenths to halves	Tenths to fifths
Tenths to one	One to tenths

Objectives: To extend understanding of multiplicative relationships of place value.

To develop Place Value system; Tenths, decimal form.

To relate common and decimal forms for tenths.

TEACHING SUGGESTIONS

Multiplicative Relationships: Tenths to 1; One to 10; Ten to 100

1. Discuss experience situations involving tenths: odometer, pedometer.

2. Use squared material to compare a strip of 2 square units ;

3 square-units □□□ 5 square-units □□□□□

with 1 square-unit □ and the reverse.

How many times larger than 1 square-unit is 2 square-units? 3 square-units? etc.

What part of 2 square-units is 1 square-unit? etc.

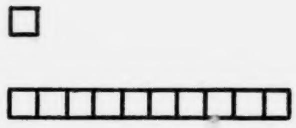
3. Compare a ten-square strip with a one unit-square; a one unit-square with a ten-square strip.

How many times as great as one unit is a ten-square strip?

[10 units is 10 times 1 unit.]

What part of a ten-square strip is one unit?

[1 is $\frac{1}{10}$ of 10]



4. Compare a unit square, a ten square strip and a one hundred square.

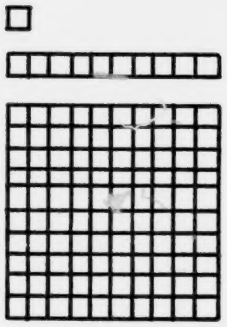
Discuss:

10 to 1 relationship.

10 is how many times 1? [10]
100 is how many times 10? [10]

1 to 10 relationship.

1 is what part of 10? $\left[\frac{1}{10}\right]$
10 is what part of 100? $\left[\frac{1}{10}\right]$



5. Compare with the values of pennies, dimes, and a one-dollar bill.

6. Have children consider 2 unit-squares.

   Cut one of the units into 10 approximately equal parts. (▯)

   What fractional number does each one of these equal parts represent?

   $$\left[\frac{1}{10}\right]$$

   Have children compare the uncut unit-square with one of the tenths; with 10 of the tenths.

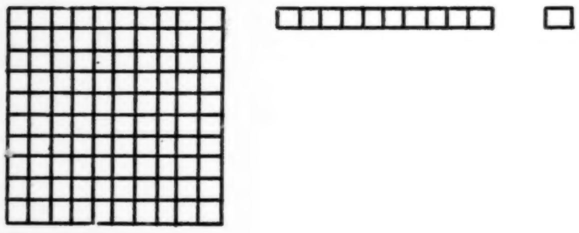
   State the relationships:

   10 tenths is equal to 1 unit.
   1 unit is 10 times as great as 1 tenth.
   1 tenth is $\frac{1}{10}$ of 1 unit.

Numeration System Extended To Tentns: Place Value

1. Place a one-hundred square, a ten-square-strip, and a unit square on the display board. Put a Place-Value Chart directly below the squared material.

   Record the numerals for the numbers represented by the material on the Place-Value Chart.



H	T	U
1	1	1

Have available one of the tenths that has been previously cut. (▯)

Ask children:

Where would you place this tenth on the display board?

[right of units place]

Why? Each place has a value one tenth of the place immediately to its left.

Where would you record the numeral for one tenth on the chart?

H	T	U	?
1	1	1	1

What would you call that place? [tenths] Why?

2. Have children label the column "Tenths".

Compare the value of 1 in units place with the value of 1 in tenths place.

H	T	U	Tenths
1	1	1	1

Note:

1 in units place is 10 times the value of 1 in tenths place.

1 in tenths place is $\frac{1}{10}$ the value of 1 in the units place.

3. Have children discuss, then record 2 tenths, 3 tenths, ... 9 tenths.

Where would you record 10 tenths? Why?

Continue to develop understanding of more than 1 unit and tenths.

H	T	U	Tenths
		3	4
		9	1
	2	6	2
1	4	8	3

4. Placing The Decimal Point

Draw a place value chart as shown.

	Units	
2	3	7

Have children label the place to the left of units place. [tens]

Label the place to the right of units place. [tenths]

T	U	Tenths
2	3	7

Have children read the numeral. [23 and 7 tenths]

Erase the labels and lines indicating columns.

> Does the numeral, 237 still indicate 23 and 7 tenths? Explain.
> How can 23 and 7 tenths be shown without labeling each place?

Ask children how tenths are shown on various instruments; odometer, pedometer.

Tell children that another way of recording $23\frac{7}{10}$ or 23 and 7 tenths is 23.7, which can be read, "twenty-three and seven-tenths".

Discuss the decimal point as a way of identifying units place.

Have children record other numerals. 64.2 29.3 7.8

> What kind of number is represented to the left of the decimal point? [Whole number]
>
> What kind of number is represented to the right of the decimal point?
> [Fractional number with a value of less than 1]
>
> What do you think 0.1 means? 0.5 means? 0.6 means?
>
> What does the zero to the left of the decimal point indicate?
>
> How does the recording of the decimal fraction differ from recording of the common fraction?
> [The denominator of the decimal fraction is not written]

5. Suggested Exercises

a. Mark the following statements true or false.

$\frac{3}{10}$ is in decimal form because the denominator is 10.

27.8 is the decimal form of $27 \frac{8}{10}$.

.5 has a value of more than 1.

.5 has a value less than 0.

b. Record each of the following as fractions in decimal form.

Three tenths
Five and one tenth
Eleven tenths
Ten tenths

Ninety-eight and six tenths
Three hundred seventy and five tenths
One hundred twenty-three and no tenths

c. Relate the reading and writing of decimal fractions to life situations, such as:

Mileage on R.R. time tables
Temperature and precipitation readings
Records of track meets

Relating Common Form and Decimal Form

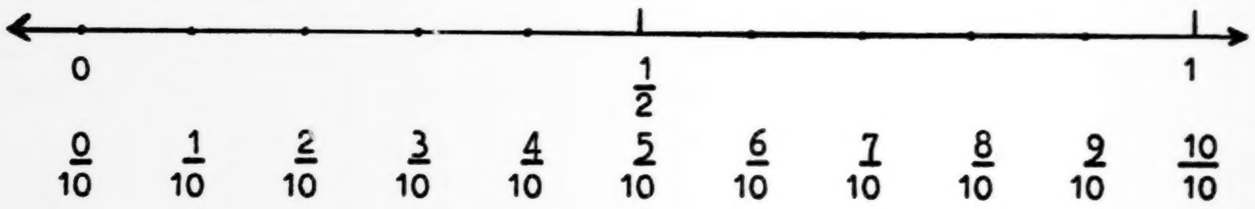
1. Using the Number Line: Halves, Fifths, Tenths

Have children:

Divide one unit of a number line into 2 equal parts.

Label 0, $\frac{1}{2}$, 1 on the number line.

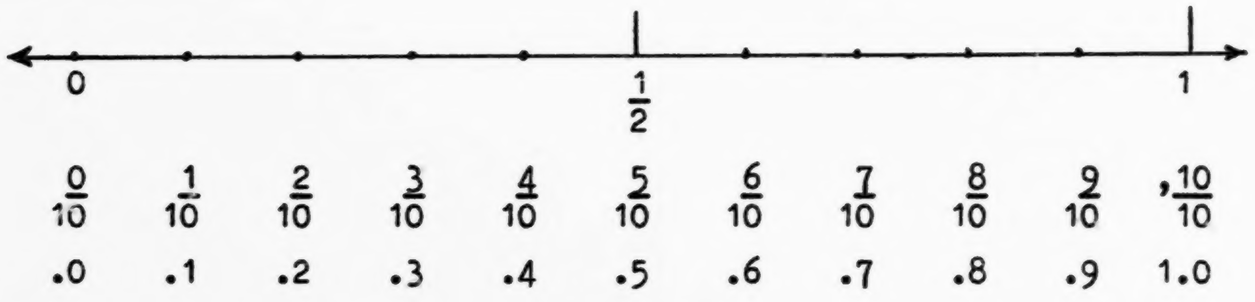
Divide each half into 5 equal parts and label tenths in fractional form.



2. In what other way can we record tenths?

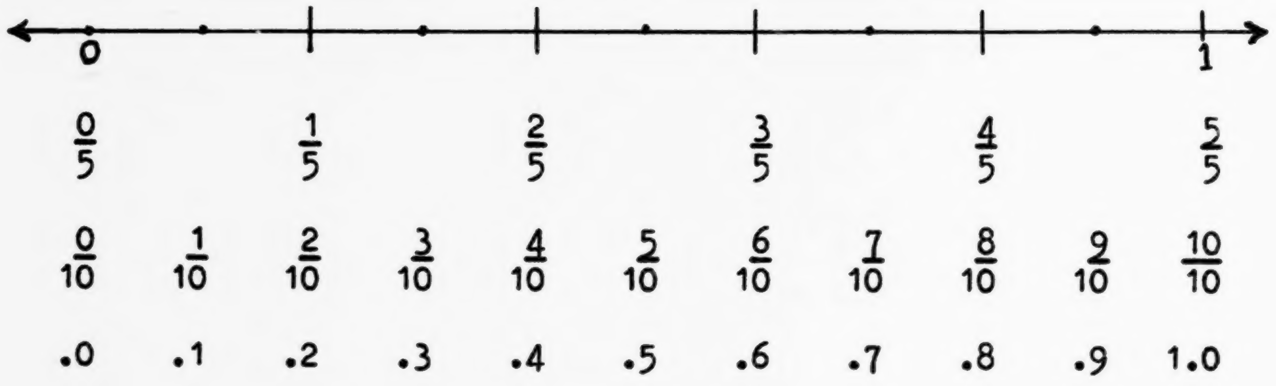
Have children record decimal form under the fractional form.

Discuss the equivalent recordings.


0
$\frac{1}{2}$
1
$\frac{0}{10}$ $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{10}{10}$
.0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0

3. **Follow the same procedure dividing the number lines into fifths, then each fifth into 2 equal parts.**

Label each part in both forms.


0
1
$\frac{0}{5}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$
$\frac{0}{10}$ $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{10}{10}$
.0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0

Have children discover:

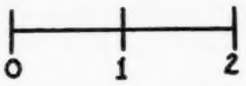
The denominator 10 is not written but is implied by the decimal point when the decimal form is recorded.

Both names $\frac{3}{10}$ and .3 are read in the same way: 3 tenths.

4. **Discuss the significance of various numerals on the number line, e.g.**

The numerals $\frac{1}{2}$, $\frac{5}{10}$, .5 all indicate the same distance from zero and name the same number.

5. Extend the lines to represent 2 units of length.



Have children indicate a variety of names for the tenth following 1.0

They may suggest: $\frac{11}{10}$, $1 + \frac{1}{10}$, $1\frac{1}{10}$, $1 + .1$, 1.1, etc.

Continue to name and discuss the meaning of each of the tenths between 1.0 and 2.0.

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Name each of the following in several ways. Refer to the number line when necessary.

$\frac{4}{5}$, $\frac{3}{2}$, $\frac{5}{5}$, $\frac{5}{2}$

one half, three fifths, nine fifths, two

$\frac{14}{10}$, $\frac{20}{10}$, $\frac{10}{10}$, $\frac{15}{10}$

2. Insert the correct symbols >, =, <, between each pair of numerals to make a true statement.

$.7 \ \square \ 1$ $1.2 \ \square \ 1$ $\frac{10}{10} \ \square \ 1$

$1.0 \ \square \ 1$ $\frac{9}{10} \ \square \ 1$ $\frac{20}{20} \ \square \ 1$

3. Complete the following open sentences.

$.5$ in. $= \frac{1}{\square}$ in. $\frac{23}{\square}$ in. $= 2.3$ in.

2.5 in. = $2\frac{1}{\square}$ in. $\frac{3}{\square}$ in. = .6 in.

$\frac{15}{\square}$ in. = 1.5 in. $1\frac{2}{\square}$ in. = 1.4 in.

4. Write true or false next to each of the following. Explain.

.8 in. is closer to 1.0 than to .5 in.
1.4 in. is nearer in value to 1.0 than to 2.0 in.
2.5 in. is halfway between 2.0 and 3.0 in.

5. Find the number of tenths in 1.0 ft.; in 4.0 ft.; in 10.0 ft.

6. Represent the decimal form of the following sums:

a. $5 + \frac{3}{10}$

b. $(4 \times 10) + (7 \times 1) + (5 \times \frac{1}{10})$

c. $0 + \frac{7}{10}$

7. Write each of the following as the sum of two numbers:

6.4   14.3   0.1

8. A foot rule calibrated in tenths may be used to:

Count by tenths.
Emphasize distances from zero. The numeral 1.5 marks the distance 1.5 units from zero.

Counting

1. Reinforce counting forward. Use the number line when helpful.

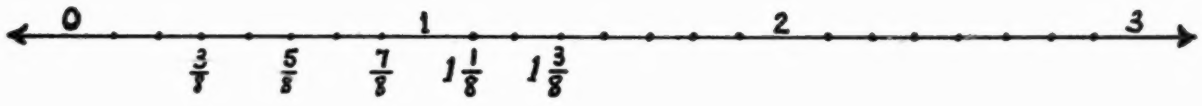
a. Have children count by steps of 1 half, 1 third, 1 fourth, 1 fifth, etc. beginning with zero. Later they can count using larger intervals. Then count again changing to simpler form.

For example: $0, \frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}, \frac{10}{5}, \frac{12}{5}, \ldots\ldots$

$0, \frac{2}{5}, \frac{4}{5}, 1\frac{1}{5}, 1\frac{3}{5}, 2, 2\frac{2}{5}, \ldots\ldots$

b. Have children count beginning at any point on the number line, counting by jumps of fractions of any size. For example:

Begin at $\frac{3}{8}$, count by jumps of fourths:



c. Discuss sets of fractions in sequence.

When counting by steps of one-tenth: $1\frac{3}{10}$ follows what number? Is followed by what number? $1\frac{2}{5}$ follows what number? etc.

What number is two greater than $3\frac{9}{10}$?

How many tenths must be added to $2\frac{1}{5}$ to reach the next whole number?

d. Have children complete the following sequence:

Count by steps of $\frac{2}{3}$: $3\frac{1}{3}$, 4, $4\frac{2}{3}$, —, —, —, $7\frac{1}{3}$

Count by jumps of $\frac{3}{10}$: $1\frac{1}{5}$, $1\frac{1}{2}$, $1\frac{4}{5}$, —, —, —, 3

Count by steps of .2: 1.3, 1.5, —, 1.9, —, 2.3, —

Count by steps of .3: 7.1, 7.4, 7.7, —, —, —, —

2. Reinforce counting backward. Use number line when necessary.

a. Children use a number line and record as they count. They begin with 1 and count backward by intervals of 1 half, 1 third, 1 fourth, 1 fifth, etc. Later using larger intervals.

They count again changing to simpler form.

First: $2,\ 1\frac{7}{9},\ 1\frac{5}{9},\ 1\frac{3}{9},\ 1\frac{1}{9},\ \frac{8}{9},\ \frac{6}{9},\ \frac{4}{9},\ \frac{2}{9},\ 0$

then: $2,\ 1\frac{7}{9},\ 1\frac{5}{9},\ 1\frac{1}{3},\ 1\frac{1}{9},\ \frac{8}{9},\ \frac{2}{3},\ \frac{4}{9},\ \frac{2}{9},\ 0$

b. Children count backward, beginning with any number on the line.

$3\frac{1}{8},\ 2\frac{5}{8},\ 2\frac{1}{8},\ 1\frac{5}{8},\ 1\frac{1}{8},\ \frac{5}{8},\ \frac{1}{8}$ (subtracting $\frac{1}{2}$)

2.1, 1.9, 1.6, 1.3, 1, .7, .4, .1 (subtracting .3)

c. Discuss fractions in sequence:

When counting backward by one-tenth: 1.3 comes before what number?

What number is two less than $3\frac{11}{20}$? than 5.1?

How many twelfths must be subtracted from $2\frac{1}{3}$ to reach the nearest whole number?

OPERATIONS

Unit 60 - SET OF FRACTIONAL NUMBERS: ADDING AND SUBTRACTING TENTHS; DECIMAL FORM; HORIZONTAL AND VERTICAL FORMAT; WITHOUT EXCHANGE

NOTE TO TEACHER

Properties of Operations that apply to fractional numbers in fractional form apply to fractional numbers in decimal form. For addition, the Associative and Commutative properties hold and zero is the Identity Element.

Objective: To add and subtract tenths: (decimal form)by:

Relating common fractional and decimal forms.
Applying The Associative Property of Addition.
Developing vertical algorithms, no exchange.

TEACHING SUGGESTIONS

Suggested experience situations to use in this unit are:

Distances walked.
Distances traveled by bus.
Using a cyclometer, etc.

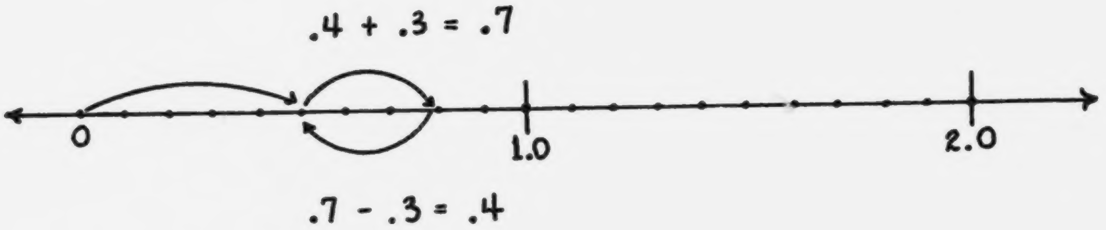
Horizontal Format

Sums and Minuends Less Than 1

Use number lines to show relationship between adding and subtracting tenths.

1. **Begin with addition:**

Adding	.3	$.4 + .3 = .7$
Subtracting	.3	$.7 - .3 = .4$



To add .4 + .3, children find .4 on the number line, then move 3 of the one-tenth spaces to the right.

To subtract .3 from .7, children locate .7 on the number line, then move 3 of the one-tenth spaces to the left.

2. **Begin with subtraction:**

Subtracting	.3	$.7 - .3 = .4$
Adding	.3	$.4 + .3 = .7$

3. **Have children replace the frame to make true statements.**

$.4 + .3 = \frac{4}{10} + \frac{\triangle}{10} = n$ $.7 - .3 = \frac{7}{10} - \frac{\triangle}{10} = n$

$.2 + .5 = \frac{2}{10} + \frac{5}{\triangle} = n$ $.7 - .5 = \frac{7}{10} - \frac{5}{\triangle} = n$

$\frac{7}{10} + \frac{1}{\triangle} = .7 + .1 = n$ $.8 - \triangle = \frac{8}{10} - \frac{1}{10} = n$

$\triangle + .6 = \frac{3}{10} + \frac{6}{10} = n$ $\triangle - .3 = \frac{9}{10} - \frac{3}{10} = n$

Sums and Minuends Greater Than 1

1. Reinforce renaming fractions in decimal form as whole number and/or a whole number plus a fraction.

$.9 + .1 = \square$ $\qquad$ $1.0 - .1 = \square$

$.8 + .2 = \square$ $\qquad$ $1.0 - .2 = \square$, etc.

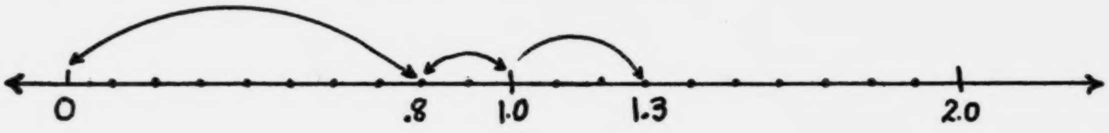
$.1 + .9 = \square$ $\qquad$ $1.0 - .9 = \square$

$.2 + .8 = \square$ $\qquad$ $1.0 - .8 = \square$, etc.

2. When children find the sum of $.8 + .5$, they may think of $.5$ as $(.2 + .3)$

$$.8 + .5 = .8 + (.2 + .3) = (.8 + .2) + .3 = 1 + .3 \text{ or } 1.3$$

(Application of Associative Property)



$.8 + .5 = 1.3$

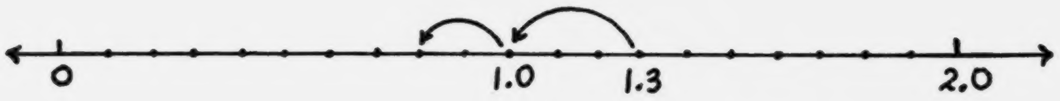
Children may record the sum in several ways.

13 tenths or 1 and 3 tenths or $1.0 + .3$ or 1.3 or $1\frac{3}{10}$

3. $1.3 - .5 = n$

When children find the remainder for $1.3 - .5$, they should think of $.5$ as $(.3 + .2)$.

$$(1.3 - .3) - .2 = 1 - .2 = .8$$



$1.3 - .5 = .8$

Have children record the remainder in several ways. 8 tenths, $\frac{8}{10}$, $.8$, etc.

4. **Provide practice exercises.**

Complete the following

A. $\frac{3}{10} + \frac{7}{10} = \triangle$ $\qquad$ $.3 + .7 = \square$

$\frac{5}{10} + \frac{\triangle}{10} = 1$ $\qquad$ $.5 + \triangle = 1$

B. $\frac{3}{10} + \frac{8}{10} = \frac{3}{10} + \frac{\triangle}{10} + \frac{1}{10} = \square$ $\qquad$ $.3 + .8 = .3 + \quad + .1 = \square$

$\frac{5}{10} + \frac{6}{10} = \frac{5}{10} + \frac{5}{10} + \frac{\triangle}{10} = \square$ $\qquad$ $.5 + .6 = .5 + .5 + \quad = \square$

Tell what property helps to solve the problems in B.

Complete the following:

$\frac{10}{10} - \frac{1}{10} = \triangle$ $\qquad$ $1.0 - .1 = \triangle$

$\frac{10}{10} - \frac{\triangle}{10} = \frac{6}{10}$ $\qquad$ $1.0 - \triangle = .6$

$1\frac{2}{10} - \frac{4}{10} = 1\frac{2}{10} - \frac{2}{10} - \triangle = \square$ $\qquad$ $1.2 - .4 = 1.2 - .2 - \triangle = \square$

$1\frac{6}{10} - \frac{8}{10} = 1\frac{6}{10} - \triangle - \frac{2}{10} = \square$ $\qquad$ $1.6 - .8 = 1.6 - \triangle - .2 = \square$

Complete the following and then explain the property involved.

$.7 + .6 = \triangle$ and $\triangle - .6 = .7$
$.6 + .7 = \triangle$ and $\triangle - .7 = .6$

Since $.6 + .9 = 1.5$, then $1.5 - .9 = \square$
Since $.9 + .6 = 1.5$, then $1.5 - .6 = \square$

Vertical Format

1. Addition: No Exchange

Suggested Problem: During April, the average rainfall was 5.3 inches. In May, it was 3.4 inches. How much rain fell during the two months?

Estimate: $n > 8$; $n < 9$ n will be between 8 and 9.

Have children compute, then record the sum. They should describe the method they used.

$$\begin{array}{ll} 5\frac{3}{10} = 5 + \frac{3}{10} & \quad 5\frac{3}{10} \\ \underline{3\frac{4}{10} = 3 + \frac{4}{10}} & \quad \underline{3\frac{4}{10}} \\ \quad 8 + \frac{7}{10} \text{ or } 8\frac{7}{10} & \quad 8\frac{7}{10} \end{array}$$

$$\begin{array}{ll} 5.3 = 5 + .3 & \quad 5.3 \\ \underline{3.4 = 3 + .4} & \quad \underline{3.4} \\ \quad 8 + .7 = 8.7 & \quad 8.7 \end{array}$$

Have children compare the sum with the estimate.
Compare the various algorithms used and discuss the most efficient.

2. Subtraction: No Exchange

Suggested Problem: John walked 1.6 miles. Tom walked .5 of a mile. How much farther did John walk than did Tom? $1.6 - .5 = n$

Estimate: About a mile.

Children compute then record the difference. They describe the method used. Encourage a variety of methods.

$$\begin{array}{lr}
1 + 6 \text{ tenths} & 1\frac{6}{10} \\
- \quad 5 \text{ tenths} & \frac{5}{10} \\
\hline
1 + 1 \text{ tenth} & 1\frac{1}{10}
\end{array}$$

$$\begin{array}{rr}
1 + .6 & 1.6 \\
\underline{\quad .5} & \underline{-\ .5} \\
1 + .1 & 1.1
\end{array}
\qquad 1.6 - 0.5 = n$$

Have children compare the remainder with the estimate. Compare the algorithms and select the most efficient.

3. Provide practice:

   a. Estimate the sum, then add. Compare the exact sum with the estimate. Verify by using the common fraction form.

$$\begin{array}{r} .2 \\ .4 \\ .3 \\ \hline \end{array}
\qquad 0.5 + 0.1 + 0.3
\qquad \begin{array}{r} 5.2 \\ 7.4 \\ \hline \end{array}
\qquad \begin{array}{r} 8.3 \\ 2.5 \\ 4.1 \\ \hline \end{array}
\qquad \begin{array}{r} 37.2 \\ 14.6 \\ \hline \end{array}
\qquad \begin{array}{r} 35.6 \\ 12.2 \\ 13.1 \\ \hline \end{array}$$

   Find the sum of 54.2 and 37.6   Add: 14.7, 23.0, 19.2

   b. Compare each of the following pairs of numbers and find the difference:

   8.3, 5.0   19.5, 0.3
   6.7, 4.3   2.6, 17.9
   0.4, 5.6   27.3, 162.5

   c. Solve the following:

   $n + 6.4 = 23.7$ Find the difference between 65.9 and 39.6

   $29.8 + n = 35.9$
   $238.6 - 129.3 = n$ 185.8 minus 28.6
   From 138.9 take 59.7   Subtract 8.0 from 53.6

d. Without doing the computation, tell whether the solution of each of the following will be a whole number, a fraction, or a whole number and fraction.

0.7 + 0.4	0.8 + 0.6
.9 + .1	34.5 - 26.5

4. Additional exercises may be found in textbooks.

SETS; NUMBERS; NUMERATION

UNIT 61 - EXTENDED UNDERSTANDING OF SETS: INTERSECTION OF SETS

NOTE TO TEACHER

The intersection of two sets is the set which contains those elements and only those elements that are common to both sets. If A = {a, b, c} and B = {b, c, e } then the intersection of sets A and B is the set whose elements are b and c; {b, c }.

The symbol for intersection is ∩. The set which is the intersection of Set A and Set B is symbolized as $A \cap B$. Here $A \cap B = \{b, c\}$.

One of the applications of intersecting sets is in finding common denominators, common factors, etc.

Objectives: To reinforce concept of sets.
To develop meaning of intersection of sets.

TEACHING SUGGESTIONS

Reinforce Understanding of Sets, Set Notation, Set Union

1. Give an example of a set, then choose a subset of that set. For example, for the set of boys in our class, a subset may be {John, Fred}.

2. State the example given in question 1 above, in set notation. [{boys in our class} {John, Fred}]

3. $\{3, 4\} \cup \{\ \} = \square$ [{3, 4}]

4. $N\{3, 4\} + N\{\ \} = \square$ [2]

5. Are {a, b, c} and {d, e, f} equal sets? equivalent sets? Explain.

6. For $S = \{0, 1, 2, 3, 4, 5\}$

   Write the subset of S such that each number in the subset is even; is odd.

   Write the subset of S such that each element is a factor of 12; of 49.

Intersection of Sets

1. Suggested problem:

   John, Mary, Tom, Ralph and Sue belong to the Science Club.
   John, Ellen, Sue, Alice, David belong to the Math Club.
   Which children are members of both clubs? [John, Sue]

   Discuss set notation for this problem.

A.

The set of children in the Science Club = {John, Mary, Tom, Ralph, Sue}
The set of children in the Math. Club = {John, Ellen, Sue, Alice, David}
The set of children who are members of both clubs = {John, Sue}
The intersection of the two sets = {John, Sue}

B.

If S is the set of members of Science Club and M is the set of members of Math. Club, then the intersection of S and M = {John, Sue}

2. Tell children that the symbol for intersection is $\cap$.

   For example:

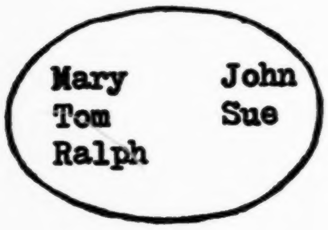
$$\{\text{John, Mary, Tom, Ralph, Sue}\} \cap \{\text{John, Ellen, Sue, Alice, David}\} = \{\text{John, Sue}\}$$

or

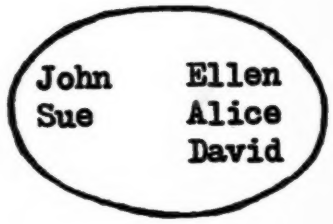
$$S \cap M = \{\text{John, Sue}\}$$

3. Using diagrams to show intersection (Venn Diagrams) present and discuss the following:

Members of Science Club


Mary
Tom
Ralph
John
Sue

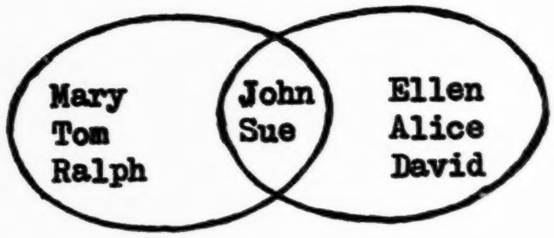
Members of Math Club


John
Sue
Ellen
Alice
David

John and Sue; Members of Both Clubs

Intersection of Science and Math Clubs

$S \cap M$


Mary
Tom
Ralph
John
Sue
Ellen
Alice
David

4. Provide practice

   a. Find the intersection of the following sets.
   Show in set notation.
   Then show using Venn Diagrams.

   {*, ○, △, □, } and {◇, △, □, ▽, ⏢} [{△, □}]

   {a,b,c,d } and {e,b,g,h,d,k } [{b, d}]

   {0,1,2,3,4,5,6,7,8,9,10} and {1,3,5,7,9} [{1,3,5,7,9}]

   b. Which states border on New York?
   Which states border on Pennsylvania?
   Which states border on both New York and Pennsylvania?

   Show this as the intersection of two sets.

SETS; NUMBER; NUMERATION

UNIT 62 - SET OF FRACTIONAL NUMBERS: FINDING GREATEST COMMON FACTOR OF TWO NUMBERS; COMMON DENOMINATOR; LEAST COMMON DENOMINATOR

Objective: To help children apply the Fundamental Principle of Fractions. ($\frac{a}{b} = \frac{a \times c}{b \times c}$; $\frac{a}{b} = \frac{a \div c}{b \div c}$ where $c \neq 0$, $b \neq 0$) to find:

The greatest common factor of two numbers.
The common denominator for two or more fractions.
The least common denominator of two or more fractions.

TEACHING SUGGESTIONS

Finding the Greatest Common Factor

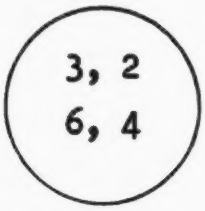
1. Reinforce renaming a fraction in simpler fractional forms.

For $\frac{8}{16}$: $\frac{8}{16} = \frac{\square}{8}$, $\frac{8}{16} = \frac{2}{\square}$, $\frac{8}{16} = \frac{\square}{2}$

What was the common factor by which you divided the numerator and the denominator in each case?

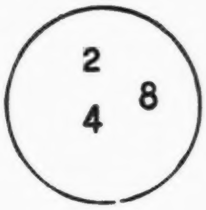
2. **Use diagrams to show the intersection of the sets of factors. For example: For $\frac{12}{16}$**

Factors of the Numerator



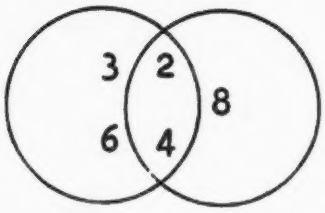
A

Factors of the Denominator



B

Factors Common to Numerator and Denominator



Children should note that:

$A = \{2, 3, 4, 6\}$
$B = \{2, 4, 8\}$
$A \quad B = \{2, 4\}$

Ask children:

What are the factors common to both?	**[2, 4]**
What is the largest factor common to both?	**[4]**

Direct children to change $\frac{12}{16}$ to simplest form.

Have children find an equivalent of $\frac{12}{16}$ using the greatest common factor as a divisor.

Children record the computation. $\left[\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}\right]$

What was the (greatest) common factor by which the numerator and the denominator were divided? [4]

What is the advantage of dividing the numerator and the denominator by the greatest common factor?

Finding A Common Denominator

1. Suggested problem: Which represents a larger number: $\frac{1}{2}$ or $\frac{7}{10}$?

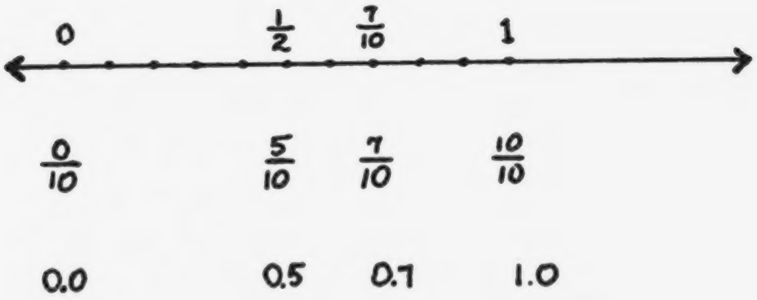
Have children draw a number line and label points corresponding to 0, $\frac{1}{2}$, $\frac{7}{10}$, 1.

They then compare the relative positions of $\frac{1}{2}$ and $\frac{7}{10}$.

Since $\frac{1}{2} = \frac{5}{10}$

and $\frac{7}{10} > \frac{5}{10}$

$\frac{7}{10} > \frac{1}{2}$



Follow the same procedure to compare $\frac{1}{4}$ with $\frac{3}{8}$, $\frac{1}{3}$ with $\frac{4}{9}$, $\frac{1}{2}$ with $\frac{5}{6}$, etc.

Discuss how the comparison was made in each case.

Tell children that: When two or more fractions have the same denominator they are said to have a common denominator.

In each set of fractions below have children select the fraction whose denominator can be used as a common denominator.

$\frac{1}{3}, \frac{1}{9}$ $\quad$ $\frac{1}{6}, \frac{1}{12}$ $\quad$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ $\quad$ $\frac{1}{10}, \frac{1}{50}, \frac{1}{100}$

2. **Problem:** To find a common denominator for $\frac{1}{2}$ and $\frac{1}{3}$.

   Children rename $\frac{1}{2}$ and $\frac{1}{3}$

   $$\frac{1}{2} = \frac{2}{\square}, \frac{3}{\square}, \frac{4}{\square}, \frac{5}{\square}$$

   $$\frac{1}{3} = \frac{2}{\square}, \frac{3}{\square}$$

   Teacher should record on chalkboard:

   $$\begin{tikzcd}[column sep=small] \frac{1}{2} = \frac{2}{4}, & \frac{3}{6}, & \frac{4}{8}, & \frac{5}{10} \\ \frac{1}{3} = \frac{2}{6}, \arrow[ur, leftrightarrow] & \frac{3}{9} & & \end{tikzcd}$$

   Children should note that:

   $\frac{1}{2}$ and $\frac{1}{3}$ can both be renamed as sixths, a common denominator.

   The common denominator, 6, is not the denominator of either fraction.

3. Follow the same procedure to find a common denominator for:

   $\frac{1}{3}$ and $\frac{1}{4}$;　　$\frac{1}{2}$ and $\frac{1}{5}$;　　$\frac{1}{3}$ and $\frac{2}{5}$

   Ask children:

   For the denominators 2 and 3, what was a common denominator? [6 or 12 or . . .]
   For the denominators 3 and 4? [12]
   for 3 and 5? [15]
   Do you see a pattern?
   How can you find common denominators without writing a set of equivalents for each fraction? [Generalization: The product of the denominators is always a common denominator.]

Finding The Least Common Denominator

1. Reinforce the concept that when adding or subtracting fractions with unlike denominators, children must rename the fraction so that the denominators are the same.

2. Suggested exercise: $\frac{1}{4} + \frac{1}{6} = n$

   Children should use the method of multiplying one denominator by the other to find a common denominator.

$$\begin{aligned} \frac{1}{4} + \frac{1}{6} &= \frac{6}{24} + \frac{4}{24} \\ &= \frac{10}{24} \\ &= \frac{5}{12} \end{aligned}$$

3. Discuss finding other denominators common to 4 and 6.

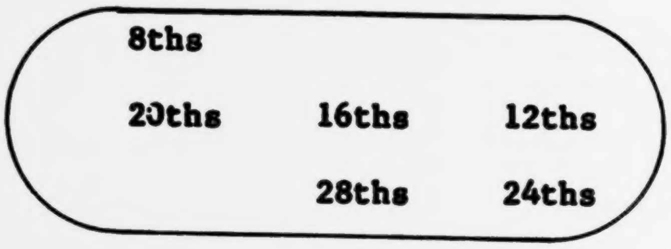
   Children rename $\frac{1}{4}$ in many ways; $\frac{1}{6}$ in many ways.

   Teacher records as shown below:

$$\begin{tikzcd}
\frac{1}{4} = & \frac{2}{8}, & \frac{3}{12}, \arrow[dl, leftrightarrow] & \frac{4}{16}, & \frac{5}{20}, & \frac{6}{24}, \arrow[dll, leftrightarrow] & \frac{7}{28}, & \frac{8}{32}, & \frac{9}{36} \arrow[dlll, leftrightarrow] & \text{etc.} \\
\frac{1}{6} = & \frac{2}{12}, & \frac{3}{18}, & \frac{4}{24}, & \frac{5}{30}, & \frac{6}{36}, & \text{etc.}
\end{tikzcd}$$

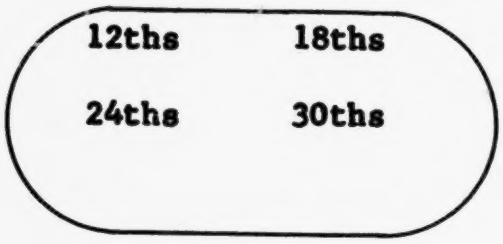
   Children note that 12, 24, and 36 are common denominators for 4 and 6, and that 12 is the Least Common Denominator.

Fourths may be renamed as


8ths
20ths
16ths
12ths
28ths
24ths

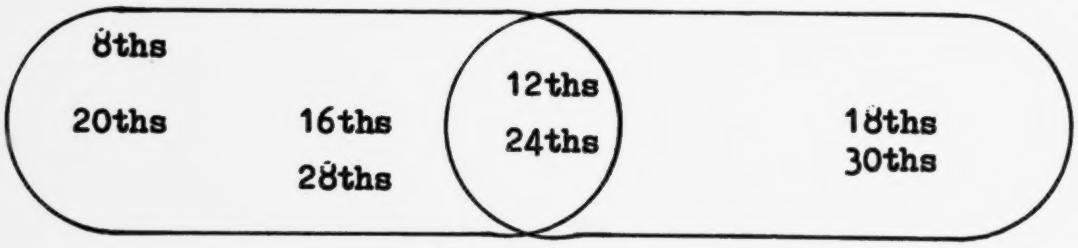
A

Sixths may be renamed as


12ths
18ths
24ths
30ths

B

Common Denominators for
Fourths and Sixths may be


8ths
20ths
16ths
28ths
12ths
24ths
18ths
30ths

Children should note that

$A = \{8, 12, 16, 20, 24, 28\}$

$B = \{12, 18, 24, 30\}$

$A \cap B = \{12, 24\}$

Have children note that common denominators may be found through the intersection of sets of multiples of the two denominators.

4. Have children solve the exercise, $\frac{1}{4} + \frac{1}{6} = n$, using 12 as the common denominator; 24 as the common denominator. They compare the computation and discover the advantage of using the least common denominator.

5. Suggested exercise: $\frac{1}{6} + \frac{1}{8} = n$.

   Children find a common denominator for $\frac{1}{6}$ and $\frac{1}{8}$.

   Is it the least common denominator?

Discuss various ways of finding the least common denominator.

Is the larger denominator (8) common to both?
If not, is 2 times the larger denominator common to both? 3 times? etc.

Children state this method for finding the least common denominator in their own words. (Multiply the larger denominator by 2, 3, etc. until a common denominator is reached.)

*6. Some children may state the above in terms of multiples. (Optional)

Ask children to write the multiples of $\{8, 16, 24, 32, \ldots\}$

Which is the smallest multiple of 8 that is also a multiple of 6?

Generalization: The least common multiple (24) of the denominators (6 and 8) is the least common denominator.

Extend to finding the least common denominator for 3 or more fractions.

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Find the least common denominator for:

   $\frac{1}{3}$ and $\frac{1}{4}$ $\frac{1}{3}$ and $\frac{1}{10}$ $\frac{1}{8}$ and $\frac{1}{5}$ $\frac{1}{12}$ and $\frac{1}{9}$

2. Which fraction is larger? Why?

   $\frac{3}{4}$ or $\frac{5}{6}$ $\frac{1}{3}$ or $\frac{2}{5}$ $\frac{3}{10}$ or $\frac{1}{6}$ $\frac{1}{4}$ or $\frac{2}{9}$

3. Find the least common denominator for:

   $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$

4. Which fraction is the largest within each set? Why?

   $\frac{2}{3}, \frac{1}{2}, \frac{3}{5}$ $\frac{5}{5}, \frac{6}{7}, \frac{2}{3}$ $\frac{3}{4}, \frac{5}{8}, \frac{2}{3}$

5. Find common denominators for $\frac{1}{8}$ and $\frac{1}{6}$, using the diagrams for intersection of sets.

6. Additional practice exercises may be found in textbooks.

OPERATIONS

UNIT 63 - SET OF FRACTIONAL NUMBERS: ADDITION AND SUBTRACTION; COMMON FORM; HORIZONTAL AND VERTICAL FORMAT

Objective: To help children add and subtract fractions, common form, using the least common denominator method.

TEACHING SUGGESTIONS

1. Reinforce counting forward and backward.

   Study each sequence below. Find the pattern that was used, then fill in the blanks.

   a. $1 \quad 1\frac{1}{4} \quad 1\frac{1}{2} \quad 1\frac{3}{4}$ - - - - -.

   b. $\frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3}$ - - - -.

   c. $1 \quad \frac{15}{16} \quad \frac{7}{8} \quad \frac{13}{16} \quad \frac{3}{4}$ - - - -.

   d. $\frac{1}{9} \quad \frac{1}{6} \quad \frac{2}{9}$ - - - - - -.

   e. $2\frac{1}{3} \quad 2\frac{1}{12} \quad 1\frac{5}{6} \quad 1\frac{7}{12}$ - - -.

2. Reinforce renaming fractions.

   a. Change each of the following to an equivalent fraction in higher terms.

      $12\frac{3}{8}$: $12\frac{\square}{16}$, $12\frac{\square}{32}$

      $6\frac{2}{3}$: $6\frac{\square}{9}$, $6\frac{\square}{6}$, $6\frac{\square}{12}$

b. Change each of the following to equivalent fractions in lowest terms.

$12\frac{10}{20}$: $12\frac{2}{\square}$, $12\frac{1}{\square}$, $12\frac{5}{\square}$

$\frac{8}{16}$: $\frac{2}{\square}$, $\frac{1}{\square}$, $\frac{4}{\square}$

3. Reinforce regrouping.

Complete the following to make the statement true:

$14\frac{3}{8} = \square\frac{11}{8}$ $26\frac{3}{10} = 25\frac{13}{\square}$ $129\frac{3}{5} = 128\frac{\square}{5}$

Addition and Subtraction: Horizontal Format

1. Reinforce adding and subtracting fractions with unlike denominators - common denominator apparent.

Suggestions for evaluation:

Find the missing number, then solve the equation.

$\frac{11}{16} + \frac{9}{16} = 1 + \frac{\square}{4}$

$5\frac{1}{2} + \frac{7}{10} = 6 + \frac{\square}{5}$

$17\frac{5}{6} + 8\frac{5}{12} = 25 + \frac{\triangle}{\square}$

$24\frac{7}{18} + 48\frac{2}{3} = n$

$15\frac{5}{8} - 3\frac{1}{2} = 12\frac{5}{8} - n$

$7\frac{1}{5} - 1\frac{4}{5} = 6\frac{1}{5} - n$

$10\frac{1}{3} - \frac{7}{9} = 9 - n$

$35\frac{3}{14} - 18\frac{3}{7} = 16 - n$

2. **Suggested exercises: Fractions with unlike denominators.**

a. $5\frac{1}{2} + 2\frac{1}{5} = n$

Children may think:

$5\frac{1}{2},\ 7\frac{1}{2},\ 7\frac{5}{10},\ 7\frac{7}{10}$ or $5\frac{1}{2},\ 5\frac{5}{10},\ 7\frac{5}{10},\ 7\frac{7}{10}$

or

$$5\frac{1}{2} + 2\frac{1}{5} = (5 + 2) + \left(\frac{1}{2} + \frac{1}{5}\right)$$

$$= 7 \quad + \left(\frac{5}{10} + \frac{7}{10}\right) = 7\frac{7}{10}$$

b. $12\frac{2}{3} - 8\frac{1}{2} = n$

Children may think:

$$12\frac{2}{3} - 8\frac{1}{2} = 4 + \left(\frac{2}{3} - \frac{1}{2}\right)$$

$$= 4 + \left(\frac{4}{6} - \frac{3}{6}\right) = 4\frac{1}{6}$$

or

$12\frac{2}{3},\ 4\frac{2}{3},\ 4\frac{4}{6},\ 4\frac{1}{6}$

or

$12\frac{2}{3},\ 12\frac{4}{6},\ 4\frac{4}{6},\ 4\frac{1}{6}$

3. **Provide practice. Suggested Exercises:**

Have children find the missing number, then solve the equation.

$\frac{2}{3} + \frac{3}{4} = \frac{8}{\square} + \frac{9}{\square}$ $\qquad$ $\frac{1}{2} - \frac{1}{7} = \frac{\Delta}{14} - \frac{\square}{14}$

$5\frac{4}{9} + \frac{1}{2} = 5 + n$ $\qquad$ $8\frac{5}{7} - \frac{1}{3} = n$

$6\frac{2}{3} + \frac{5}{8} = 7 + n$ $\qquad$ $\frac{2}{3} - \frac{1}{2} = \frac{4}{\square} - \frac{3}{\square}$

$\frac{5}{6} + \frac{1}{4} = \frac{10}{\square} + \frac{3}{\square}$ $\qquad$ $16\frac{2}{3} - 7\frac{3}{5} = n$

Vertical Format

1. **Reinforce adding and subtracting fractions with unlike denominators: common denominator apparent.**

Suggested exercises:

a. **Find the sums:**

$$\begin{array}{r} 36\frac{4}{5} \\ 18\frac{3}{10} \\ \hline \end{array} \qquad \begin{array}{r} 175\frac{6}{8} \\ 89\frac{3}{16} \\ \hline \end{array}$$

b. **Find the remainders:**

$$\begin{array}{r} 59\frac{2}{3} \\ -\ 23\frac{3}{9} \\ \hline \end{array} \qquad \begin{array}{r} 61\frac{7}{12} \\ -\ 36\frac{5}{6} \\ \hline \end{array}$$

2. **Reinforce finding Least Common Denominator for:**

Eighths and tenths
Sixths and fourths
Sixths and ninths **etc.**

3. **Develop addition and subtraction of fractions with unlike denominators: Least Common Denominator Method**

Children should estimate sums and remainders before computing. They find a least common denominator before computing. Illustrative exercises:

a. $6\frac{4}{5} + 9\frac{2}{3} = n$

Estimate: $6\frac{4}{5} + 9 = 15\frac{4}{5}$, therefore $6\frac{4}{5} + 9\frac{2}{3} > 15\frac{4}{5}$

or

$6\frac{4}{5} > 6$, $9\frac{2}{3} > 9$, $6\frac{4}{5} < 7$, $9\frac{2}{3} < 10$; therefore

$6\frac{4}{5} + 9\frac{2}{3}$ must be greater than 15 and less than 17

Compute:

$$\begin{array}{rcl} 6\frac{4}{5} & = & 6\frac{12}{15} \\ \underline{9\frac{2}{3}} & = & \underline{9\frac{10}{15}} \\ & & 15\frac{22}{15} = 16\frac{7}{15} \end{array}$$

Have children compare solution with estimate.

b. $45\frac{5}{6} - 27\frac{2}{9} = n$

Estimate: $45\frac{5}{6} - 27 = 18\frac{5}{6}$, therefore $45\frac{5}{6} - 27\frac{2}{9} < 18\frac{5}{6}$

Compute:

$$\begin{array}{rcl} 45\frac{5}{6} & = & 45\frac{15}{18} \\ \underline{-\ 27\frac{2}{9}} & = & \underline{27\frac{4}{18}} \\ & = & 18\frac{11}{18} \end{array}$$

Have children compare solution with estimate.

c. **Subtraction exercise where regrouping is necessary for exchange:** $57\frac{1}{4} - 39\frac{5}{6} = n$

Children should estimate before computing.

Compute:

$$\begin{array}{rcrcr} 57\frac{1}{4} & = & 57\frac{3}{12} & = & 56\frac{15}{12} \\ \underline{-\ 39\frac{5}{6}} & = & \underline{39\frac{10}{12}} & = & \underline{39\frac{10}{12}} \\ & & & & 17\frac{5}{12} \end{array}$$

Have children explain why $57\frac{3}{12}$ should be renamed as $56\frac{15}{12}$.

4. **Answers should be verified. A suggested algorithm for verification follows:**

Computation (Vertical Format)

Exercise: $17\frac{2}{3} + 25\frac{3}{4} = n$

$$\begin{array}{rcl} 17\frac{2}{3} & = & 17\frac{8}{12} \\ \underline{25\frac{3}{4}} & = & \underline{25\frac{9}{12}} \\ & & 42\frac{17}{12} = 43\frac{5}{12} \end{array}$$

Verification (Horizontal Format)

$$\begin{aligned} 17\frac{2}{3} + 25\frac{3}{4} &= (17 + 25) + \left(\frac{2}{3} + \frac{3}{4}\right) \\ &= 42 + \left(\frac{8}{12} + \frac{9}{12}\right) \\ &= 42 + \frac{17}{12} \\ &= 43\frac{5}{12} \end{aligned}$$

5. **Continue to provide practice in addition and subtraction of fractions using both horizontal and vertical formats. Additional suggestions may be found in textbooks.**

6. Verbal problems may be chosen from situations involving geometric figures such as finding the perimeter of squares, rectangles. For example:

    a. The perimeter of a square is 7 in. Its width is $1\frac{3}{4}$ in. Find the length of the rectangle. $3\frac{1}{2} - 1\frac{3}{4} = \square$

    b. The base of an equilateral triangle is $1\frac{3}{4}$ in. long. Find its perimeter. (Solve through addition)

OPERATIONS

UNIT 64 - SET OF FRACTIONAL NUMBERS: ADDING AND SUBTRACTING TENTHS; DECIMAL FORM; WITH EXCHANGE

Objectives: To help children maintain skill in adding and subtracting decimal fractions mentally.

To develop vertical algorithms involving decimal fractions; with exchange.

TEACHING SUGGESTIONS

Horizontal Format

1. As children add "mentally" they may begin with the entire first addend. When they subtract "mentally" they may begin with the entire minuend.

   For $3.4 + 5.8 = n$ children may think:

$$\begin{aligned} & 3.4 + 5.8 \\ &= (3.4 + 5) + .8 \\ &= (8.4 + .6) + .2 \\ &= 9 + .2 \\ &= 9.2 \end{aligned}$$

or

$$\begin{aligned} & 3.4 + 5.8 = n \\ &= (3 + 5) + (.4 + .8) \\ &= 8 + 1.2 \\ &= 9.2 \end{aligned}$$

   For $9.2 - 5.8 = n$ children may think:

$$\begin{aligned} & 9.2 - 5.8 \\ &= (9.2 - 5) - .8 && \text{Regrouping } 5.8 \text{ as } 5 + .8 \\ &= (4.2 - .2) - .6 && \text{Subtracting } 5 \text{ from } 9.2\text{;} \\ & && \text{regrouping } .8 \text{ as } .2 \text{ and } .6 \\ &= 4 - .6 && \text{Subtracting } .2 \text{ from } 4.2 \\ &= 3.4 \end{aligned}$$

   Encourage other ways of arriving at solutions.

2. Suggested exercises:

Addition

26.7 + 5.7 = n [26.7, 31.7, 32, 32.4]
142.9 + 3.2 = n [142.9, 145.9, 146, 146.1]
32.5 + 41.4 = n
16.9 + 12.2 = n

Subtraction

32.4 - 5.7 = n [32.4, 27.4, 27, 26.7]
146.1 - 3.2 = n [146.1, 143.1, 143, 142.9]
45.8 - 24.5 = n
29.1 - 12.2 = n

Vertical Format

1. Refer to Unit 59 to reinforce adding and subtracting tenths without exchange.

2. Introduce addition with exchange.

Suggested problem: Tom rode 3.5 miles to Frank's house and then 2.7 miles to the library. How far did Tom travel?

$$3.5 + 2.7 = n$$

Estimate: $n > 5$; n is between 5 and 7.

Have children solve using fractional form first.

$$\begin{array}{r} 3\frac{5}{10} \\ 2\frac{7}{10} \\ \hline 5\frac{12}{10} = 6\frac{2}{10} \end{array}$$

Develop the algorithm for decimal form. Discuss as you proceed.

$3.5 = 3 + 5$ tenths
$\underline{2.7} = \underline{2 + 7 \text{ tenths}}$ (Renaming)
$5 + 12$ tenths (Sum)
$= 5 + 1 + 2$ tenths (Renaming 12 tenths)
$= 6 + 2$ tenths
$= 6.2$

As you discuss each step above, record in decimal form as shown below.

A	B
$3.5 = 3 + 5$ tenths	$3.5 = 3 + .5$
$\underline{2.7} = \underline{2 + 7 \text{ tenths}}$	$\underline{2.7} = \underline{2 + .7}$
$5 + 12$ tenths	$5 + 1.2$
$= 6 + 2$ tenths	$= 6.2$

Solve the problem again, using the concise form.

$$\begin{array}{r} 3.5 \\ \underline{2.7} \\ 6.2 \end{array}$$

Compare solution with the estimate.

3. Introduce subtraction with exchange.

   a. Reinforce renaming the minuend.

$6.3 = 5 + 13$ tenths	$4.7 = 3 + \square$ tenths
$7.5 = 6 + 15$ tenths	$5.2 = \square + 12$ tenths
$9.2 = 8 + 12$ tenths	$3.8 = 2 + \square$ tenths

b. Suggested problem: The distance from Tom's house to the park is 6.2 miles. His cousin's house is 2.9 miles from the park. How much farther away is Tom's house?
$6.2 - 2.9 = n$

Estimate: About 3 miles; $n < 4$; between 3 and 4 miles

Have children solve the problem, using fractional form first.

$$\begin{array}{rcll} 6\frac{2}{10} & = & 5\frac{12}{10} & \text{(Renaming)} \\ -\ \underline{2\frac{9}{10}} & = & \underline{2\frac{9}{10}} & \\ & & 3\frac{3}{10} & \end{array}$$

Then solve the same exercise using decimal form. Discuss each step.

$$\begin{array}{rcccccccr} & & & \text{A} & & & & \quad & \text{B} \\ 6.2 & = & 6 + .2 & = & 5 + 1.2 & & & & 6.2 \\ -\underline{2.9} & = & \underline{2 + .9} & = & \underline{2 + \ .9} & & & & -\underline{2.9} \\ & & & & 3 + \ .3 & = & 3.3 & & 3.3 \end{array}$$

Compare the answer with the estimate.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. Find the sum of: 9.5, 2.7, 23.9
   Write in a column and add: 25.8, 67.0, 48.6, 0.3
   Find the missing numeral. $628.9 + 23.5 = n$

2. Verify sums by converting to fractional form.

3. Verify remainders by using the fractional form or by adding:

   From 20.0 subtract 7.6
   From 124.7 subtract 58.9
   128.5 minus 73.9
   Subtract 24.7 from 132.2
   Find the difference between 135.8 and 352.3
   Find the missing numeral:
   $53.2 - n = 34.6$
   $20.4 + n = 100.2$
   $n + 236.8 = 325.0$

4. In the following pairs of numbers, subtract the smaller from the larger.

   36.3, 18.8
   2.4, 21.3
   60.4, 307.2
   4.2, 0.5

5. Additional exercises may be found in textbooks.

GEOMETRY AND MEASUREMENT

UNIT 65 - GEOMETRY: CIRCLE

Objectives: To extend understanding of simple closed curves in a plane to include circles.

To develop the definition of a circle as a simple closed plane curve formed by the set of all points which are the same distance from a fixed point in the plane called the center.

To extend understanding of geometric terms to include: circumference, radius, diameter, chord, arc.

TEACHING SUGGESTIONS

Characteristics of a Circle

1. Reinforce meaning of simple closed curve; meaning of plane; meaning of polygon.

2. Circumference

   Have children mark a point on a paper. Label it C. Mark several points, all 1 inch from C.

   Ask children:

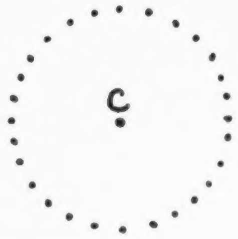
   How many points can you mark 1 inch from C?

   What figure is being formed by all of these points?

   Is a circle a simple closed figure? Explain.

   How does a circle differ from a polygon?

   How would you describe point C in relation to the points of the circle?



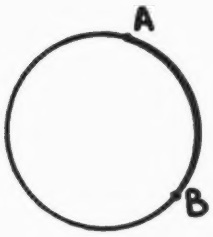
Tell children that:

A circle is a simple closed curve on a plane, formed by the set of all points which are the same distance from a fixed point in the plane called the center.

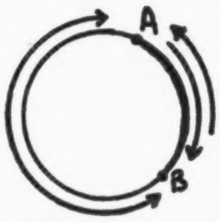
Circumference is the distance around the circle or the length of the circle.

3. **Arc**

Have children use a plastic compass to draw a circle.
Label a point on the circle A.
Name another point on the circle B.



Trace the two paths on the circle from A to B;
from B to A.



Tell children that the part of a circle between any two points is called an arc of the circle.
Discuss the length of the arc in relation to the circumference of the circle.

4. **Radius**

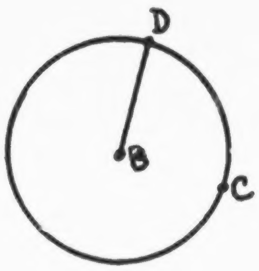
Children:

Draw a circle.

Label the center point B.

Label point D on the circle.

Draw line segment $\overline{BD}$.



Tell children $\overline{BD}$ is called a radius of the circle. (Here "radius" refers to the line segment. It can also be used to refer to the length of the segment.)

Have children:

Label another point C on the circle.
Draw $\overline{BC}$.

Discuss:

Is $\overline{BC}$ a radius of the circle? Why?

Can you draw another radius?

How many radii does a circle have?

Are all the radii the same length?

How can you find out?

Children describe a radius as:

A line segment from the center to any point of the circle. The length of any radius is also referred to as the "radius" of the circle.

5. Chord

Have children:

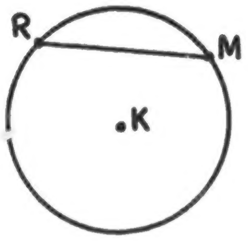
Draw a circle with a plastic compass.

Label its center K.

Mark two points on the circle.

Label them R and M.

Draw the line segment $\overline{RM}$



Have children:

Choose any other two points on the circle.

Label them.

Draw a line segment with those points as endpoints.

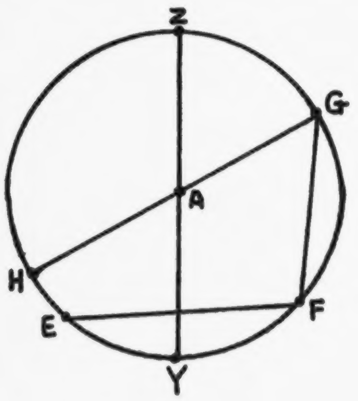
Tell children that any line segment that has its two endpoints on the circle is called a chord.

Is a radius a chord?

Is there a largest chord?

6. Diameter

Children examine the diagram of the circle whose center is A.



Name some of the chords.

What is the difference between chords $\overline{GF}$ and $\overline{GH}$?;

between $\overline{EF}$ and $\overline{ZY}$?

[$\overline{GH}$ and $\overline{ZY}$ have endpoints on the circle and pass through the point at the center.]

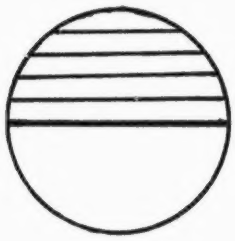
Tell children that:

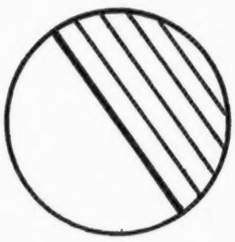
A chord that passes through the center is called the diameter.

The length of any diameter is also referred to as the diameter of the circle.

Have children:

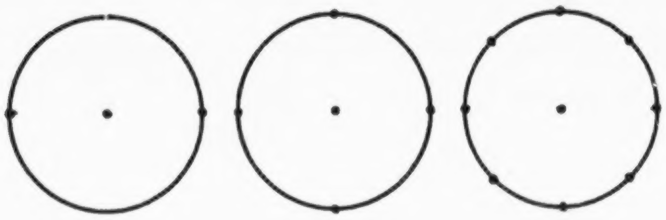
Draw a circle. Draw several diameters.
Discover that the longest chord passes through the center.
Diameters are the longest chords of a circle.



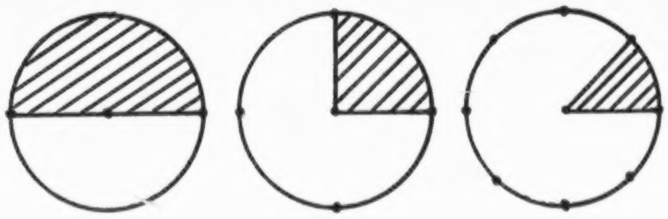


7. Central Angle

a. Provide a sheet of rexographed circles.
Have children divide successive circles in halves, fourths, eighths, thirds, sixths, etc.



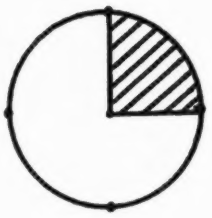
Draw and shade one central angle in each as shown:



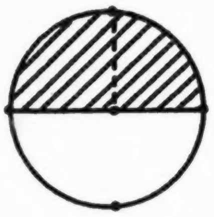
Discuss the meaning of central angle as the angle formed by 2 radii. (Radii is the plural of radius). Since two radii are involved, the vertex is at the center of the circle.

b. Describe and compare the central angles:

The arc that has the measure of:

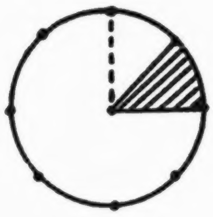


$\frac{1}{4}$ of the circle subtends a right angle.

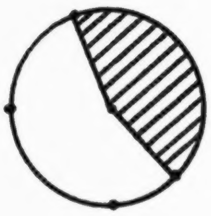


$\frac{1}{2}$ of the circle subtends a straight angle. (twice the size of a right angle)

$\frac{1}{2}$ of a circle is called a semicircle



$\frac{1}{8}$ of the circle subtends an acute angle. (smaller than a right angle)



$\frac{1}{3}$ of the circle subtends an obtuse angle. (greater than a right angle and less than a straight angle)

Discuss central angles subtended by $\frac{1}{16}$ of a circle; $\frac{1}{5}$ of a circle; $\frac{3}{8}$ of a circle; $\frac{5}{8}$ of a circle, etc.

Discuss central angles formed by the two hands of a clock:

2 P.M. (acute)
3 P.M. (right)
4 P.M. (obtuse) etc.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

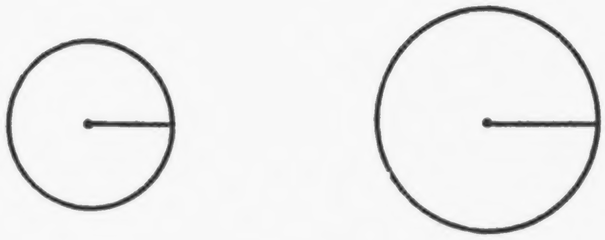
1. Draw a diagram to show whether the sentences below are true or false.

   a. A circle has all its points the same distance from a point inside called the center.

   b. All the radii of a circle have the same length.

2. Draw two different circles so that the radius of one has the same length as the radius of the other.

   Draw two different circles so that one has a radius of a different length from the other.



   Measure the length of each radius.

   Extend each radius through the center to form a diameter.

   Measure the length of the diameters.

   Determine the relationship between a diameter and a radius of each circle.

3. Draw a circle. Draw chords to form a square within the circle; a rectangle within the circle. Draw the diagonals of the square. What kind of central angles do they form?

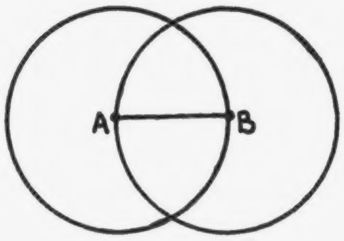
   Draw the diagonals of the rectangle. What kind of central angles do they form?

   What is true of the chords that form the square?

   [They are all the same length]

4. If the radius of a circle is 6 inches long, what is the length of the diameter?

5. Draw a line segment $\overline{AB}$. Then draw the two circles which have $\overline{AB}$ as a radius.



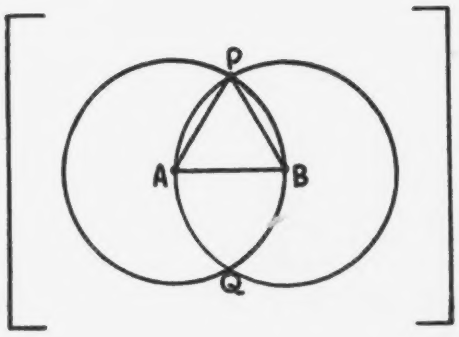
Ask children:

What is the center of one circle? [A]

Of the other circle? [B]

At how many points do these circles intersect? [2]

Label these points P and Q. Draw $\overline{AP}$ and $\overline{BP}$.



Use your compass or a ruler to determine:

Is $\overline{AP}$ equal to $\overline{AB}$?

Is $\overline{BP}$ equal to $\overline{AB}$?

Then, is $\overline{AP}$ equal to $\overline{BP}$?

What kind of triangle is Δ APB? [Equilateral]

GEOMETRY AND MEASUREMENT

UNIT 66 - MEASUREMENT: GRAPHIC REPRESENTATION; BAR GRAPH

NOTE TO TEACHER

A Graph pictures a type of numerical relationship between 2 kinds of information.

There are several kinds of graphs: Pictographs, Bar Graphs, Line Graphs, Graphs of Solution Sets of Open Sentences, etc.

A pictograph pictures information by means of pictures; bar graphs show comparisons by means of bars; line graphs show trends or changes in data by means of lines.

Children should be taught to interpret graphs before they are taught to construct graphs.

Objectives: To teach children the meaning of Scale and Axis.

To help children:

Read and interpret bar graphs.

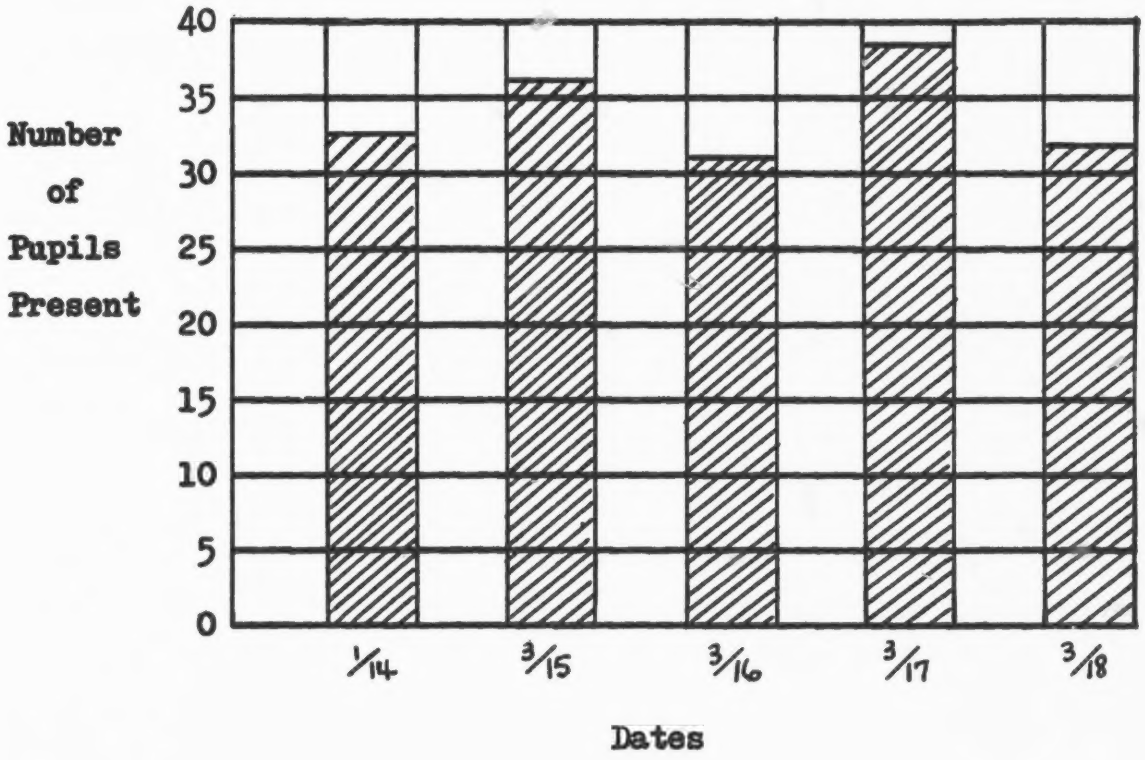
Construct bar graphs.

TEACHING SUGGESTIONS

Reading and Interpreting Bar Graphs

1. Present a graph such as the following:

WEEKLY ATTENDANCE RECORD



Discuss:

What does this graph show?

What is the title of the graph?

What do the dates at the bottom indicate?

What do the numbers at the left show?

On what date was the attendance highest?

About how many children were present on March 17?

On what date were the most children absent?

How many more children were absent on March 16 than on March 15?

On what days was the attendance perfect?

On what days was the attendance between 35 and 40?

How could we obtain the data for this graph?

Tell children that each bar graph has a scale.

Discuss:

Meaning of scale. [Scale is a measure shown by markings at regular intervals.]

The scale on the graph above. What are the intervals?

Where do they begin? [0]

Scale may be shown on horizontal or vertical line segment.

Tell children that the vertical line segment and the horizontal line segment are each called an axis. (Plural is axes).

2. Ask children to bring bar graphs cut from magazines or newspapers.

Have children study their bar graphs and note:

Bar Graphs usually have a scale which starts at zero.

Graphs should have a title, and a label for each axis.

Bars should be the same width.

Depending upon the interval selected, the numbers of the data may have to be shown approximately to the nearest 10, 100, 1000, etc.

The scale is marked on one of the axes.

Each bar should be labeled.

3. Have children compare a bar graph with a chart to determine the advantages of each way of showing records or other numerical information.

WEEKLY ATTENDANCE RECORD

Dates	No. of Pupils Present	
Mar. 14	34	
15	36	
16	32	
17	38	
18	36	(data same as bar graph)

Children should understand that:

BAR GRAPHS are used to facilitate making general comparisons. They are usually not meant to convey information with any high degree of precision. This can be shown much more accurately by charts that list data.

Constructing Bar Graphs

Bar Graphs can be used to show comparisons of:

Temperature readings	Incomes
Attendance records	Populations, etc.

The necessary data may be found in charts or tables, or may be gathered by the children.

1. Suggested Problem: On Tuesday, 432 people came to our dance festival, 549 came on Wednesday and 324 on Thursday. Show this on a chart, on a horizontal bar graph, on a vertical bar graph.

Table

Attendance at Dance Festival

Day	Number of People
Tues.	432
Wed.	549
Thurs.	324

Questions to ask children as they construct graph:

What shall we label each axis?

What scale shall we use?

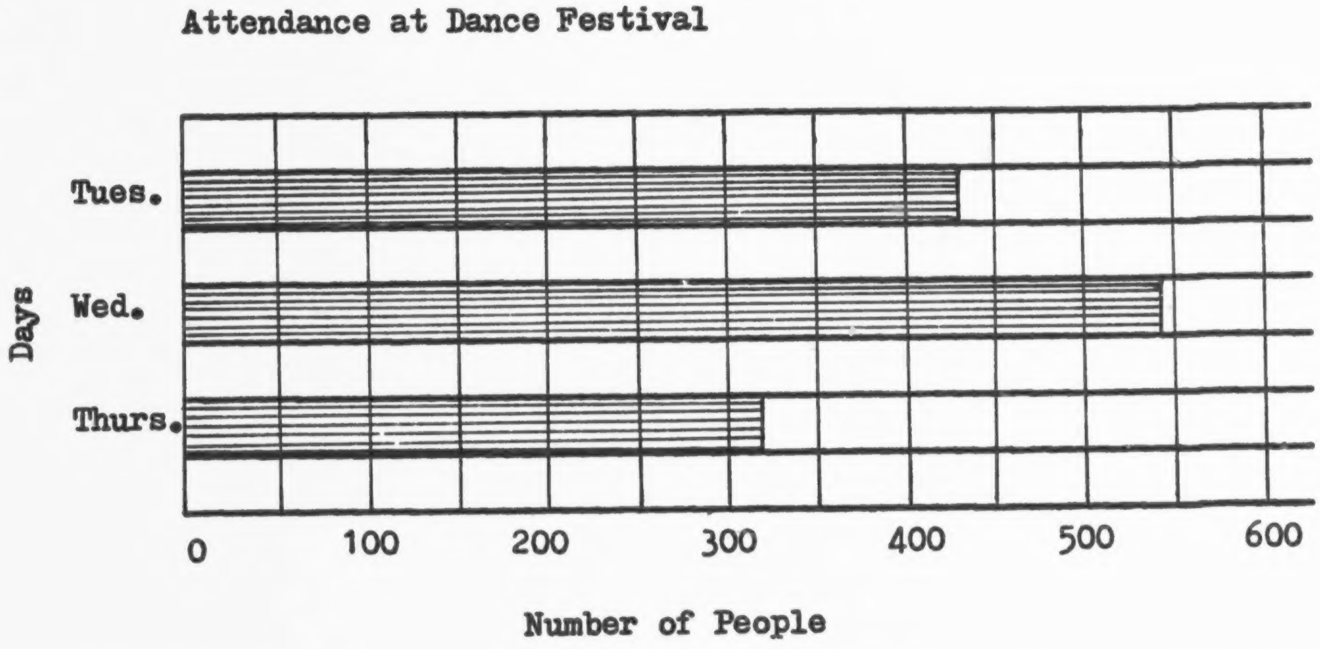
How many spaces will we need?

How shall we mark the intersection of the vertical and horizontal axes?

How can we plan so that the bars are the same width and the spaces are the same distance apart?

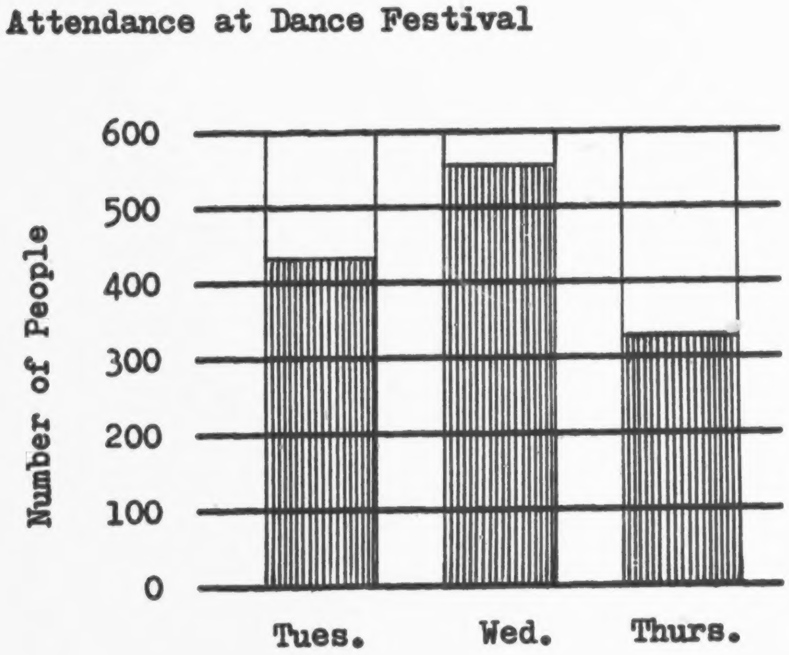
How shall we decide upon the size of the graph?

Horizontal Bar Graph



Note the placement of the numerals at the intersection of vertical and horizontal lines.

Vertical Bar Graph



2. Suggested Practice:

Have children construct bar graphs based on experiences and data found in mathematics textbooks, newspapers, and social studies textbooks.

Compare bar graphs with the tables and discuss the value of both ways of showing sets of data.

Which is of greater value pictorially? numerically?

Questions may be based on information shown in the graph.

On what day was the attendance highest? lowest? etc.

OPERATIONS

UNIT 67 - STATISTICS: FINDING AVERAGES

NOTE TO TEACHER

An average is one number that gives information about a set of data. An average is a balance point.

Although children often use the word "average," the concept is generally not understood. Finding an average means a reorganization of unequal groups to arrive at an equal although imaginary, distribution. An average can be obtained by a redistribution process. Finding averages is a use of partitioning division.

Objectives: To help children understand the meaning of average and its use in the interpretations of data.

To develop the procedure for finding the average of a set of numbers.

TEACHING SUGGESTIONS

Finding an Average When the Average is One of the Numbers in the Data

Using Objects

1. Suggested problem: Three committees brought in reference books for research in Social Studies. Committee A brought 11 books, Committee B, 5 books and Committee C, 8 books. If each committee is to receive the same number of books, how many would be in each group?

2. Have children arrange books brought by each committee and record the number.

A	B	C
11 books	5 books	8 books

3. Children should estimate the number each committee is to receive before attempting to find the answer.

   Discuss:

   Will each committee receive as many as 11 books?

   As few as 5? Why not?

   Somewhere between 5 and 11? Where?

4. Have children "even up" the stacks of books.
   Compare the number in the re-distributed groups with the original number of books in each group.

   Children should note that there are 8 books now in each stack.

   They compare 11 with 8. It is 3 more.

   They compare 5 with 8. It is 3 less.

   They compare 8 with 8. It is the same.

   They should see that there were 3 more books in one stack and 3 less in the other stack.

   8 is the "in-between" number.

   8 is the balance point or a "midpoint."

5. Tell children that the number of books in the redistributed books is the mid-value or an average.

6. Tell children that there is a faster way to find an average.

   a. Have children arrange the books as they were when each committee brought them in $\begin{array}{ccc} A & B & C \\ 11 & 5 & 8 \end{array}$.

      Then have children pile the books into a single stack.

      As they stack they add the number of books. [11 + 5 + 8]

      Help children, through questioning, to discover that stacking the books into one group and then dividing them into 3 equal parts will give the average.

On the chalkboard record:

$$\begin{array}{r} 11 \\ 5 \\ \underline{8} \\ 24 \end{array} \qquad 3\overline{)24 \text{ books}}^{\;8 \text{ books}}$$

8 books is the average number of books each committee would have brought in if each had brought the same number of books.

b. Suppose 4 committees brought in reference books. Committee A brought in 11 books, Committee B, 5 books, Committee C, 8 books and Committee D 12 books. If each committee is to receive the same number of books, how many would each receive?

Have children follow the same procedure as for items 2 - 6 above.

Children discuss after the recording:

$$\begin{array}{r} 11 \\ 5 \\ 8 \\ \underline{12} \\ 36 \end{array} \qquad 4\overline{)36 \text{ books}}^{\;9 \text{ books}}$$

that 9 books is the average number of books each committee would have brought in if each had brought the same number of books.

Help children, through questioning, to see that stacking the books into one group and then dividing them into 4 equal parts will give the average.

Children state the generalization for the method of finding an average as it applies to the situations above. For example, add and then divide by the number of piles.

Using Numbers Only

1. Suggested problem: Sally's arithmetic test marks were 70, 80, 75, 70, 80. What was her average arithmetic test mark?

Suggest that children arrange the numbers in order and use the middle value as a first estimate.
For: 70, 70, 75, 80, 80
Estimate: 75

2. Children compare each number with the estimate, 75. They note:

   70 is 5 below the middle value

   70 is 5 below the middle value

   75

   80 - - - - - - - - - - - - - - - is 5 above the middle value

   80 - - - - - - - - - - - - - - - is 5 above the middle value

   10 below — 10 above

   Solution: 75, since the amount below (10) balances the amount above (10). Find the solution by adding the marks and dividing the sum into 5 equal parts.

3. Children should state the generalization for the method of finding an average.

4. Find the average of each of the following sets of numbers and check your solution.

   17, 7, 12   39, 15, 27   8, 12, 10, 4, 6

   56, 62, 68, 59, 65

Average Is Not One of the Numbers in the Data

1. Suggested problem: Find the average temperature for the school day if our recordings show that it was 68° in the morning, 74 ° at noon and 77° in the afternoon.

2. Children try the midvalue at 74° and observe that it is not the average.

   Children note:

   68° is 6° below 74°

   74°

   77° - - - - - - - - is 3° above 74°

   6° below — 3° above

Children should note that:

6° and 3° do not balance.
Their estimate of 74° was too large because there are more degrees below it than above it.

3. Choose a lower average in order to have few degrees below and more degrees above. They might choose 72°.

   Record the temperatures again and compare each with 72°.

   68° is 4° below 72°
   74° - - - - - - - - - is 2° above 72°
   77° - - - - - - - - - is 5° above 72°
   [illegible] below   7° above

   Children should realize that the balance has changed. There are too many above and not enough below.

4. Children should discuss the estimates chosen, 72° and 74°. Note that the estimate was too large in one instance and too small in the other. Try 73°.

   68° is 5° below 73°
   74° - - - - - - - - - is 1° above 73°
   77° - - - - - - - - - is 4° above 73°
   5° below   5° above

   Children note that 5 below balances 5 above.

   Solution: 73° is the average temperature for the day.

5. Children should then find the average by adding the three numbers and dividing by 3.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. Find the average of each of the following sets of numbers and check the solutions.

   4, 5, 9, 10   60, 60, 70, 80, 90
   6, 9, 12, 17   45, 60, 75, 85, 95

2. On our vacation trip we drove 432 miles the first day, 386 miles the second day and 364 miles the third day. How many miles did we average each day?

3. For a Scholarship Collection, 86 people contributed various amounts totaling $1978. What was the average amount contributed?

4. Discuss the meaning of averages in other situations: Test results, game scores, seasonal temperatures, rainfall statistics, etc.

5. Present other problems requiring children to find averages.

6. Look up the meanings of the word "average" in different dictionaries.

OPERATIONS

UNIT 68 - SET OF WHOLE NUMBERS: DIVISION; VERTICAL FORMAT

Objectives: To maintain skill in using the division algorithm.

To give practice in interpreting and then solving problem situations involving division.

TEACHING SUGGESTIONS

Maintain skill in dividing with divisors through 9.
Use oral drills, games, patterns, etc.

Maintain skill in dividing with divisors through 99; Digit in units column less than 5. For example, $24\overline{)268}$; $53\overline{)718}$; etc.

Divisors Through 99; Digit in Units Column Greater Than 5

Children who have developed skill in dividing with divisors through 99 with units digit less than or equal to 5, should use more difficult divisors through 99, where the digit in units column is greater than 5.

1. Reinforce multiplying by 10 and multiples of 10.

   Suggested problem: At the end of the year, Class 5-1 helped to store textbooks in the library. Each shelf holds 27 books. How many shelves had to be reserved for 891 books.

   Have children interpret the problem. (How many twenty-sevens in 891?)

   Children record the problem in equation form: $n \times 27 = 891$.

To estimate quotient children may first write:	Estimate:
$10 \times 27 = 270$	
$20 \times 27 = 540$	$891 > 20 \times 27$; $n > 20$
$30 \times 27 = 810$	
$40 \times 27 = 1080$ (too large)	$891 < 40 \times 27$; $n < 40$; n is between 20 and 40.

Have children use division algorithm with first partial quotient of 30 to complete the computation.

$$\begin{array}{r|l} 27\overline{)891} & \\ \underline{810} & 30 \\ 81 & \end{array}$$

Children should record first partial quotient and product from above.

They continue to find and record partial quotients and products.

$10 \times 27 = 270$ (too large)

$5 \times 27 = 135$ (too large)

$4 \times 27 = 108$ (too large)

$3 \times 27 = 81$

$$\begin{array}{r|l} 27\overline{)891} & \\ \underline{810} & 30 \\ 81 & \\ 81 & \underline{\ \ 3} \\ \hline & 33 \end{array}$$

Solution: $n = 33$. The number of shelves needed is 33.

2. Children should check solutions.

Have children refer to the original problem and interpret their answer.

> If we discovered that 33 shelves are needed and each shelf holds 27 books, how many books are there altogether?
> $33 \times 27 = n$

Discuss why multiplying by 27 checks dividing by 27. (Inverse Operation).

Does the number 33 answer the original question?

Suggested Division Exercises With Remainders

Exercise: Divide 1852 by 28

```
     66              66
28)1852            x 28
   1680  60         528
    172            1320
    168   6        1848
      4  66        + 4
                   1852
```

or $(28 \times 66) + 4 = (20 \times 66) + (8 \times 66) + 4$

$= 1320 + 528 + 4$

$= 1852$

Discuss verifying solutions when remainders are involved.
Why do we add 4? (Refer to Unit 39)

Answer: Quotient 66, Remainder 4

1. Solve the following divisions. Verify the solutions by multiplication.

   $48\overline{)1687}$ ☐ x 56 = 2408 $79\overline{)1829}$ ☐ x 87 = 5978

2. Extend to divisions with more difficult divisors; with larger dividends.

   $39\overline{)8085}$ $17\overline{)3013}$ $59\overline{)31147}$ $48\overline{)14712}$ etc.

   Children check their solutions.

3. Interpret some of the above division exercises in problem situations.

Dividing Larger Numbers

Ability to estimate quotients helps children to divide larger numbers.

1. Suggested Problem: $43\overline{)92,851}$

   Necessary Background: Ability to multiply by 1000 and by multiples of 1000.

   1000 x 43 = ☐
   2000 x 43 = ☐
   3000 x 43 = ☐
   4000 x 43 = ☐

   1000 x 43 = ☐
   2000 x 43 = ☐
   4000 x 43 = ☐
   8000 x 43 = ☐

   2000 x 43 = ☐
   4000 x 43 = ☐
   6000 x 43 = ☐
   7000 x 43 = ☐

Estimate: $1000 \times 43 = 43{,}000;\ 43{,}000 < 92{,}851$
$2000 \times 43 = 86{,}000;\ 86{,}000 < 92{,}851$
$3000 \times 43 = 129{,}000;\ 129{,}000 > 92{,}851$

Therefore the quotient is between 2000 and 3000 and 2000 is our first partial quotient.

2. Children then compute as above and check the solution.

They should discover that the underlying meaning and method of dividing numbers does not change with the size of the numbers.

EVALUATION AND/OR PRACTICE

SUGGESTED EXERCISES

1. Study the problem and computation below. Then answer the questions.

Mary divided 856 by 24, and checked her work in two ways. Her thinking looked like this:

$$\begin{array}{rr|l} & \underline{35} & \\ 24) & 856 & \\ & \underline{720} & 30 \\ & 136 & \\ & \underline{120} & \underline{5} \\ & 16 & 35 \end{array} \qquad \begin{array}{r} 720 \\ 120 \\ \underline{16} \\ 856 \end{array} \qquad \begin{array}{r} 24 \\ \underline{\times 35} \\ 120 \\ \underline{720} \\ 840 \\ \underline{16} \\ 856 \end{array}$$

Solution: Quotient 35
Remainder 16 or 35 R 16

Explain each step in the solution and in each check.

In the division, which number is the dividend? The quotient? The divisor?

Explain the meaning of the 30; the 720; the 5; the 16; the 35.

In the multiplication, which number is the same as the dividend, the quotient, the divisor?

What is each called in the multiplication?

Explain how the multiplication can verify the division.

Explain how addition will have helped to verify the division.

Which of the two checks Mary used, do you think is better? Why?

2. The Parents Association wants to order 1494 bottles of soft drinks for their meetings for the entire year. If they are packaged in cartons that hold 6 bottles each, how many cartons should the parents order? ($n \times 6 = 1494$)

3. The thrush migrates from Louisiana to Alaska, covering the distance of 4000 miles in about 30 days. About how many miles does it fly in one day? ($30 \times n = 4000$)

4. The district library has 9000 books. New book shelves are to be ordered, each of which will hold 36 books. How many shelves will be needed?

5. If one pair of socks cost $.59, how many pairs can mother buy for $4.00? What does the remainder tell?

6. John said, "I weigh 98 lb. The British unit of weight for 14 lb. is called a stone. How many stones do I weigh?"

7. Jane filled 16 birthday baskets with 12 pieces of candy each. Four of her guests did not come to the party. Jane put their share equally into each of the other baskets. How many pieces of candy did each guest receive in her basket? (2 step problem)

8. The 36 children of our class will send a basket to John who is ill. They will include books which cost $3.30, games for $1.74, a box of stationery for $1.98 and $.90 worth of fruit. How much should each child pay as his share?
($\$3.30 + \$1.74 + \$1.98 + \$.90) \div 36 = n$

9. Joe's graduation is 9 weeks away. He is saving money to pay for his expenses. He needs $12.50 for a class ring, $3.50 for the class party, $1.00 for the class gift and $2.50 for extra expenses. How much should he save each week to meet these costs?

10. Sally wants to do all the 336 problems in her arithmetic book during her vacation. She has finished one half of them. How many must she do each day to finish the problems in the 12 days left?

11. There were 214 children in the 8 classes of the fifth grade at the beginning of the school year. A new housing development opened in December and 58 more children were registered in the grade. What was the new average size of the classes on the grade?

12. Our class was moved to another room. The children helped to carry the books. If 426 books were moved, what was the average number of books each child carried? (missing information)

13. All the schools of the district were having a dance festival. The cost of using the auditorium for the 3 evenings was $385. The cost of the costumes for the 28 schools was $1267. The charge for 28 buses to transport the children was $700. How much money did each school have to contribute to cover the total expenses?

* 14. Astronaut John Glenn took 88 minutes to complete one orbit around the earth.

If he was aloft for 4 hours and 55 minutes, how many orbits did he make?

If the distance traveled in each orbit was about 27,016 miles, how many miles did he cover in one minute?

SETS; NUMBER; NUMERATION

UNIT 69 - SET OF FRACTIONAL NUMBERS: HUNDREDTHS; COMMON AND DECIMAL FORMS

Objectives: To introduce concept of hundredths, fractional and decimal forms.

To develop relationships: units, tenths, fifths, etc. to hundredths; cents to one dollar.

To develop concept of whole number plus fraction (mixed form) in decimal form.

TEACHING SUGGESTIONS

Extend understanding of relationships among fractions to include fractions through ninetieths.

1. Children should draw line and other diagrams to discuss.

   $\frac{1}{20}$ as $\frac{1}{2}$ of $\frac{1}{10}$; $\frac{1}{30}$ as $\frac{1}{3}$ of $\frac{1}{10}$; to $\frac{1}{90}$ as $\frac{1}{9}$ of $\frac{1}{10}$

2. Suggested exercises:

   a. $\frac{1}{20}$ = What part of $\frac{1}{10}$ $\quad \frac{1}{60} = \frac{1}{\square}$ of $\frac{1}{20}$ $\quad \frac{1}{20} = \square \times \frac{1}{40}$

   $\frac{1}{40}$ = What part of $\frac{1}{20}$ $\quad \frac{1}{60} = \frac{1}{\square}$ of $\frac{1}{30}$ $\quad \frac{1}{20} = \square \times \frac{1}{60}$

   etc. etc. etc.

   b. $\frac{1}{3}$ of $\frac{1}{10} = \square$ $\quad \frac{1}{3}$ of $\frac{1}{20} = \square$ $\quad \frac{1}{3}$ of $\frac{1}{30} = \square$ etc.

   $\frac{1}{5}$ of $\frac{1}{10} = \square$ $\quad \frac{1}{6}$ of $\frac{1}{10} = \square$ $\quad \frac{1}{7}$ of $\frac{1}{10} = \square$ etc.

c. What part of $\frac{1}{30}$ is $\frac{1}{60}$?

Can you think of a way to derive $\frac{1}{80}$? $\left[\frac{1}{8} \text{ of } \frac{1}{10}\right]$

d. Insert the correct numeral to make true statements.

$\frac{2}{20} = \frac{\square}{10}$, $\frac{4}{20} = \frac{\square}{10}$, $\frac{6}{20} = \frac{\square}{10}$, etc.

$\frac{10}{20} = \frac{\square}{10}$, $\frac{12}{20} = \frac{\square}{10}$, etc.

$\frac{40}{40} = \square$ $\frac{20}{40} = \square$ $\frac{10}{40} = \square$ $\frac{60}{60} = \square$ $\frac{20}{60} = \square$ etc.

e. Draw a number line. Count on the number line by steps of one-twentieth from 0 to 1.

$\left[\frac{1}{20}, \frac{1}{10}, \frac{3}{20}, \frac{2}{100}, \text{ etc.}\right]$

Then do the above changing to tenths as you proceed.

$\left[\frac{1}{20}, \frac{1}{10}, \frac{3}{10}, \frac{2}{10}, \text{ etc.}\right]$

Concept of Hundredths

Extend the understanding of fractions through hundredths.
Relate hundredths to tenths.

Using Squared Material.

Consider a"hundred square"as one unit.

Children should use the hundred square to discover relationships between:

> The unit (the whole square) and tenths; the unit and hundredths; tenths and hundredths; etc.

Children should recognize that:

> The unit is made up of ten rows with 10 small squares on each row.
>
> The unit is made up of 100 small squares.

Each row is $\frac{1}{10}$ of the unit.

Each small square is $\frac{1}{100}$ of the unit. Why?

Each small square is $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{100}$ of the unit.

Outline other fractional parts of the unit; e.g. 20 squares (using 2 rows).

Children should record names for this fractional part in terms of tenths; in terms of hundredths; (2 tenths, $\frac{2}{10}$, .2, 20 hundredths, $\frac{20}{100}$)

Proceed similarly with 3 rows, 7 rows, 9 rows, 10 rows, (the entire unit)

Discuss the entire unit as 10 tenths; 100 hundredths: $\frac{10}{10} = 1$; $\frac{100}{100} = 1$

Discuss 28 squares, 37 squares, 3 squares, etc. as 28 of the one hundred, 37 of the one hundred, or as $\frac{28}{100}$, $\frac{37}{100}$, etc.

Hundredths in Decimal Form

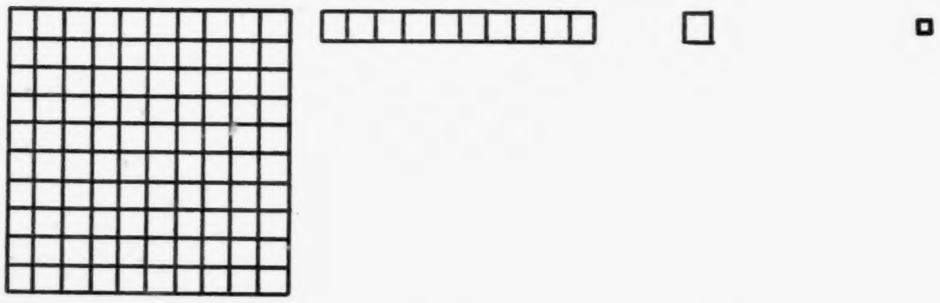
1. Place Value

   Consider a small square as 1 unit.

   a. Place a hundred square, a ten-square strip, a unit-square and a one-tenth strip (the result of dividing a unit-square into 10 equal parts) on the display board.

      Draw a Place Value Chart directly below the squared material.

Record the numerals for the number of unit squares represented by the material on the Place Value Chart.



Hundreds	Tens	Units	Tenths
1	1	1	1

Cut one of the tenths into approximately 10 equal parts.

Ask children:

What should we call this part?
Where would you place this hundredth on the display board?
[right of tenths place]
Why? [Each place has a value one tenth of the place immediately to its left.]
Where would you record the numeral for one hundredth on the chart?

H	T	U	Tenths	?
1	1	1	1	1

What would you call that place? [Hundredths] Why?

b. Children label the column "Hundredths".

They discuss the value of the 1 in the tenths place and the value of the 1 in the hundredths place.

H	T	U	Tenth	Hundredths
1	1	1	1	1

They note:

a "1" in the tenths place has 10 times the value of a "1" in the hundredths place.

a "1" in the hundredths place has $\frac{1}{10}$ the value of a "1" in the tenths place.

Children should discuss, then record 2 hundredths, 3 hundredths,... 9 hundredths on the place value chart.
Where should 10 hundredths be recorded?

Children show 3 hundredths on the place value chart.
They discuss how to record 3 hundredths without using a place value chart.

Record .03 on the chalkboard. Compare with .3.
What does the zero in .03 indicate?

Continue to 10 hundredths or .10 and compare with .1.

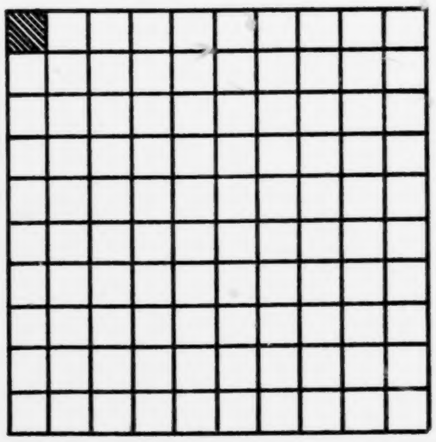
Continue to develop understanding and recording of more than 1 tenth or 10 hundredths.

Relationship: Units, Tenths, Fifths, etc. to Hundredths

Have children outline several "hundred-square" blocks on graph paper.
Have each hundred-square block represent one unit.

Children make comparisons by shading several fractional parts of the hundred-square blocks.

Relationship of One Unit to Hundredths



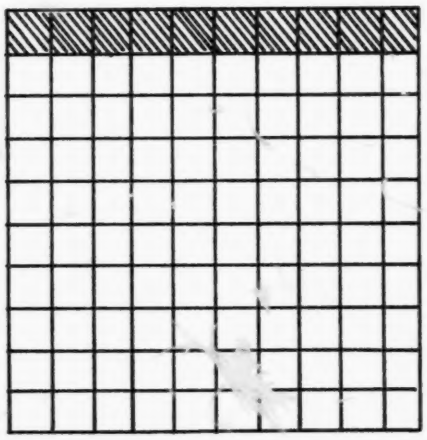
Have children shade one hundredth of the unit.

Compare the value of 1 unit with the shaded part.
Which is larger? How many times larger?
Each part is $\frac{1}{100}$ of the unit.
Each part is .01 or 0.01

Discuss the significance of the two zeros.

How many units are there?
How many tenths are there?

Relationship of Tenths to Hundredths



Have children shade one tenth of the unit.

Compare the value of 1 unit with the shaded part.
Which is larger? How many times larger?

$\frac{1}{10} = \frac{10}{100}$

0.1 = 0.10

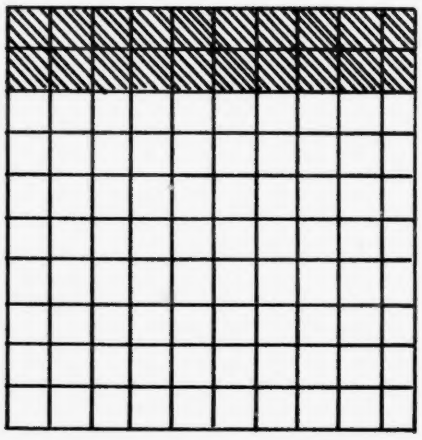
Have children shade additional hundred-square blocks to show:

$\frac{2}{10} = \frac{20}{100} = 0.2 = 0.20$

through

$\frac{10}{10} = \frac{100}{100} = 1 = 1.0 = 1.00$

Relationship of Fifths to Tenths; of Fifths to Hundredths



Have children shade on fifth of the unit.

Each part = $\frac{2}{10}$ of the unit.

Each part contains 2 tenths; $\frac{2}{10} = \frac{1}{5}$

Each part contains 20 hundredths

$\frac{1}{5} = \frac{2}{10} = \frac{20}{100}$

$\frac{1}{5} = 0.2 = 0.20$

Have children outline additional hundred-square blocks to show:

$\frac{2}{5} = \frac{4}{10} = \frac{40}{100}$ through

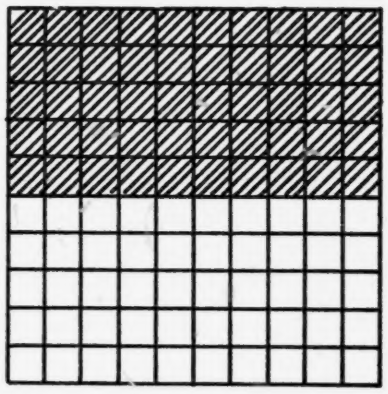
$\frac{5}{5} = \frac{10}{10} = \frac{100}{100} = 1$

$\frac{2}{5} = 0.4 = 0.40$ through

$\frac{5}{5} = 1 = 1.0 = 1.00$

Ask children: Do .6 and .60 name the same number? [Yes] Justify your answer. $\left[\frac{6}{10} = \frac{60}{100}\right]$

Relationship of Halves to Tenths; of Halves to Hundredths



Have children shade one half of the unit.

Discuss:

$$\frac{1}{2} = \frac{5}{10} = \frac{50}{100}$$

$$\frac{1}{2} = 0.5 = 0.50$$

$$\frac{2}{2} = \frac{10}{10} = \frac{100}{100} = 1$$

$$\frac{2}{2} = 1 = 1.0 = 1.00$$

Extend to other subdivisions; $\frac{1}{4}$, $\frac{1}{5}$, etc.

Relationship of Cents to One Dollar

1. Reinforce:

   1 cent as $\frac{1}{10}$ of 1 dime or 10¢

   1 dime (10¢) as $\frac{1}{10}$ of 1 dollar

   Use real money if necessary.

2. Suggested Exercises:

   1 cent = $\frac{1}{\square}$ of a dime.

   1 dime has a value of ___ times as large as 1 cent.

   1 dime = $\frac{1}{\square}$ of a dollar

   1 dollar = ___ dimes

   1 dollar has a value of ___ times as large as 1 dime.

3. Children know that one dollar = 100 cents and are now ready to understand and use such relationships as 1 cent is 0.01 of a dollar; a dime is 0.10 of a dollar; etc.

Children derive relationships such as:

1 cent $= \frac{1}{100}$ of a dollar = \$.01

1 nickel $= \frac{5}{100}$ of a dollar = \$.05

1 dime $= \frac{10}{100}$ of a dollar = \$.10

1 quarter $= \frac{25}{100}$ of a dollar = \$.25

1 half dollar $= \frac{50}{100}$ = \$.50

1 dollar $= \frac{100}{100}$ of a dollar = \$1.00

Have children write decimal symbols for the following amounts of money:

$\frac{3}{100}$ of a dollar; $\frac{75}{100}$ of a dollar; *$\frac{137}{100}$ of a dollar

Have children write the fraction symbols for:

\$.15 \$.27 \$5.00

Ask children:

Why do you think we use decimal symbols for money, as \$.20 rather than the fractional symbol $\frac{20}{100}$ of a dollar?

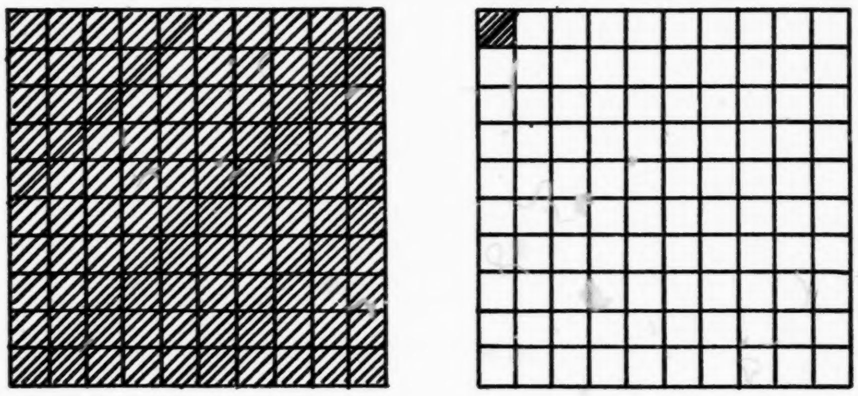
[It is easier to speak of, to write and to use in computations.]

*Some years ago, India changed to $\frac{1}{100}$ of a Rupee. (Optional)
England is planning to change, too.
Let children look this up in an Almanac, Encyclopedia, etc., and report on this information.
Have them explore Decimal System for money denominations in different countries.

Numbers In Mixed Form (Whole Number Plus Fraction): Common and Decimal Forms

Use 2 hundred-square blocks, one shaded to represent one whole, the other to represent hundredths of one whole.

Represent $1\frac{1}{100}$ as illustrated.



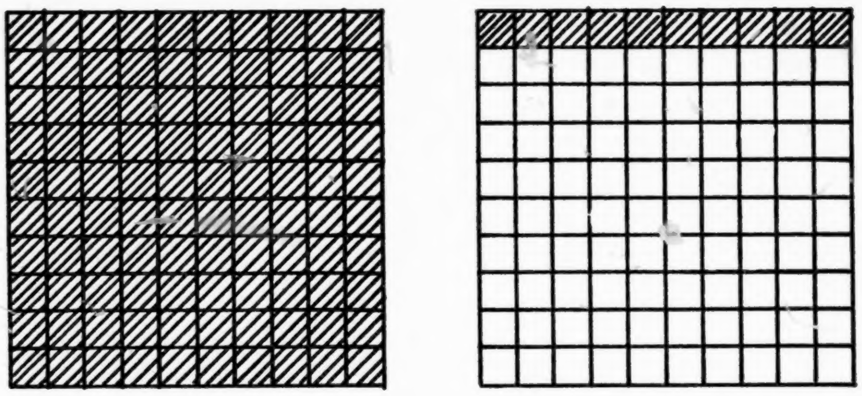
Children record the value represented as:

$1 + \frac{1}{100} = 1\frac{1}{100}$

$1. + 0.01 = 1.01$

Continue through $1\frac{9}{100}$

Represent $1\frac{10}{100}$ as illustrated.



Children record the value represented as:

$1 + \frac{10}{100} = 1\frac{10}{100} = 1.10$

$1 + \frac{1}{10} = 1\frac{1}{10} = 1.1$

Explain: $1.10 = 1.1$

Follow a similar procedure with numbers such as:

$$1 + \frac{15}{100} = 1 + \frac{10}{100} + \frac{5}{100} = 1 + .10 + .05 = 1.15$$

$$1 + \frac{15}{100} = 1 + \frac{1}{10} + \frac{5}{100} = 1 + .1 + .05 = 1.15$$

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Replace the frame or placeholder with a numeral to make true statements of each of the following:

.34 = 3 tenths + □ hundredths
.56 = □ tenths + 6 hundredths
*8 tenths + □ hundredths = $\frac{87}{100}$

.08 = △ tenths + 8 hundredths
4.97 = □ ones + △ tenths + ◇ hundredths
7 tenths + □ hundredths = $\frac{87}{100}$

2. Rename the following decimals in common fractional form.

4.05 8.17 28.27 60.06

6.75 9.60 13.30 27.25

3. Record the following in decimal form.

30 hundredths 14 and 3 tenths $\frac{9}{100}$ $23\frac{23}{100}$

7 and 36 hundredths 69 and 7 hundredths

4. Arrange the following in order of value from the smallest to the largest.

4.4 44 .44 4.44
.60 .24 .9 .53
.11 .90 .07 .1

5. Between each pair of decimal fractions insert > or < or = to form a true sentence.

.5 .45 5.5 .55
.7 .69 .62 .9
1.0 .99 .03 .3

OPERATIONS

UNIT 70 - SET OF FRACTIONAL NUMBERS: ADDING AND SUBTRACTING HUNDREDTHS; DECIMAL FORM

Objective: To help children develop the ability to add and subtract numbers involving hundredths, without and then with exchange; decimal form.

TEACHING SUGGESTIONS

Addition: No Exchange

Suggested Problem: The amount of rainfall in a month may be expressed in hundredths of an inch. Newton had 3.26 inches of rainfall in April and 1.71 inches in May. What was the total amount of rainfall for those two months? $3.26 + 1.71 = n$

Estimate: $3.26 > 3$; $1.71 > 1$ $n > 4$ More than 4 inches of rainfall.
$3.26 < 4$; $1.71 < 2$ $n < 6$ Less than 6 inches of rainfall.

Children should solve in a variety of ways, some of which are:

$$\begin{array}{r} 3\frac{26}{100} \\ 1\frac{71}{100} \\ \hline 4\frac{97}{100} \end{array} \qquad \begin{array}{r} 3.26 = 3 + .2 + .06 \\ 1.71 = 1 + .7 + .01 \\ \hline 4 + .9 + .07 = 4.97 \end{array} \qquad \begin{array}{r} 3.26 \\ 1.71 \\ \hline 4.97 \end{array}$$

Total amount of rainfall: 4.97 in.

Children should compare the sum with the estimated sum.

They should decide which algorithm is most efficient.

Addition: With Exchange

Suggested exercise: $2.56 + 3.78 = n$

Children rewrite as:

$$\begin{array}{r} 2.56 \\ \underline{3.78} \end{array}$$

Method 1

Step 1 Have children:

Record the hundredths and add

$$\begin{array}{r} .06 \\ \underline{.08} \\ .14 \end{array}$$

Record the tenths and add

$$\begin{array}{r} .5 \\ \underline{.7} \\ 1.2 \end{array}$$

Record the whole numbers and add

$$\begin{array}{r} 2 \\ \underline{3} \\ 5 \end{array}$$

Step 2 Find the sum of hundredths, tenths and whole numbers.

$$\begin{array}{r} .14 \\ 1.2 \\ \underline{5} \\ 6.34 \end{array}$$

Solution: $n = 6.34$

Method 2

Children should examine the hundredths column.

They note:

$.06 + .08 = .14$

$.14 = .10 + .04$ They should record the 4 hundredths in the hundredths column.

$$\begin{array}{r} 2.56 \\ \underline{3.78} \\ 4 \end{array}$$

Since ten hundredths = one tenth, .10 is renamed as .1 to be added to the tenths.

They note the tenths column and add the tenths.

$.1 + .5 + .7 = 13$ tenths

13 tenths is renamed as 1.3. Why? They record the 3 tenths in the tenths column.

$$\begin{array}{r} 2.56 \\ \underline{3.78} \\ 34 \end{array}$$

The "1" is added to the whole numbers and the sum recorded.

$$\begin{array}{r} 2.56 \\ \underline{3.78} \\ 6.34 \end{array}$$

Children should observe that fractional numbers in decimal form are added just as whole numbers are added. They may verify the sum by changing to the common fractional form and/or using subtraction. Discuss the role and place of the decimal point.

Suggested Practice Exercises

1. Find the sum of each and verify each sum by changing to the common fractional form.

(a) $\begin{array}{r} 39.25 \\ \underline{8.72} \end{array}$ (b) $\begin{array}{r} 115.37 \\ \underline{26.56} \end{array}$ (c) $\begin{array}{r} 135.92 \\ \underline{87.36} \end{array}$ (d) $\begin{array}{r} 94.86 \\ \underline{86.39} \end{array}$

(e) $1.26 + 5.86 + 7.42$

2. Add 24.51, 43.09, 216.97

3. Find the sum for each set of numbers

(a) 7.35, 12.68, 9.08

(b) $98.97, $25.75, $13.86

(c) 29.08, 208.45, 8.67

4. Add: using the common fractional form. Convert to decimal form and verify your solutions.

(a) $\begin{array}{r} 75\frac{1}{4} \\ \underline{26\frac{1}{2}} \end{array}$ (b) $\begin{array}{r} 93\frac{4}{5} \\ \underline{37\frac{1}{2}} \end{array}$ (c) $\begin{array}{r} 68\frac{3}{4} \\ \underline{72\frac{2}{5}} \end{array}$ (d) $\begin{array}{r} 56\frac{9}{10} \\ \underline{84\frac{7}{100}} \end{array}$

5. Carl added 32.14 and 26.2 as shown below and of course, his sum was incorrect.

$$\begin{array}{r} 32.14 \\ \underline{26.2} \\ 34.76 \end{array}$$

Can you explain his error? (Hint: Use common fractions and expanded notation)

$$\begin{array}{l} 30 + 2 + \frac{1}{10} + \frac{4}{100} = 32.14 \\ \underline{20 + 6 + \frac{2}{10} \qquad\quad = 26.2} \\ 50 + 8 + \frac{3}{10} + \frac{4}{100} = 58.34 \end{array}$$

What would you advise Carl to do when adding decimals?

Subtraction: No Exchange

Suggested Problem: Each foot in a surveyor's tape is divided into tenths and hundredths. Paul's father used a surveyor's tape to measure the length and width of the kitchen. The length was 10.63 feet and the width was 8.22 feet. How much greater than the width is the length of the kitchen? $10.63 - 8.22 = n$

Estimate: $10.63 > 10$; $8.22 > 8$; $.63 > .22$
The difference is greater than 2 feet.

Children should solve in a variety of ways, for example:

$$\begin{array}{r} 10\,\frac{63}{100} \\ -\ 8\,\frac{22}{100} \\ \hline 2\,\frac{41}{100} \end{array} \quad \text{or} \quad \begin{array}{rl} 10.63 = 10 + .6 + .03 & \\ -\ \underline{8.22 = 8 + .2 + .02} & \\ 2 + .4 + .01 & = 2.40 + .01 = 2.41 \end{array} \quad \text{or} \quad \begin{array}{r} 10.63 \\ -\ 8.22 \\ \hline 2.41 \end{array}$$

Children should compare the exact difference with the estimated difference.

They should decide which algorithm is most efficient.

Subtraction: With Exchange

Suggested exercise: $9.62 - 3.45 = n$

Children should rename the minuend.

$$\begin{array}{rcl} 9.62 = 9 + .6 + .02 & = & 9 + .5 + .12 \\ -\ \underline{3.45} = \underline{3 + .4 + .05} & = & \underline{3 + .4 + .05} \\ & & 6 + .1 + .07 = 6.17 \end{array}$$

Why did we change .6 to .5?

$$n = 6.17$$

Children should observe that fractions in decimal form are subtracted just as whole numbers are subtracted.

Suggested Practice Exercises

1. Verify remainders by using the common fractional form or by adding.

$$\text{(a)}\ \begin{array}{r} 9.69 \\ -\ \underline{3.27} \end{array} \qquad \text{(b)}\ \begin{array}{r} 8.92 \\ -\ \underline{3.47} \end{array} \qquad \text{(c)}\ \begin{array}{r} 18.04 \\ -\ \underline{3.92} \end{array} \qquad \text{(d)}\ \begin{array}{r} 29.54 \\ -\ \underline{15.86} \end{array}$$

(e) From 8.34 subtract 2.93 (f) 127.00 minus 39.73

(g) Subtract

$$\begin{array}{r} \$875.25 \\ \underline{350.98} \end{array}$$

(h) Find the difference between 63.43 and 82.27.

2. Find the missing numeral.
   (a) $73.28 - n = 32.57$
   (b) $25.43 + n = 131.12$
   (c) $n + 524.85 = 913.72$

3. Subtract using the common fractional form. Convert to decimal form and verify your solution.

   (a) $\begin{array}{r} 15\frac{3}{4} \\ -\ 9\frac{1}{2} \\ \hline \end{array}$ (b) $\begin{array}{r} 28\frac{3}{5} \\ -\ 16\frac{1}{4} \\ \hline \end{array}$ (c) $\begin{array}{r} 85\frac{3}{10} \\ -\ 27\frac{7}{100} \\ \hline \end{array}$

4. Find the value of n.
   (a) $(27.3 + 6.9) - (8.5 + 3.6) = n$
   (b) $(42.8 + 5.7) - (16.2 + 4.8) = n$
   (c) $(127.5 + 32.8) - (29.3 + 15.7) = n$

EVALUATION AND/OR PRACTICE
SUGGESTED EXERCISES
ADDITION AND SUBTRACTION: COMMON AND DECIMAL FORMS

1. When Joan visited Barbara at the hospital, she walked $\frac{3}{4}$ of a mile to reach the hospital and $\frac{7}{10}$ of a mile from the hospital to her home. How many miles did she walk? ($\frac{3}{4} + \frac{7}{10} = n$)

2. It took Barbara $\frac{1}{4}$ hour to walk to the hospital from school. She stayed for $1\frac{1}{3}$ hours and then took $\frac{1}{2}$ hour to reach home. (Incomplete)

3. Linda has $1\frac{3}{8}$ yards of material. The pattern for the skirt she wants to make calls for $2\frac{1}{3}$ yards. How much more material must she buy.

4. A pail filled with a gallon of water weighs $10\frac{2}{15}$ lb. If the pail weighs $1\frac{8}{10}$ lb., how much does the gallon of water weigh?
($10\frac{2}{15} - 1\frac{8}{10} = n$)

5. The table in the clubhouse is $11\frac{1}{3}$ ft. long. We have one cloth $4\frac{1}{2}$ ft. long, another $7\frac{3}{4}$ ft. long. After the table is covered how much is left for overlap and drop at the ends?

6. A tank contained 92.4 gallons of oil. During the day, 15.75 gallons were used. How much oil remained in the tank?

7. With $85.00 in cash, Mr. Bates set out to pay two bills. One bill amounted to $25.15. The other was $19.75. After Mr. Bates paid these bills, how much cash did he have left?

8. In 1961, the winning racing car traveled at an average speed of 139.44 miles per hour. In 1962, the winning car traveled at an average speed of 142.29 miles per hour. What was the difference in speed?

OPERATIONS

UNIT 71 - SET OF FRACTIONAL NUMBERS: MULTIPLICATION; COMMON FORM; PROPERTIES APPLIED

Objective: To help children use properties of multiplication (Distributive and Commutative) in multiplication of fractions.

TEACHING SUGGESTIONS

Distributive Property Applied: Suggested Development

Problem: John wants to make a book shelf. He needs $2\frac{1}{3}$ feet of wood for each shelf. How much lumber will he need for 4 shelves? $(4 \times 2\frac{1}{3} = n)$

Estimate: $n > 8$; $n > 9$ (Why?) $[4 \times \frac{1}{3} > \square]$

Compute: Children may solve in a variety of ways. They should observe that the Distributive Property is applied.

Horizontal Format

A. $4 \times 2\frac{1}{3} = (4 \times 2) + (4 \times \frac{1}{3}) = 8 + \frac{4}{3} = 8\frac{4}{3} = 9\frac{1}{3}$

B. $4 \times 2\frac{1}{3} = (4 \times \frac{1}{3}) + (4 \times 2) = \frac{4}{3} + 8 = 9\frac{1}{3}$

C.
$$\begin{array}{rl} 4 \times 2 = & 8 \\ 4 \times \frac{1}{3} = & \underline{\frac{4}{3}} \\ & 8\frac{4}{3} = 9\frac{1}{3} \end{array}$$

D.
$$\begin{array}{rl} 4 \times \frac{1}{3} = & \frac{4}{3} \\ 4 \times 2 = & \underline{8} \\ & 8\frac{4}{3} = 9\frac{1}{3} \end{array}$$

Vertical Format

E.

$$\begin{array}{rl} 2\frac{1}{3} & \\ \times\, 4 & \\ \hline 8 & (4 \times 2) \\ \frac{4}{3} & (4 \times \frac{1}{3}) \\ \hline 8\frac{4}{3} = 9\frac{1}{3} & \end{array}$$

F.

$$\begin{array}{rl} 2\frac{1}{3} & \\ \times\, 4 & \\ \hline \frac{4}{3} & (4 \times \frac{1}{3}) \\ 8 & (4 \times 2) \\ \hline 8\frac{4}{3} = 9\frac{1}{3} & \end{array}$$

Children may check by adding. $2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} + 2\frac{1}{3} = 8\frac{4}{3} = 9\frac{1}{3}$

Suggested Practice:

$3 \times 4\frac{1}{5} = 12 + \frac{\square}{5} = n$ $5 \times 3\frac{2}{3} = 15 + \frac{\square}{3} = n$

$6 \times 4\frac{1}{9} = \square + \Delta = n$ $4 \times 5\frac{2}{7} = \square + \Delta = n$

$5 \times 2\frac{5}{6} = 10 + \frac{\square}{6} = n$

$3 \times 6\frac{5}{12} = \square + \Delta = n$

Commutative Property of Multiplication Applied: Suggested Development

1. Reinforce the generalization for multiplication of fractions. (Multiplying the numerator of the fraction by the whole number gives the numerator of the product. The denominator remains the same.)

   a. Have children solve:

   $4 \times \frac{1}{5} = \square$ $\frac{1}{5} \times 4 = \square$

   Ask children:

   Since $4 \times \frac{1}{5} = \frac{4}{5}$, and $\frac{1}{5} \times 4 = \frac{4}{5}$, then $4 \times \frac{1}{5} = \frac{1}{5} \times \square$

b. Have children solve:

$\frac{3}{4} \times 8 = \square$ $\qquad$ $8 \times \frac{3}{4} = \square$

Ask children:

Since $\frac{3}{4} \times 8 = 6$, and $8 \times \frac{3}{4} = 6$, then $\frac{3}{4} \times 8 = 8 \times \square$

c. Ask children to explain the property involved.

2. Have children complete the following to make each statement true. Explain.

$8 \times \frac{1}{6} = \frac{1}{6} \times \square = n$ $\qquad$ $\frac{1}{8} \times 46 = 46 \times \square = n$

$\square \times \frac{2}{3} = \frac{2}{3} \times 5 = n$ $\qquad$ $\square \times 60 = 60 \times \frac{7}{10} = n$

$12 \times \frac{3}{10} = \square \times 12 = n$ $\qquad$ $\frac{2}{9} \times 84 = \square \times \frac{2}{9} = n$

$35 \times \square = \frac{7}{12} \times 35 = n$ $\qquad$ $\frac{3}{20} \times \square = 95 \times \frac{3}{20} = n$

Since $2 \times 4\frac{2}{3} = 9\frac{1}{3}$, $4\frac{2}{3} \times 2 = \square$ Why?

Since $6 \times 2\frac{3}{10} = 13\frac{4}{5}$, $2\frac{3}{10} \times 6 = \square$ Why?

Multiplication: Changing Mixed Form to Fractional Form

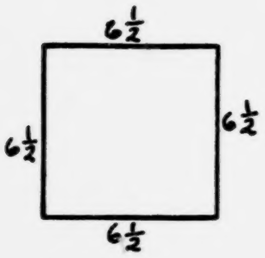
1. Suggested Development

Problem: Each side of John's square garden plot is $6\frac{1}{2}$ ft. long. How much fencing must he buy to build a fence around it.

Reinforce the formula for the perimeter of a square. $\left[\begin{array}{l} P = S + S + S + S \\ P = 4S \end{array}\right]$

Have children write the sentence to solve the problem. $\left[4 \times 6\frac{1}{2} = n\right]$

Reinforce: $6\frac{1}{2} = \frac{\square}{2}$



Rewrite the problem: $4 \times 6\frac{1}{2} = n$ as $4 \times \frac{13}{2} = n$

Then compute: $4 \times \frac{13}{2} = \frac{52}{2} = 26$

2. Present problem: Mother needs to fill $2\frac{3}{4}$ books of trading stamps. If each book contains 1200 stamps, how many stamps does she need to collect?

Method I: $2\frac{3}{4} \times 1200 = (\square \times 1200) + (\Delta \times 1200) = n$ (Applying Distributive Property of Multiplication with respect to Addition)

Method II: $2\frac{3}{4} \times 1200 = \frac{11}{4} \times 1200 = n$ (Changing Mixed Form to Fractional Form)

EVALUATION AND/OR PRACTICE
SUGGESTED EXERCISES

1. Children compute the following using methods 1 and 2.

a. $35\frac{2}{3} \times 900 = n$ b. $56\frac{3}{8} \times 1432 = n$

2. Complete the following:

a. $5\frac{2}{3} \times 6 = \frac{\square}{3} \times 6 = n$ d. $4\frac{1}{5} \times 18 = \frac{\square}{5} \times 18 = n$

b. $4 \times 2\frac{1}{8} = 4 \times \frac{\square}{8} = n$ e. $7 \times 4\frac{2}{3} = 7 \times \frac{\square}{3} = n$

c. $8\frac{3}{4} \times 10 = \frac{\square}{4} \times 10 = n$ f. $6 \times 1\frac{5}{16} = 6 \times \frac{\square}{16} = n$

3. Complete and find the product.

$$\begin{array}{r} 8\frac{2}{3} \\ \underline{\times\ 6\phantom{\frac{2}{3}}} \\ (6 \times \square) \\ \underline{(6 \times \Delta)} \end{array}$$

4. Find the product $5 \times 7\frac{3}{8}$ by renaming $7\frac{3}{8}$ as a sum.

5. Interesting Number Patterns

Show that the following statements are true:

$$1\frac{1}{2} \times 3 = 1\frac{1}{2} + 3$$

$$1\frac{1}{3} \times 4 = 1\frac{1}{3} + 4$$

$$1\frac{1}{4} \times 5 = 1\frac{1}{4} + 5$$

Can you continue the pattern?

OPERATIONS

UNIT 72 - SET OF FRACTIONAL NUMBERS: FRACTION AS AN INDICATED DIVISION OF TWO NUMBERS; DIVISION OF FRACTIONS

NOTE TO TEACHER

Fraction As A Number

The sets of numbers that we consider in Elementary School Mathematics are:

1. The Set of Counting Numbers (also called Natural Numbers)

   $\{1, 2, 3, \ldots\}$

2. The Set of Whole Numbers

   $\{0, 1, 2, \ldots\}$

3. The Set of Integers

   $\{\ldots, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, \ldots\}$

4. The Set of Rational Numbers

Rational numbers may be defined as numbers which can be symbolized as an indicated quotient, $\frac{a}{b}$, of two integers (the denominator not zero). Thus one-half, which can be written as $\frac{1}{2}$ is a rational number, as in three, which can be written as $\frac{3}{1}$ (or $\frac{6}{2}$).

Fraction As A Numeral

Technically, a fraction, like $\frac{1}{2}$ or $\frac{2}{1}$ is a numeral, naming or symbolizing a rational number.
It is understood that we will use the term "fraction" whether we are considering the number (the idea) or the numeral (the symbol).
Children need rarely refer to this distinction in grade 5.

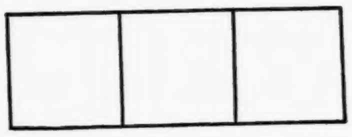
Objectives: To extend the meaning of fractional numbers.
To develop dividing a fraction by a whole number.

TEACHING SUGGESTIONS

Fraction As An Indicated Quotient

1. Suggested problem: 3 boys are to share a candy bar equally. What part of it will each boy get?

Show, using a diagram, how the boys shared the candy bar.



What did the boys do to the candy bar? [They divided it into 3 equal parts.]

What part did each boy get? $\left[\frac{1}{3}\right]$

Ask children to record the action in a mathematical sentence.

$$\left[1 \div 3 = \frac{1}{3}\right]$$

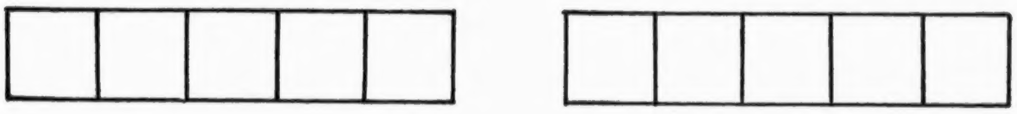
Is $\frac{1}{3}$ another symbol for 1 ÷ 3? Can $\frac{1}{3}$ be read as 1 ÷ 3?
What can the line separating the numerator and the denominator represent?

[the bar of the division symbol;
• ←numerator
— ←indicated division
• ←denominator]

2. Suggested problem: 2 candy bars were to be divided equally among 5 boys. How much should each boy get?

Ask children:

To draw a diagram to show the action.



To write a mathematical sentence to show the action.

$$\left[2 \div 5 = \frac{2}{5}\right]$$

What is the result of the division represented by $2 \div 5$ or $\frac{2}{5}$

$$\left[\frac{2}{5} \text{ or 2 fifths}\right]$$

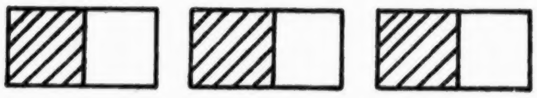
To read $\frac{2}{5}$ in two different ways.

Children discover that:

$\frac{2}{5}$ read as $2 \div 5$ represents the result of the operation of division on 2 and 5.

$\frac{2}{5}$ read as 2 fifths represents the result of that operation on the answer.

3. Suggested problem: 3 candy bars are to be divided equally between 2 boys. Show this with a diagram and record the action with symbols.

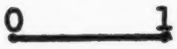


$$\left[3 \div 2 = \frac{3}{2}\right]$$

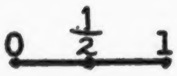
Have children interpret $\frac{3}{2}$ in two different ways.

4. Using number lines and symbols.

Have children draw a line segment of 1 unit length.



They divide it into 2 equal parts and label the midpoint.



Write the sentence that tells how you arrived at 1 half the unit of length. $1 \div 2 = \frac{1}{2}$

Is $\frac{1}{2}$ the same as $1 \div 2$? Why?

Is $1 \div 2$ another name for $\frac{1}{2}$? Why?

Have children draw a line segment of 3 units of length.

They divide it into 2 equal parts.

They show that $3 \div 2 = \frac{3}{2}$ and that $\frac{3}{2}$ is another name for $3 \div 2$.

5. Suggested exercises for practice.

   a. Complete the following equations and change to mixed form where possible:

   $1 \div 6 = \frac{\square}{\triangle}$ $\frac{\square}{\triangle} = 4 \div 5$ $7 \div 3 = \frac{\square}{\triangle}$ $25 \div 8 = \frac{\square}{\triangle}$

   $\frac{3}{5} = \square \div \triangle$ $\frac{5}{10} = \square \div \triangle$ $100 = \square \div \triangle$ $\frac{400}{100} = \square \div \triangle$

   b. What operation does the symbol for a fraction indicate?

   c. State another interpretation of a fraction. [A fraction indicates the quotient of two numbers.]

Dividing a Fraction by a Whole Number

1. Suggested problem: $\frac{1}{4}$ of a large candy bar is divided equally between 2 children. How much will each receive?

Children may use representative materials if necessary.

Discuss:

What part of the candy bar must be shared? $\left[\frac{1}{4}\right]$

Into how many parts must $\frac{1}{4}$ of the candy bar be divided? [2]

To have children understand how to record the division of a fraction by a whole number use the following pattern:

Ask children to record:

8 divided into 2 equal parts; $2\overline{)8}$; $8 \div 2$

4 divided into 2 equal parts; $2\overline{)4}$; $4 \div 2$

2 divided into 2 equal parts; $2\overline{)2}$; $2 \div 2$

1 divided into 2 equal parts; $2\overline{)1}$; $1 \div 2$

$\frac{1}{2}$ divided into 2 equal parts; $2\overline{)\frac{1}{2}}$; $\frac{1}{2} \div 2$

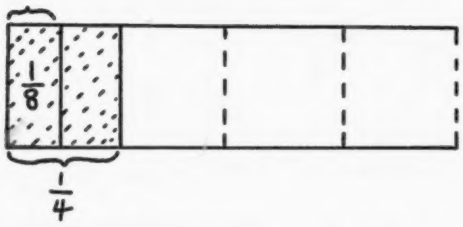
$\frac{1}{4}$ divided into 2 equal parts, $2\overline{)\frac{1}{4}}$, $\frac{1}{4} \div 2$

How then may the problem be recorded in a mathematical sentence? $\left[\frac{1}{4} \div 2 = n\right]$

What will "n" equal? $\left[n = \frac{1}{8}\right]$

Solution: $\frac{1}{4} \div 2 = \frac{1}{8}$

Have children derive or check answers by drawing diagrams.



2. Suggested problem: For the school garden, $\frac{1}{3}$ of a plot of land is to be shared equally by 4 classes. What part of the entire plot will each class cultivate? $\frac{1}{3} \div 4 = n$

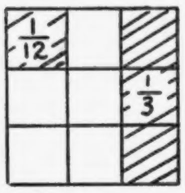
Discuss:

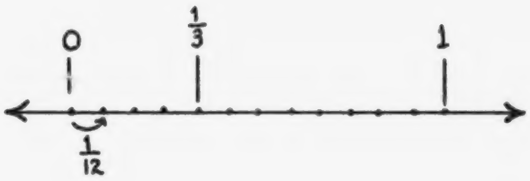
What part of the plot must be shared?

Into how many parts must $\frac{1}{3}$ of the plot be divided?

Record: $\frac{1}{3}$ divided into 4 parts = n

Children should represent the situation using a diagram or a number line.





$\frac{1}{3}$ divided into 4 equal parts = $\frac{1}{12}$ $\quad n = \frac{1}{12}$

3. Suggested practice exercises.

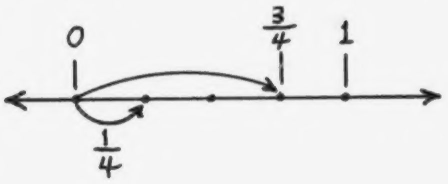
Children may use diagrams or number rays to derive the answer.

$\frac{1}{2} \div 2 = n \quad \frac{1}{4} \div 2 = n \quad \frac{1}{2} \div 3 = n \quad \frac{1}{4} \div 3 = n$, etc.

Dividing a Fraction With a Numerator Greater Than One by a Whole Number.

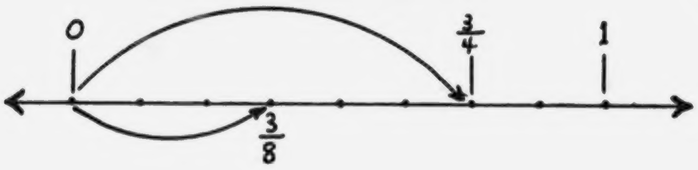
1. Suggested Exercise: $\frac{3}{4} \div 3 = n$

Discuss: $\frac{3}{4}$ divided into 3 equal parts. Use diagrams or number line.



$\frac{3}{4} \div 3 = \frac{1}{4}$

2. **Suggested problem:** $\frac{3}{4}$ of a yard of ribbon is to be divided into 2 equal parts for apron strings. How long will each string be? $\frac{3}{4} \div 2 = n$



Discuss:

$\frac{1}{4}$ divided into 2 equal parts equals $\square$? $\left[\frac{1}{8}\right]$

$\frac{3}{4}$ divided into 2 equal' parts equals $\square$? $\left[\frac{3}{8}\right]$ Why?

$\left[\frac{3}{4}\text{ is 3 times as much as }\frac{1}{4}\right]$

3. **Suggested practice exercises:**

Since $\frac{1}{3} \div 3 = \frac{1}{9}$	Since $\frac{1}{6} \div 2 = \frac{1}{12}$	Since $\frac{1}{5} \div 2 = \frac{1}{10}$
$\frac{2}{3} \div 3 = \square$	$\frac{5}{6} \div 2 = \square$	$\frac{4}{5} \div 2 = \square$

SETS; NUMBER; NUMERATION

*UNIT 73 - SYSTEMS OF NUMERATION: BASE FIVE (Optional)

NOTE TO TEACHER

The Hindu-Arabic system of numeration is a decimal or base 10 system.
It uses 10 symbols called digits:

{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

and 10 is called the Base.
Any whole number can be recorded using a combination of these ten symbols and the principle of place value. This can be extended to include fractional numbers expressed as decimal fractions.

A similar system of numeration can be based on any number of symbols greater than 1.
For example:

Base	No. of Symbols	Symbols
2	2	0,1 (Binary System of Numeration)
3	3	0,1,2
4	4	0,1,2,3
5	5	0,1,2,3,4 (Quinary System of Numeration)
10	10	0,1,2,3,4,5,6,7,8,9 (Decimal System of Numeration)
11	11	0,1,2,3,4,5,6,7,8,9,T
12	12	0,1,2,3,4,5,6,7,8,9,T,E (Duodecimal System)

The reason for studying a system of numeration based on another number such as 2 or 5 is to give children a better understanding of the Base 10 system and the use of the Place Value concept.

When introducing and developing the brief study of the Base 5 system of numeration suggested below, the emphasis should be only on understanding and appreciation and on comparison with the decimal (Base 10) system. Emphasis should not be on computation, speed, skill or memorization.

Objective: To extend understanding of the decimal system of numeration by introducing numeration in Base 5.

TEACHING SUGGESTIONS

1. Reinforce characteristics of the decimal system of numeration. Discuss:

   The names of the symbols (digits)

   The number of symbols

   Name of the system [Decimal system of numeration] Why?

   How numbers greater than 9 are recorded

   Place Value - Use the Place Value Chart to record and explain the value of the digits in each column. For example:

H	T	O
4	3	6

Tell children that the decimal system of numeration is also called a Base 10 system. Why?

2. Introduce Base Five

Develop with children:

What would you call a system similar to our Base 10 system, with only five symbols? [Base 5 or Quinary System]

Have children make up their own symbols. For example:

Δ, □, *, O, X

If we use Hindu-Arabic symbols, which of them should we use? [0, 1, 2, 3, 4] Why?

How would you represent the numbers one, two, three, four? [1, 2, 3, 4]

What would the next number be? [five]

In the Base 10 system, have we a single symbol for ten, the base?

How do we represent ten? [10]

In the Base Five system, we would represent five by 10_{five}. There is no single symbol for five in Base Five, just as there is no single symbol for ten in Base 10.

We read 10_{five} as "one, zero; Base 5"

Discuss:

The verbalization of "10" when it represents the quantity five. [One, zero; Base Five]

The reason why "one, zero; Base Five" should not be called ten. [It is too easily confused with the number 10.]

The recording of "one, zero; Base Five" to distinguish it from "one, zero; Base 10."

10_{five} read as: One, zero; Base Five

10 read as: Ten. (Base need not be indicated in Base Ten, although it may also be read as "One, zero; Base Ten)

Continue to discuss, record and read Base Five quantities through 9.

Number	Numeral	Read as
five	10_{five}	one, zero; base five
six	11_{five}	one, one; base five
seven	12_{five}	one, two; base five
eight	13_{five}	one, three; base five
nine	14_{five}	one, four; base five

How shall we continue?

3. **Place Value Chart for Base Five**

Discuss how to label the columns. Begin with "ones."
Have children volunteer. Teacher records symbols for column labeled "Ones."
They record numbers through 4.

	Ones
	1
	2
	3
	4

Extend chart to the next column.

In Base Ten system what did we label the column to the left of unit's column? Why?
[Ten times as large as the one's unit]

In Base Five system what can we label the column to the left of "One's" column? [Fives]
Why? [Five times as great as the "One's" unit]

Have children:

Record "Five" in the Place Value Chart.

Tell what is the number after 4. [five]

Record it in its proper place.

Fives	Ones
	1
	2
	3
	4
1	0 (one, zero)

(one group of five, 𝍸)

Discuss:

Number of ones, number of groups of fives. Explain.
[Four is the largest number that can be represented in the ones column]

What is the relationship between 10_{five} and 1_{five}?

[10_{five}: read as one, zero; is five times the size of 1_{five}.]

4. Comparing numerals in Base Five with numerals in Base 10

a. Use chart to make comparisons:

BASE FIVE		BASE TEN or DECIMAL EQUIVALENT	
Fives	Ones		
	1	1	𝍷
	2	2	𝍷𝍷
	3	3	𝍷𝍷𝍷
	4	4	𝍷𝍷𝍷𝍷
1	0	5	𝍸

b. Have children continue to record numerals in Base Five and to compare them with the decimal equivalents.

$11_{five} = 6_{ten}$; $12_{five} = 7_{ten}$; $13_{five} = 8_{ten}$; $14_{five} = 9_{ten}$

BASE FIVE		DECIMAL EQUIVALENT			
Fives	Ones				
1	0	5	𝍸		
1	1	6	𝍸	𝍷	
1	2	7	𝍸	𝍷𝍷	
1	3	8	𝍸	𝍷𝍷𝍷	
1	4	9	𝍸	𝍷𝍷𝍷𝍷	
2	0	10	𝍸	𝍸	(2 sets of five)

Discuss:

20_{five} [Two, zero; Base Five represents 2 sets of five and no sets of one, which is equal to 10 in Base 10.]

Continue through 44 (24) [Represents 4 sets of five and 4 sets of one]

c. Extend the Place Value Chart.

Relate the Place for 10 tens (hundreds) to the Place for 5 fives.

Record 25 in Base Five (100). Children should see the need for another column.

BASE FIVE			DECIMAL EQUIVALENT
Twenty-fives	Fives	Ones	
	4	4	24
1	0	0	25

What is the largest quantity that can be shown in two columns in the decimal system? [99]
Why? There is no single symbol to represent a value larger than 9.

What is the largest quantity that can be shown in two columns in the quinary system? [44]
Why? [We cannot use the symbol 5 in Base Five]

d. Discuss relationship between column headings in the Quinary System.

Fives and ones; ones and fives
[Five times as large]
[One fifth as large]

Twenty-fives and fives; fives and twenty-fives

Twenty-fives and ones; ones and twenty-fives

e. Compare these relationships with relationships in the decimal system; tens and ones; hundreds and tens; etc.

Discuss:

Value of 42_{five} as 4 fives and 2 ones. [$(4 \times 5) + 2$, in base 10]

Method of changing 42_{five} to Base Ten.

$$42_{five} = (4 \times 5) + 2 = 22 \text{ in Base Ten}$$

Have children change the following numerals in Base Five to numerals in Base Ten.

33_{five} 24_{five} 101_{five}

5. **Suggested practice exercises:**

a. Write each of the following Base Five numerals as a Base Ten numeral.

BASE FIVE	BASE TEN
0	[0]
10	[5]
24	[14]
42	[22]
103	[28]

b. Construct a chart to compare numbers from zero to fifty in Base Five with numbers from zero to fifty in Base Ten.

c. Insert the correct relation symbol: >, =, <

$17 \square 11_{\text{five}}$ [>] $29 \square 120_{\text{five}}$ [<]

$19 \square 34_{\text{five}}$ [=] $1000 \square 131_{\text{five}}$ [>]

*d. Some children may be able to add and subtract in Base Five.

*e. Encourage children to develop numeration systems with other bases.

GEOMETRY AND MEASUREMENT

UNIT 74 - MEASUREMENT: GRAPHIC REPRESENTATION; LINE GRAPH

Objectives: To interpret Line Graphs.
To construct Line Graphs.

TEACHING SUGGESTIONS

Reading and Interpreting Line Graphs

1. Reinforce:

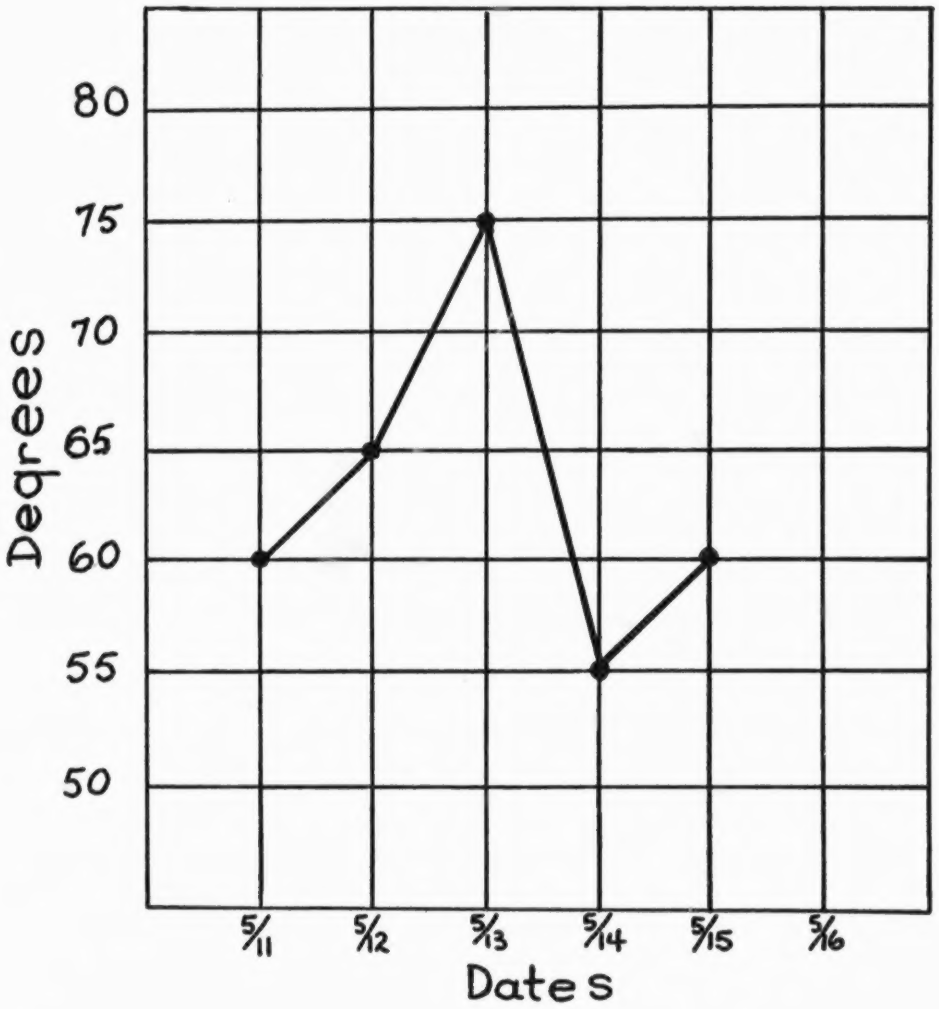
   Interpretation of bar graphs as showing a way of comparing data.

   Meaning of vertical axis, horizontal axis, scale.

2. Present a graph as below, and discuss the following:

   What is the title of the graph?
   What do the dates at the bottom tell us?
   What do the numerals at the left tell us?
   On what day or date was it warmest? - coldest?
   On what dates was the temperature the same?
   How much warmer was it on May 13th than on May 11th?
   How many degrees did the temperature drop between May 13th and May 14th?

Record of 2 P.M. Temperature—Week of May 11th



Discuss the advantage of using this Line Graph rather than a Bar Graph.

Line graphs are useful to show general trends and changes as well as to facilitate comparisons. A line graph may show changes over a period of time; changes in weight as heights change; etc.

3. Study line graphs and note:

   a. A line graph should have a title.

   b. Each axis should be labeled.

   c. The graph has two axes.

   d. A line graph scale does not necessarily start at zero.

   e. The lines are numbered, not the spaces.

   f. The mark used to show data on a line graph is a "dot" which will become the endpoint of one or two line segments.

   g. The dot represents an ordered pair. On the chart above each day is paired with the average temperature at 2 P.M. on that day.

      On the graph above each dot represents one of the ordered pairs: 12, 65; 15, 60; etc.

   h. Dots are connected by line segments from left to right. The line segments make it easier to interpret the graph and to observe a trend.

   i. When one of the scales of a line graph represents units of time, these are usually marked on the horizontal axis.

4. Children should interpret other line graphs found in newspapers; periodicals and textbooks.

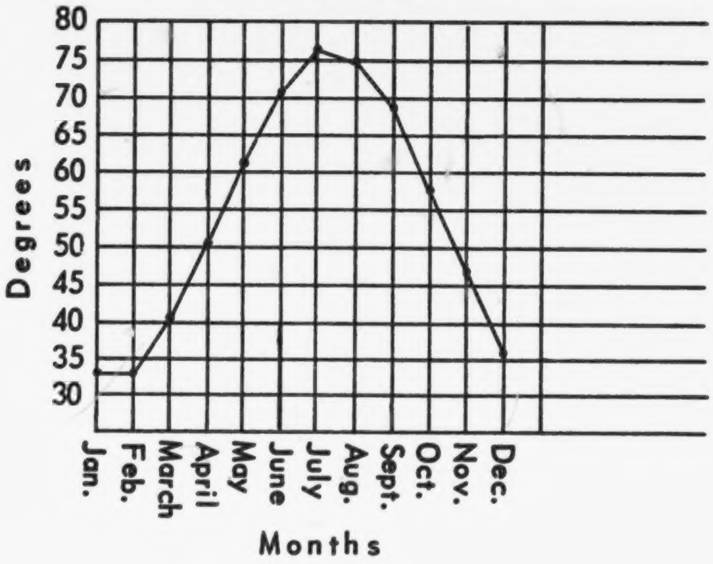
Constructing Line Graphs

1. Children should use graph paper (4 or 5 boxes to the inch are suggested) to construct a line graph from information organized on a chart.

For example:

Normal Monthly Temperatures for New York City

Month	Temp.	Month	Temp.
Jan.	33	July	77
Feb.	33	Aug.	75
Mar.	41	Sept.	69
April	51	Oct.	58
May	62	Nov.	47
June	71	Dec.	36


Normal Monthly Temperatures for New York City

As children construct the graph of the information on the chart, discuss:

> Why was a scale of 5° for each space selected for the degrees of temperature?
>
> What other scale could have been chosen? Explain.
>
> Why were the months written on the horizontal axis?

What determined the placement of the dots?
[Ordered pairs as shown on chart]

During what 2 months was there no change in temperature?

If there is a sharp rise upward in the line from 1 month to the next, what does that tell about the increase?

Compare the use of a bar graph vs. a line graph for this data.

2. Have children construct and interpret other line graphs based on information from daily experiences, textbooks, etc.

GEOMETRY AND MEASUREMENT

UNIT 75 - MEASUREMENT: AREA OF A RECTANGULAR REGION

NOTE TO TEACHER

In measuring area by counting the number of units, we follow the same pattern as when we measured length by counting hand spans, etc.

We:

1. Select a unit.
2. Count the number of units that fit into the object being measured.
3. This number is called the measure; in this case, the area.

Objectives: To develop concept of area.
To introduce standard units of measure for area.
To develop formula for finding area.

TEACHING SUGGESTIONS

1. Reinforce meaning of point, line, line segment, ray, plane, simple closed figure, polygon.

2. To understand area as the measure of the interior surface of a polygon, children should review properties of some geometric figures.

a. Reinforce: Right Angle
  Angle: Meaning of angle; kinds of angles.
  Right Angles are formed when 2 intersecting lines result in 4 angles with equal measure.
  Perpendicular lines are 2 lines that intersect to form right angles.

b. Reinforce properties of Rectangles.
  A rectangle is a polygon with 4 sides (quadrilateral).
  Both pairs of opposite sides are parallel.
  Both pairs of opposite sides are equal in length.
  All four angles are right angles.
  A rectangle encloses a region called a Rectangular Region.

2. Develop measurement of a Rectangular Region.

a. Discuss the need for measuring a rectangular region: for example, the amount of paper, tile, carpet, etc. needed to cover a surface.

Ask children:

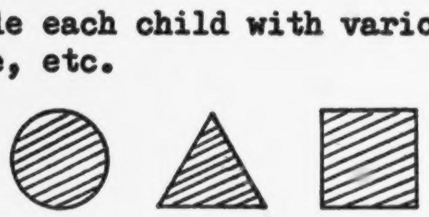
What is meant by the "play area" of the playground?
What is meant by the "floor space" of a room?
How do you think you can find the measure of an enclosed region?

Tell children:
The measure of an enclosed space (the region) is called its area. Area measures surface just as length measures curves or lines.
It can sometimes be found by counting the number of times a selected unit is contained in the region being measured.

b. Non-Standard Units of Measure of Area.

Suggested problem: To find the area of the drawing paper.

Provide each child with various shapes: circular, triangular, square, etc.



Consider each in turn as a unit of area.

(The square should be contained an exact number of times in the sheet of drawing paper.)

Have children experiment with the various shapes to find how many units of any one of them are contained within the surface.

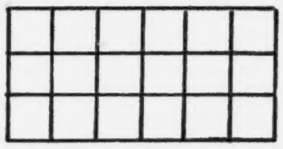
Discuss the fact that to measure the surface area, the units must not overlap, and spaces must not be left between units.

Which unit of area was most convenient for measuring the surface? [the square unit] Why?

Which unit of area was impossible to use for measuring the surface? [the circle] Why?

Have children lay off the square unit and count the number of times the unit is repeated to cover the whole surface.

How many times was the square unit contained in 1 row across the top of the drawing paper?



For how many rows was this repeated?

If 6 square units were laid off horizontally for 3 rows, how many square units were contained on the surface?

Have children discover that the area is 6 square units repeated 3 times, or 3 times 6 square units, or 18 square units.

Children discover that the area is 6 x 3 unit squares.

Suggested problem: To measure a section of the floor using a square unit.

Have children determine the number of times the length of the square is contained in 1 horizontal row, and the number of rows needed.

Discuss various ways of finding the area of the section of the floor by:

Counting the number of units the section of the floor will contain.
Multiplying the number of units in each row by the number of rows.

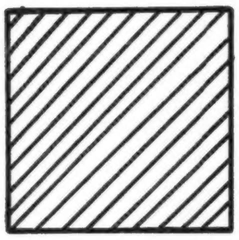
c. Standard Units of Measure

Square Inch

Discuss:

One inch as a unit of length.

A one inch square as enclosing 1 square inch of area.



The enclosed region has an area of 1 square inch.

One square inch as a standard unit of measure for area.

How to construct a square inch.

Difference between a "1 inch square" and "1 square inch".

In a "1 inch square" we are talking about the square, which is the boundary; in "1 square inch" we are talking about the measure of the interior region of a "1 inch square".

Suggested problem: Find the area of a sheet of drawing paper measuring 9" x 12".

Have children:

Mark the paper into one inch squares.

Count to find the number of square inches in one row and the number of rows.

Find the area by counting the number of one inch squares that the paper contains. [108 inch squares]

Discover that they can also find the area in square inches if they multiply the number of inches in one row (length) by the number of rows (width) 9 x 12 = 108. [108 square inches]

Square Foot

Discuss:

one foot as a unit of length.
one square foot as a unit of area.

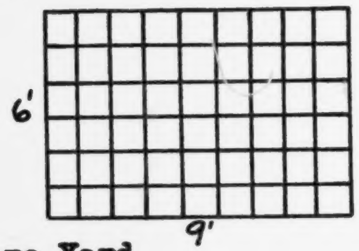
How would you construct a square foot using a foot rule? Using the square inch?

How many square inches are there in a square foot? [144] Why?

What is the area in a 1 foot square in square inches? in square feet?

Suggested problem: What is the area in square feet of a dinette floor that measures 6' by 9'?

Have children draw a diagram to show the area marked off in square feet.



Find the area:
6 x 9 sq. ft. = 54 sq. ft.

What does the 6 represent? the 9?

Square Yard

Discuss need for using larger units of square measure, such as square yard, to measure larger surfaces, e.g. carpeting a floor.

Children discover that:

144 sq. in. = 1 sq. ft. Explain. - Note that $144 = 12 \times 12 = 12^2$

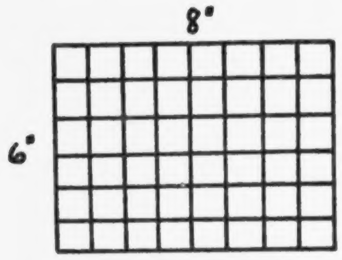
9 sq. ft. = 1 sq. yd. Explain. - Note that $9 = 3 \times 3 = 3^2$

1296 sq. in. = 1 sq. yd. Explain. - Note that $1296 = 36 \times 36 = 36^2$

Discuss why $1296 = 9 \times 144$.

3. Formula for finding Area of a Rectangular Region

Consider a rectangle 8" long and 6" wide.



Have children:

Count the number of one inch squares in one row. [8]

Measure the length of the row. [8 inches]

Relate the measure of the length (8 inches) to the number of inch squares. [8; the number is the same.]

Count the number of rows. [6; width]

Measure the width. [6 inches]

Relate the measure of the width (6 inches) to the number of rows. [6; the number is the same]

Arrive at the formula for finding the area of a rectangular region:

If A = Area; l = length; w = width

then $A = l \times w$ or $A = lw$

Apply the formula to find the area of the rectangular region above.

$A = l \times w$ $A = 8 \times 6$ $A = 48$ Area = 48 sq. inches

4. Discuss:

Need for square mile as a unit to measure large areas of land.

Acre as a unit of area. (Not "square acre" since acre is already a unit of area.)

Relationship of area of square to square numbers.

Need for using same unit of measure of length in finding areas.

Children should understand that when the dimensions are stated in two different units of measure, such as feet and inches, either the number of larger units may be changed to smaller units, or the number of smaller units may be renamed as fractional parts of the larger unit.

Children should note that when:

There are more of the smaller units; the number of those units is larger.

There are fewer of the larger units; the number of those units is smaller.
For example: 1 ft. = 12 in., $1 < 12$

EVALUATION AND / OR PRACTICE

SUGGESTED EXERCISES

1. Find the area of the following rectangular regions.

   12' by 15'   14" by 14"   25 yd. by 32 yd.

   5' by 8'9"   8' by 6 yd.   4'3" by 2'7"

2. Additional exercises may be found in textbooks.

GEOMETRY AND MEASUREMENT

UNIT 76 - GEOMETRY: SOLID GEOMETRIC SHAPES

Objectives: To help children distinguish between plane and solid figures.

To distinguish solids by their characteristics

TEACHING SUGGESTIONS

1. Reinforce children's understanding of the following terms.

Line
Line segment
Ray
Parallel
Perpendicular
Surface
Region
Dimensions
Angle
Right angle
Obtuse angle
Acute angle

Vertex
Simple closed figure
Plane
Polygon
Parellelogram
Quadrilaterial
Square
Rectangle

Circle
Arc
Degree
Area
Trapezoid
Rhombus
Perimeter

2. Discuss the meaning of dimension.

Children should know intuitively that:

A point has no dimensions.

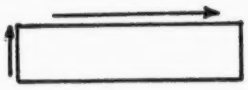
A line segment has one dimension; length

A closed plane figure such as a rectangular region has two dimensions; length and width.

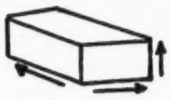
Compare a plane surface, e.g. a desk top with a solid, e.g. individual cereal box.

Have children note:

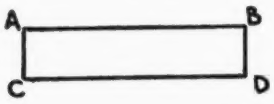
The boundaries of the desk top are line segments.
The boundaries of the box are parts of planes.



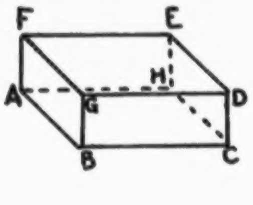
That the desk top has length and width.
That the box has length, width, and depth; the box encloses a volume.



The number of corners (vertices) of the desk top. (4)



The number of corners (vertices) of the box. (8)



That both are closed figures.

Discuss the dimensions of the desk top and the dimensions of the box.

Tell children that the box is an example or model of a geometric solid.

Show children:

filled cereal box; empty cereal box
brick; empty chalk box

Are all of these models of geometric solids? Why?

Discuss:

3 - D films
Photographs as plane figures (2 dimensional) that suggest solid or 3 dimensional figures.

3. Characteristics of some solid geometric figures.

Display items that are examples of solids:

Orange, ball, globe, pyramid, cylinder or can, building block, cube, triangular prisms, cone, etc.

Identify each as an example of a solid and compare them.

Note that some solids:

Have spherical shapes. (orange and ball have curved surface only.)

Have curved and plane surfaces. (cylinder, cone)

Have only plane surfaces. (block, pyramid, prism)

Continue the development using only the following:

Brick, block, a square box, pyramid, triangular prism.

Emphasize the similarities of each. (Each has surfaces that are enclosed by polygons).

Discuss:

The surfaces as faces

The line segments as edges

The intersections of the line segments as vertices

Emphasize the differences

Compare a cube with other rectangular solids. (boxes)

Compare a pyramid with the other solids.

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Show diagrams of plane and solid figures. Identify each.

2. Find and list objects in the classroom that suggest geometric solids.

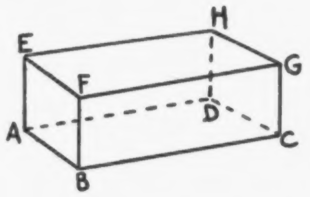
3. What geometric figure is suggested by each of the following:

can	ice cream cone	pup tent
carton	ball	tepee

4. Write the name of an object which suggests each of the geometric terms listed below:

sphere	cube
rectangular solid	cone

5. Draw a picture of a rectangular solid. Label each vertex with a letter.



Identify each edge by two letters. [AB, etc.]

Identify each face by four letters. [EFGH, ABFE, etc.]

Identify each vertex by one letter. [A, B, etc.]

Use letters to identify the faces of the solid that have the largest area, the smallest area.

Use letters to identify the edges that are parallel. [$\overline{AB} \parallel \overline{DC}$, etc.]

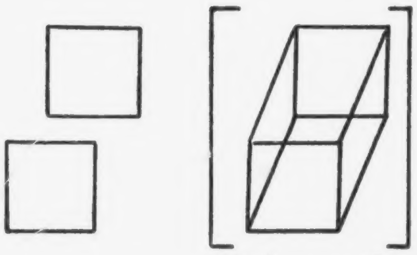
6. Indicate which of the following suggest surface regions only; which represent geometric solids:

an ice cube	a counter top
a can of peas	the outside of the can of peas

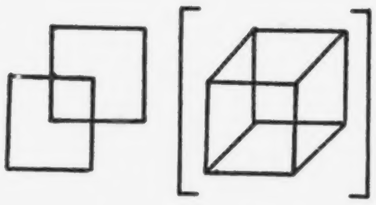
Visualizing, Drawing, Constructing of 3 dimensional figures may help children to understand better their properties.

1. Starting with the following, can you finish the drawing to make a model of

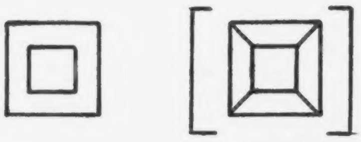
   a. A cube:



Not a cube

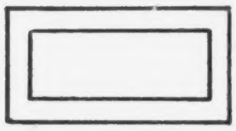


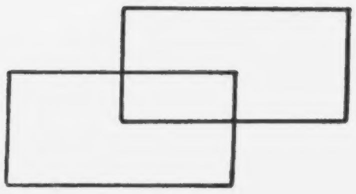
A cube

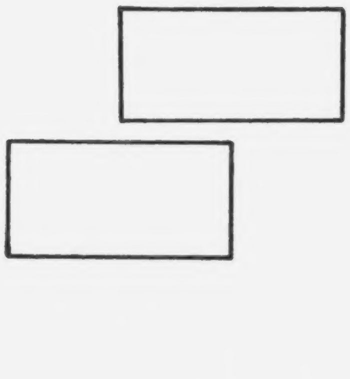


A cube

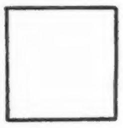
   b. A rectangular solid





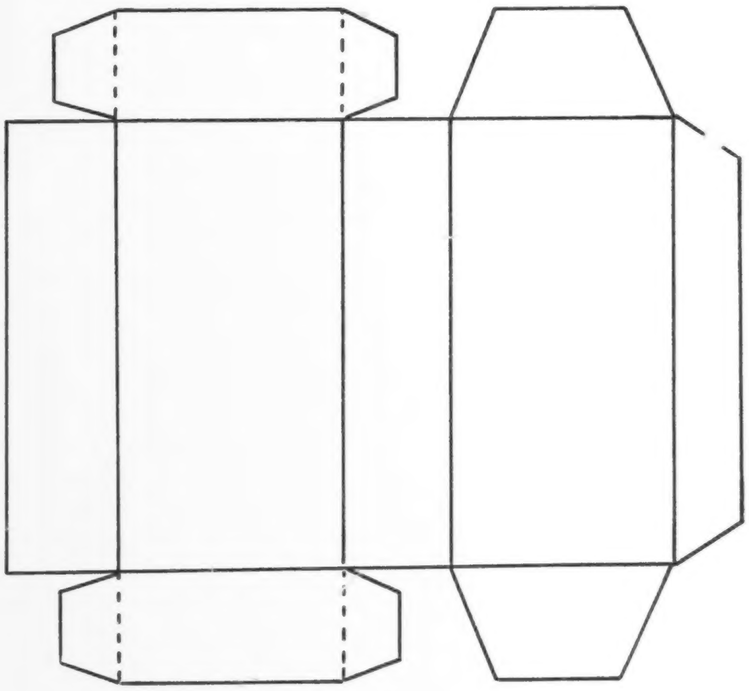


c. A pyramid: (starting with a square and a point outside the square)



2. Visualize and then draw a picture of some 3 dimensional figures as they would look flattened out.

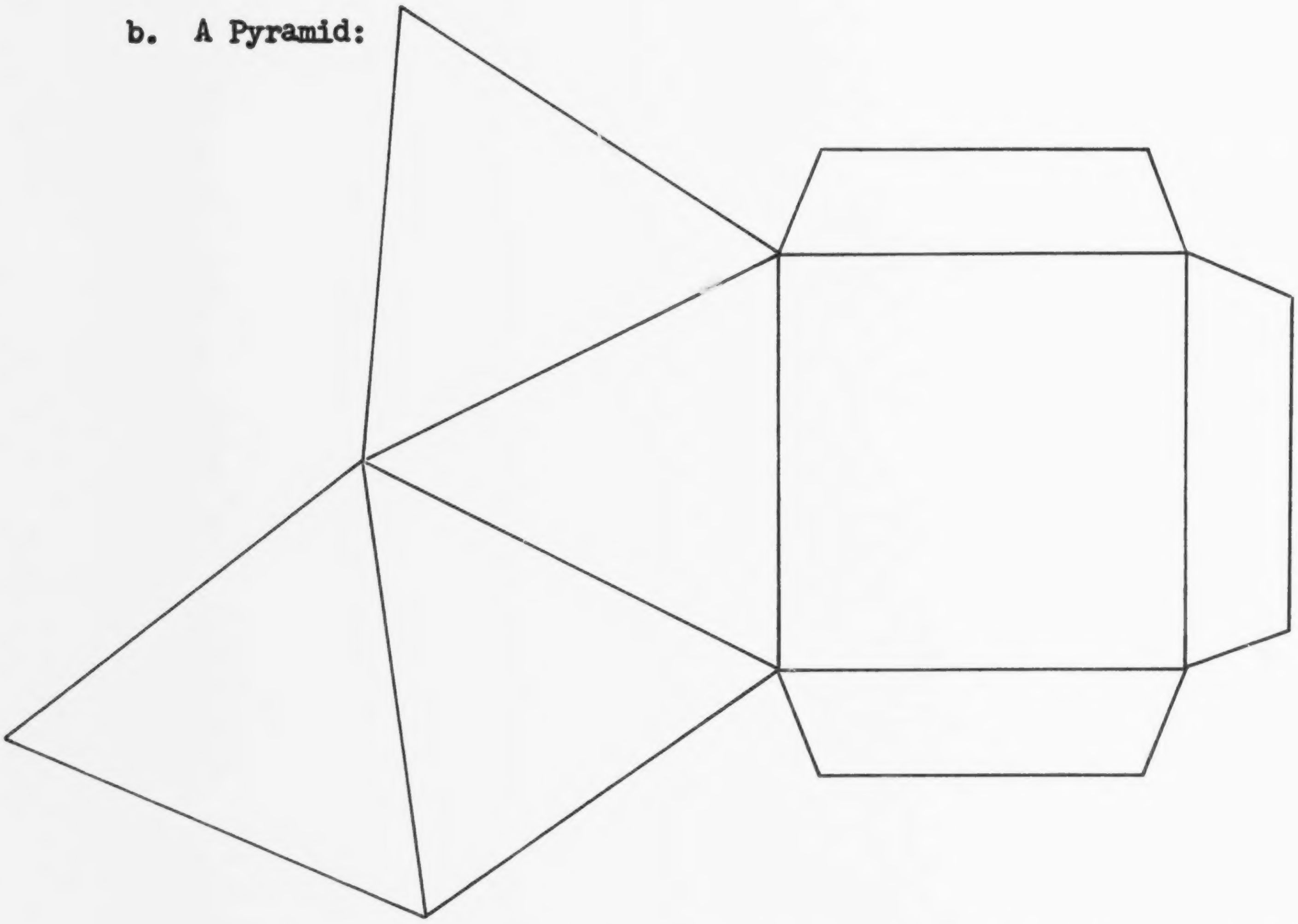
   a. A Rectangular solid:



Describe a rectangular solid as you visualize it from the flattened out model.

Add tabs to your model, cut it out and construct the model of a rectangular solid.

b. A Pyramid:

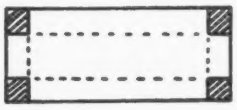


Describe a pyramid as you visualize it from the flattened out model.

Add tabs to your model, cut it out and construct the model of a Pyramid.

c. Start with a rectangular sheet of paper or card board.

Cut squares of the same dimensions out of each corner.



Make an open box by folding up the four sides. Use tape to connect the sides.

Experiment with cutting squares of different sizes out of each corner. Describe your results.

Experiment with cutting rectangles out of each corner. Describe your results.

SETS; NUMBER; NUMERATION

UNIT 77 - NUMERATION: EXPONENTIAL NOTATION

NOTE TO TEACHER

Children should be aware that in scientific notation, numbers are expressed using exponents and powers of 10. For example:

$10{,}000 = 10 \times 10 \times 10 \times 10 = 10^4$ can be read as: Ten to the fourth power.

10 is referred to as the "base". The term "base" has different meanings in systems of numeration, (Base 10, Base 5) and in Geometry.

For 10^4, the 4 is referred to as the Exponent. The exponent 4 tells how many times the base 10 is used as a factor.

10^4 is called the fourth power of 10.

Zero, used as an exponent is a special case.

$10^0 = 1$, by definition as is a^0 for any $a \neq 0$

One, used as an exponent is a special case.

$10^1 = 10$, by definition: $a^1 = a$ for all "a".

Objective: To help children understand meaning and use of exponents.

TEACHING SUGGESTIONS

Exponent for Squares

1. Reinforce understanding of the meaning of the square of a number.

   Ask children:

   What are the two equal factors that result in the product

   25 [5 x 5]   36 [6 x 6]   64 [8 x 8]
   1 [1 x 1]

   What number multiplied by itself equals 100?
   How many times does 5 appear as a factor of 25? [2 times]
   6 as a factor of 36? [2 times] etc.

2. Tell children that:

   When the same factor is used 2 times to arrive at a product, the expression may be written as:

   $5 \times 5 = 25$ or 5 square = 25 or $5^2 = 25$
   $6 \times 6 = 36$ or 6 square = 36 or $6^2 = 36$

   The ^{2} in 5^2, 6^2, etc. is called the exponent;
   The 5, 6 is the factor that is being repeated.

   This type of recording may be called the exponential form.
   5^2 is called the second power of 5.

   25 can be written as:

   25 in Whole Number Form
   5×5 in Product Form
   5^2 as a power of 5 or in Exponential Form

3. Suggested practice exercises:

   a. Express the following as the product of 2 equal factors, and then in exponential form.

      1, 4, 16, 49, 81, 100

b. Complete the following, then state in exponential form.

9 = ☐ x ☐ = ☐ square = ____ [3^2]
121 = ☐ x ☐ = ☐ square = ____ [11^2]
400 = ☐ x ☐ = ☐ square = ____ [20^2]

c. 12^2 = ☐ 13^2 = ☐ 14^2 = ☐ 15^2 = ☐

d. If $n^2 = 100$ then n = ☐ If $n^2 = 121$ then n = ☐ If $n^2 = 144$ then n = ☐

If $n^2 = 169$ then n = ☐

* e. If one side of a square is 4 inches, what is the area of the square? Write this in exponential form.

* f. If we let one side of a square be "s", write a formula for the area of a square, using exponential form. [$A = S^2$]

Extended Exponential Notation

1. Discuss:

In how many ways can we name 16 as a product?
Record some of these ways. For example:

a. 16 x 1 b. 8 x 2 c. 4 x 4 d. 4 x 2 x 2
e. 2 x 2 x 2 x 2

In which of the above are only equal factors used?

What are the factors of 16 in c? [4 x 4]; in e? [2 x 2 x 2 x 2]

How many times is the factor 4 used in c? the factor 2 used in e?

Rename the following, repeating the same factor:

25 = __ x __ [5 x 5]
64 = __ x __ [8 x 8]
64 = __ x __ x __ [4x4x4]
27 = __ x __ x __ [3x3x3]
81 = __ x __ x __ x __ [3x3x3x3]
100 = __ x __ [10x10]

Ask children to consider 25 as 5 x 5.
Which factor is repeated? [5]
How many times is it used as a factor? [2 times]
How can we write this in exponential form? [5^2]

Ask children to consider 81 as 3 x 3 x 3 x 3
Which factor is repeated? [3]
How many times is it used as a factor? [4 times]

2. Tell children that 3 x 3 x 3 x 3 can be written as 3^4 .
3^4 can be read "three to the fourth power".

Ask children:

What do you think the 3 in 3^4 means? The factor that is to be repeated.

What do you think the ^{4} in 3^4 means? [The number of times 3 is used as a factor.]

If this form of renaming numbers is called exponential notation, which number do you think we should call the repeated factor? [3]; the exponent? [4]; the base? [3]

Read 6^2 in two different ways. [6 to the second power, or 6 square]

Read 5^3 in two different ways. [5 to the third power, or 5 cube]

Why do we call it "6 - square"; "5 - cube" ?

3. Provide practice in reading numerals written in exponential notation.

3^5 , 8^3 , 7^4 , etc.

4. Provide practice in renaming numbers by repeating a factor, then stating the exponential form.

e.g. $4 = 2 \times 2 = 2^2$, $8 = 2 \times 2 \times 2 = 2^3$, etc.

5. Express in words and tell the meaning of:

$23'$, $43'$, n' [In n' we cannot think of "n used as a factor once", so what we do is to define n to equal n.]

EVALUATION and / or PRACTICE
SUGGESTED EXERCISES

1. Fill in the frames with the correct numeral.

$8 = 2 \times 2 \times 2 = 2^{\square}$ [2^3] $25 = 5 \times 5 = \square^2$ [5^2]

$49 = 7 \times 7 = 7^{\square}$ [7^2] $512 = 8 \times 8 \times 8 = \square^3$ [8^3]

$10{,}000 = 10 \times 10 \times 10 \times 10 = 10^{\square}$ [10^4]

$32 = 2 \times 2 \times 2 \times 2 \times 2 = \square^{\triangle}$ [2^5]

2. Express each of the following in words and tell the meaning of:

8^2 [eight to the second power, or eight square]

5^3 [five to the third power, or five cube]

6 [six to the seventh power]

3. Write the following in exponential form and tell the value of each:

9 to the fourth power. [$9^4 = 9 \times 9 \times 9 \times 9 = 6561$]

Repeated factor 4, exponent 2 [$4^2 = 16$]

Repeated factor 10, exponent 3 [$10^3 = 1{,}000$]

3 to the fifth power [$3^5 = 243$]

4. Solve

$3^4 = \_\_ \times \_\_ \times \_\_ \times \_\_ = n$ [$n = 81$]

$4^3 = \_\_ \times \_\_ \times \_\_ = n$ [$n = 64$]

$10^6 = \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_ = n$ [$n = 1{,}000{,}000$]

5. Solve

$64 = 4^{\square}$ [4^3] $81 = 9^{\square}$ [9^2]

$64 = 2^{\square}$ [2^6] $81 = 3^{\square}$ [3^4]

6. Fill in the spaces in the following chart:

Word Name	Numeral	Repeated Factor Form	Exponential Form
Twenty-five	25	5 x 5	5^2
One hundred twenty-five		5 x 5 x 5	
	625	5 x 5 x 5 x 5	
Sixty-four	64		
Eight			

7. Which of the following represents the largest number? the smallest number? Explain.

53, 3^5, 5^3,   35,   5 x 3,   5 + 3

[largest: $3^5 = 243$; smallest: $5 + 3 = 8$]

8. Which number is larger? How much larger?

3^4 or 4^3

5^2 or 2^5

2^4 or 4^2

9. Extend the Place Value Table - Base Five to include exponential notation.

One Hundred Twenty-fives	Twenty-fives	Fives	Ones
125	25	5	1
5 x ☐ x ☐	5 x ☐	5	1
5	5	5	1

SETS; NUMBER; NUMERATION

*UNIT 78 - NUMBER: SET OF INTEGERS (Optional)

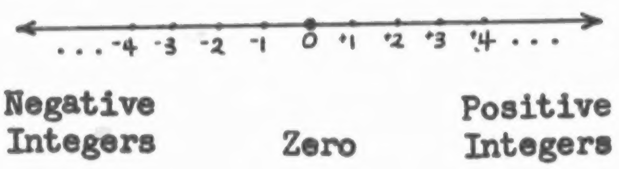
NOTE TO TEACHER

Children have worked with the set of Counting or Natural Numbers $\{1, 2, 3, \ldots\}$ and with the set of Whole Numbers $\{0, 1, 2, \ldots\}$. We are ready now to extend the number system to include the set of Integers.

The Set of Integers consists of:

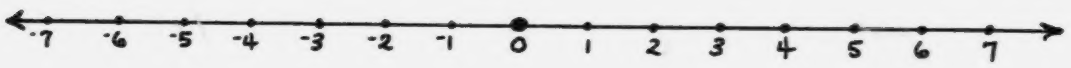
> The Set of Counting Numbers, Zero, and for each number, n, of the set of Counting numbers, another number, ^-n, such that:
>
> $$n + {^-n} = 0$$

The Set of Integers



A number line represents a one-to-one correspondence between a set of numbers and their associated points on a line. It may be a horizontal line, a vertical line or a line in any other direction extending indefinitely in both directions.

Positive and negative integers may be represented by evenly spaced dots on the number line to the right and to the left (or above and below) the point that designates the number Zero.



6 shown as $^{+}6$ is read "positive 6"

8 shown as $^{+}8$ is read "positive 8"

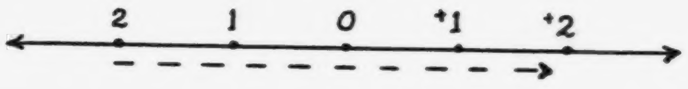
When the sign of the number is not shown, it is understood that the number is positive. $8 = {}^{+}8$; $6 = {}^{+}6$; etc.

$^{-}4$ is read "negative 4"

$^{-}3$ is read "negative 3"

Zero is a number which is neither positive nor negative. It is the reference point or origin on the number line.

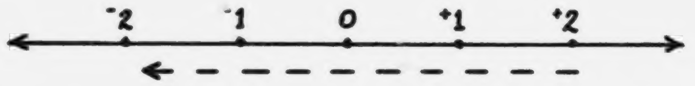
On the number line, positive numbers usually are to the right of the zero. The positive direction here, indicates that we proceed to the right from any point on the number line.



Positive Direction

The Negative direction indicates that we proceed to the left from any point on the number line.

a



Negative Direction

We sometimes call integers "directed numbers" because of the way we represent them on the number line. Often the term "signed Numbers" is used because of the $^+$ or $^-$ sign in the numeral.

Objectives: To introduce the meaning of and symbolism for integers.

To introduce concept of negative and positive direction on the number line.

TEACHING SUGGESTIONS

Set of Integers: Concepts

1. Reinforce reading a thermometer. Consider the thermometer as a vertical number line.

   Have children discuss and record temperatures above zero, below zero, at zero.

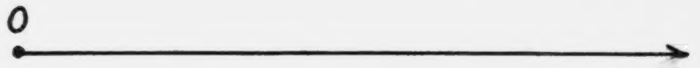
   Ask children:

   What is the origin or reference point on the number line? [zero]

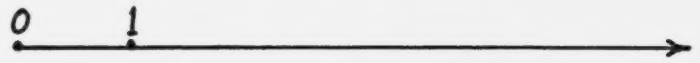
   How many times does zero appear on the number line? [one]

   How many times does every other numeral appear on the number line? [2 times] Where?

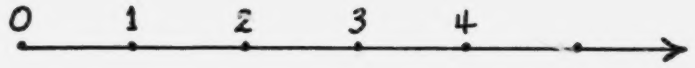
2. Have children draw a horizontal Number Ray and indicate zero.



They should then mark off unit to the right of zero and assign "1" to that point.

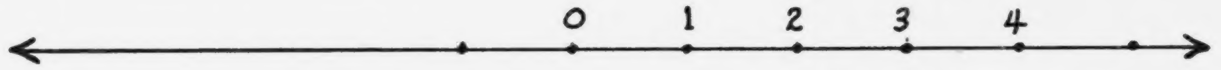


Have the children indicate, discuss and assign "2", "3", "4", etc. to points on the line to the right of zero.



Introduce the Number Line by extending the Number Ray in the other direction.

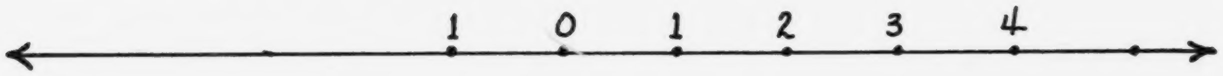
Ask children to indicate 1 unit to the left of zero.



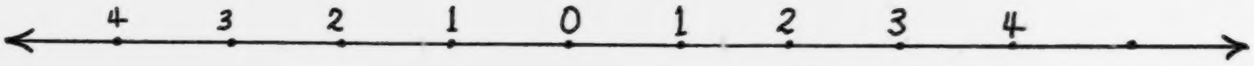
They discuss naming points in this direction to the left of zero.

What would you name the new point? [1]

Label it.



Children should than indicate, discuss and label points 2, 3, 4 to the left of zero.



Have children compare the two points labeled "1".

> What is true of the distance from zero to each of the points labeled "1" ? [same]
>
> What can you tell about the direction of those points from zero? [opposite]

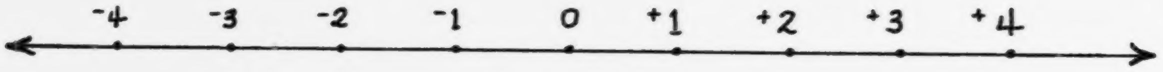
Have children continue to compare pairs of points to discover equal distances from zero; different directions.

3. Develop symbols for positive and negative numbers.
Children discuss the need to label these pairs of points on the number line to indicate their differences.

Ask children:

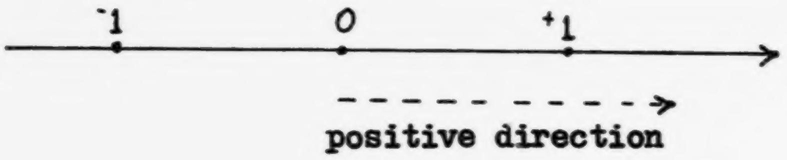
> If we label points: 1, 2, 3, 4, etc. to right of zero $^{+}1$, $^{+}2$, $^{+}3$, $^{+}4$, what do you think we could label the corresponding points: 1, 2, 3, 4, to the left of zero? [$^{-}1$, $^{-}2$, $^{-}3$, $^{-}4$] Why?

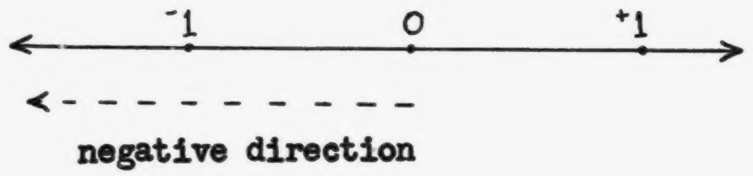
Children record these symbols on the number line.



Discuss Direction:

> If $^{+}1$ is said to be in a positive direction from zero, how would you name the direction of $^{-}1$? [negative direction]





If the direction of $^{+}1$ is a positive direction $^{+}1$ is called positive 1, what would you call $^{-}1$? [negative 1]

Would you call zero a positive or negative number? [neither]

Discuss pairs of numbers as to direction and distance from zero.

$^{+}2$ and $^{-}2$ $^{+}4$ and $^{-}1$
$^{-}3$ and $^{+}3$ $^{-}3$ and 5 etc.

Children note that:

$^{+}2$ is opposite $^{-}2$ (from zero); $^{-}2$ is opposite $^{+}2$ (from zero); etc.

5. Discuss the Whole Number System {o, 1, 2, 3, . . .} with which children have been operating until now.

   Which is the first numeral in the set?
   How many numerals are in the set?
   What other name can be given to 1, 2, . . . [positive 1, etc.]

6. Tell children that this new set of numbers that includes *positive* and *negative* numbers and *zero* is called the *Set of Integers*.

   Tell children that we *sometimes* call the Set of Integers

   The Set of *Directed Numbers*. Why? or

   The Set of *Signed Numbers*. Why?

7. Children note that:

   The Number Ray maps the Set of Counting Numbers.
   The Number Line is needed to map the Set of Integers.

8. Suggested practice exercises.

   a. Write the symbol which represents the numbers:

      negative twenty-six [$^{-}26$]
      positive sixty-four [$^{+}64$]
      negative two hundred seven [$^{-}207$]

   b. Write the words (number names) for:

      $^{+}15$ $^{-}64$ $^{+}314$ $^{-}708$

   c. If $^{+}6$ represents 6 degrees above zero, what does its opposite $^{-}6$ represent?

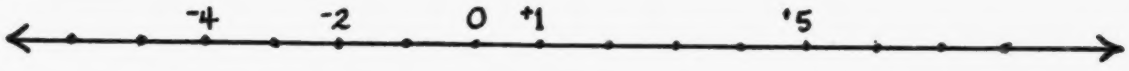
      If $^{+}3$ represents 3 hours after noon, what does its opposite $^{-}3$ represent?

      If $^{+}5$ represents 5 steps to the right, what does its opposite $^{-}5$ represent?

      If $^{-}15$ represents a loss of 15 pounds, what does its opposite $^{+}15$ represent?

      If $^{-}100$ represents a loss of \$100, what does its opposite $^{+}100$ represent?

   d. On the number line below, label the unmarked points.



e. Complete the sequence:

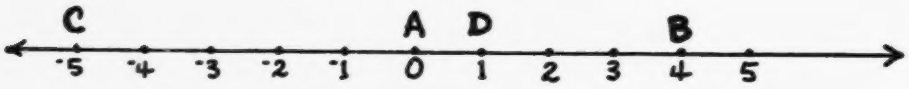
$^{+}2$, $^{+}1$, ___ , $^{-}1$, ___ , $^{-}3$

$^{-}2$, $^{-}4$, $^{-}6$, ___ , ___ , ___

$^{-}6$, $^{-}4$, $^{-}2$, ___ , ___ , ___

$^{-}4$, $^{-}1$, $^{+}2$, ___ , ___ , ___

f. Use the number line to answer the question below.



What is the endpoint of each of the following and what number does it represent?

$\overrightarrow{DC}$	[D, $^{+}1$]
$\overrightarrow{AB}$	[A, 0]
$\overrightarrow{BC}$	[B, $^{+}4$]
$\overrightarrow{CB}$	[C, $^{-}5$]

Making Comparisons

1. Have children draw a number line to compare positive integers in order to determine which is greater, which is less.

   Children note that:

   As we move to the right on the number line, the numbers become greater.

   As we move to the left the numbers become smaller.

2. Extend the number line to the left to include negative integers.

3. Have children explain how the number line shows the following relationships:

$^{+}3 > {}^{+}2$

$^{+}2 < {}^{+}3$

$^{+}3 > 0$

$0 < {}^{+}3$

$^{+}3 > {}^{-}1$

$^{-}1 < {}^{+}3$

$0 > {}^{-}1$

$^{-}1 < 0$

$0 > {}^{-}2$

$^{-}2 < 0$

$0 > {}^{-}4$

$^{-}4 < 0$

$^{-}1 > {}^{-}2$

$^{-}2 < {}^{-}1$

$^{-}3 > {}^{-}4$

$^{-}4 < {}^{-}3$

$^{-}1 > {}^{-}4$

$^{-}4 < {}^{-}1$

4. After many comparisons children note:

   Any integer represented on the number line is greater than any integer to its left.

   Any integer represented on the number line is less than any integer to its right.

   Any positive integer is greater than zero.

   Zero is greater than any negative integer.

   Any positive integer is greater than any negative integer.

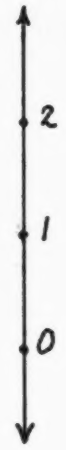
5. Since $^{+}6 > {}^{+}4$ and $^{+}4 > {}^{+}2$, how would you compare $^{+}6$ and $^{+}2$?

   Since $^{-}4 < {}^{-}3$ and $^{-}3 < {}^{-}2$, how would you compare $^{-}4$ and $^{-}2$?

* UNIT 79 - RECTANGULAR COORDINATE SYSTEMS (Optional)

NOTE TO TEACHER

When children represented numbers on a number line they were dealing with a set of points and a set of numbers. A one-to-one correspondence was set up between each of the sets of points and the number associated with it. By selecting and marking off a scale on a number line any point may be located by its corresponding number on the number line. For example:



When children studied line graphs they were dealing with the elements of two sets and the correspondence between the elements of the sets. For example: In a graph dealing with rainfall, the number of inches of rainfall are matched with dates or with locations.

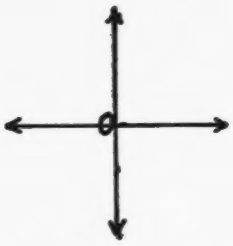
The development has been

1. Points and numbers represented on one line.
2. Inches of rainfall and corresponding dates represented on a grid.

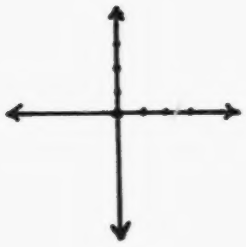
Now we proceed to

3. Coordinates for using an ordered pair of numbers by which any point in a plane may be located.

Two number lines called axes are drawn at right angles and the point of intersection is the zero point of each line.



Both axes are marked off in equal intervals.



To identify a point in the plane of these lines, we assign to it 2 numbers. The first shows its relationship to the horizontal axis; the second shows its relationship to the vertical axis. The horizontal axis is called the X axis. The vertical axis is called the Y axis. The point of intersection O is called the origin.

To locate a point A on the plane, lines are drawn perpendicular to each axis.

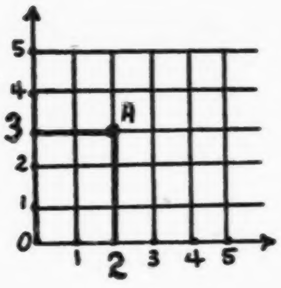


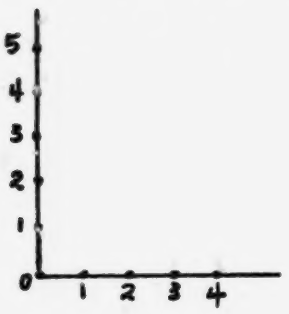
Fig. I

The perpendicular to the horizontal axis in this graph intersects the horizontal axis at the point corresponding to 2.

The perpendicular to the vertical axis intersects the vertical axis at the point corresponding to 3.

2 is called the first or X coordinate.
3 is called the second or Y coordinate.

The coordinate of a point on a number line is defined as the number which corresponds to that point.



Coordinates on the horizontal axis are:
. . ., 0, 1, 2, 3, 4, . . .

Coordinates on the vertical axis are:
. . ., 0, 1, 2, 3, 4, 5, . . .

The location of A in Figure I is described by the ordered pair of numbers (2, 3).

The order of the pair of numbers is of utmost importance.

Suppose the location was listed as (3,2).

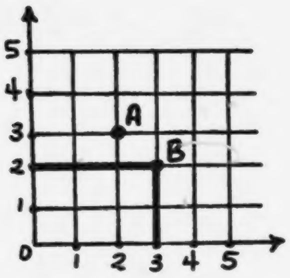
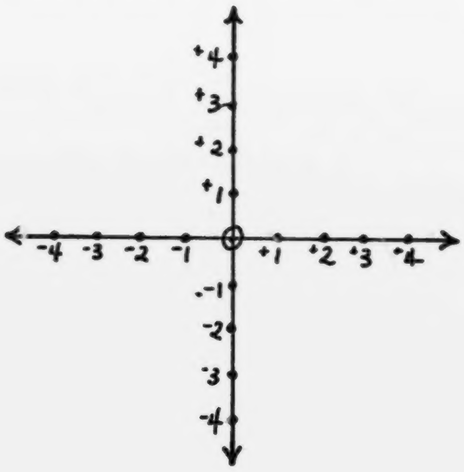


Fig. II

Then a point B, (3,2) shown in Figure II, not the same as point A (2, 3), has been described above.

The first number of an ordered pair is, by convention, the coordinate of the X (horizontal) axis and is called the X-coordinate.
Note that the horizontal and vertical axes are number lines intersecting at zero and including the positive and negative numbers. The direction from zero determines the sign of the number assigned to a point.



It is convenient to use graph paper to help in locating points on a plane since equally spaced horizontal and vertical lines are already printed.

Objectives: To help children understand the graph of a number.

To help children understand the use of an ordered pair of numbers in locating a point on a plane.

To help children understand the graphing of a truth set for an open sentence involving one or two place holders.

TEACHING SUGGESTIONS

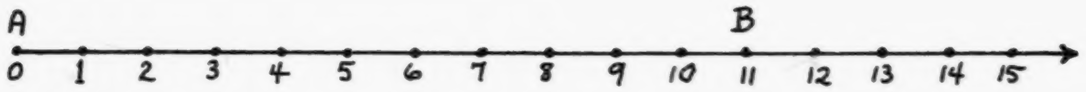
Extending Concepts of Graph to the Graphing of a Number in a Plane

1. Reinforce locating a point on a number line.

Have children:

Observe the number ray below.

Fig. I

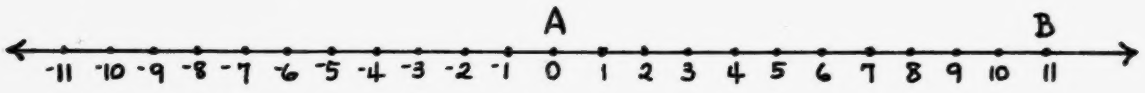

A
B
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Show where point C is on the number ray if the distance of C from A in a positive direction is 3 units.

Mark point C.

Tell how many measurements were required to locate point C.

2. Present a number line such as the one below.


A
B
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11

Ask children:

If you know the distance of point C from A and that C is on $\overleftrightarrow{AB}$, how many different points could be named C, if C is 3 units from point A?

[Two points, either in the negative or positive direction from A.]

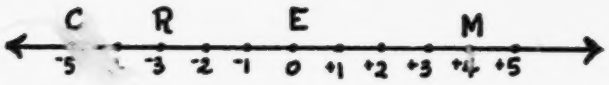
What did we have to know to locate point C in Figure I?

[distance and direction]

What kind of numbers tell both the direction and distance of a point from A. [positive and negative]

Discuss: The number that tells both distance and direction of a point on a line from the "0" point is called the coordinate of the point on the number line.

For example: On the number line below the coordinate of R is $^{-}3$; the coordinate of M is $^{+}4$.



3. Tell children that each point in a number line can be called the graph of the number to which it corresponds. If we wish to consider the graph of a set of points it helps to darken the points corresponding to their numbers.

For example:

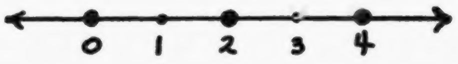


Fig. II

Figure II is the graph of the points whose coordinates are 0, 2, and 4, or the set { 0, 2, 4 }.

Ask children to graph the following sets of points whose coordinates are:

$^{-}3$ 0 4 9

Graph each on a separate horizontal and vertical number line.

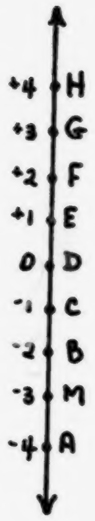
Present a vertical number line.

Direct children:

Show the graph of 2.

Circle the coordinate of point B.

Graph the solution set for
n = 0 + 4 on another number line



Locating Points In a Plane

1. Ask children to observe the number line and points R and S below and answer the questions.

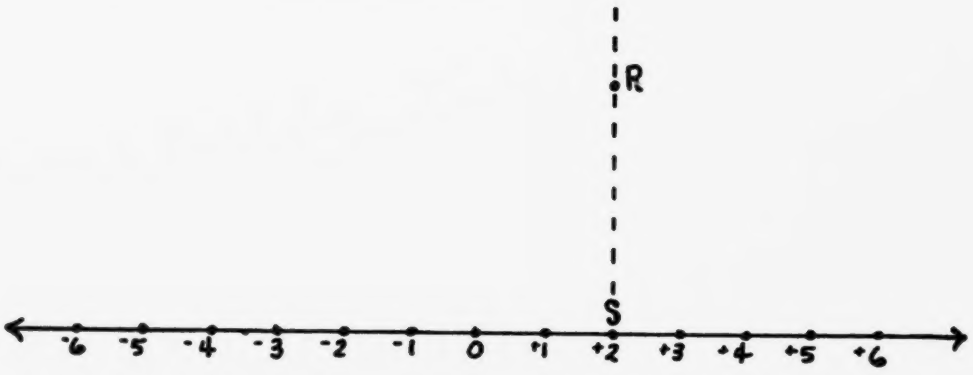


Fig. I

What is the coordinate of point S?
How can we state the position of point S?
Can we tell the direction of point R from point S?
Can we state the position of point R? Why or why not?

[No, we do not know how far above the line it is.]

2. Tell children that we can state its position by using a second (vertical) number line which intersects the horizontal number line forming right angles and which has the same zero point.

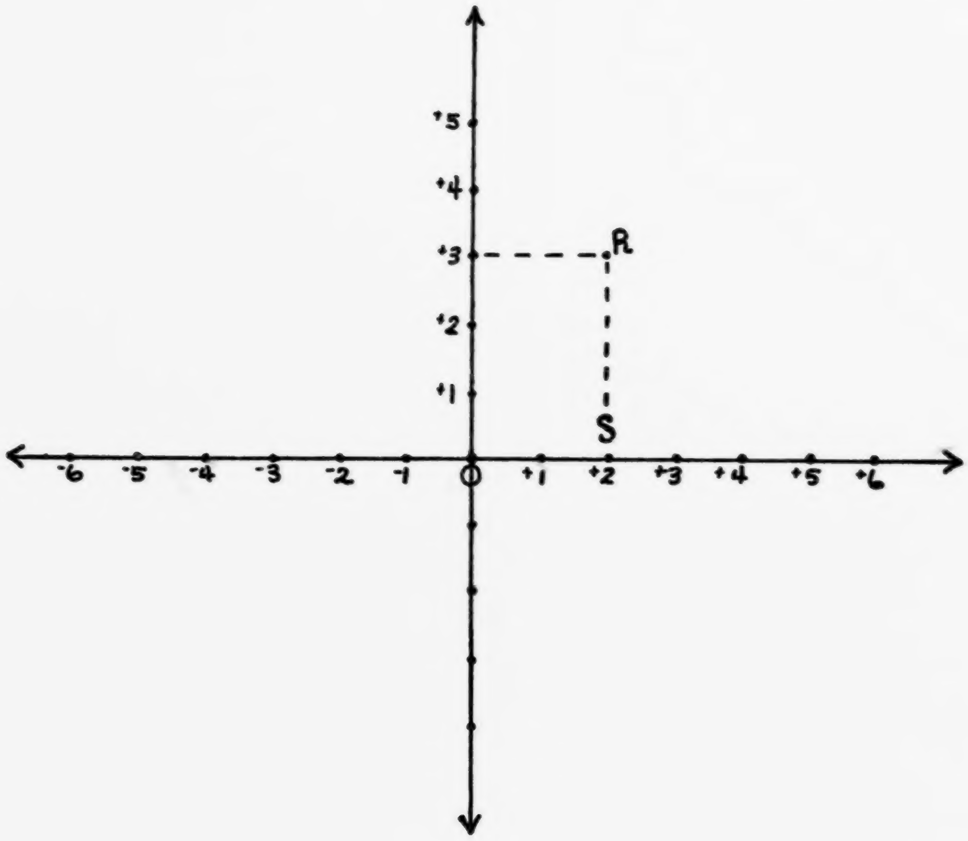


Fig.II

Children should compare Figure I and Figure II.

Direct children:

How did we describe the position of point R in Figure I?
[above $^{+}2$]

Observe Figure II. Can we state the position of point R more exactly now?
[Yes, it is above $^{+}2$ on the horizontal axis and to the right of $^{+}3$ on the vertical axis.]

How do we describe direction on a number line?

[positive direction
negative direction]

Is there any point except R which is exactly above $^{+}2$ on the horizontal number line and also exactly to the right of $^{+}3$ on the vertical number line?

[NO]

How many numbers are required to describe the position of point R?

[2]

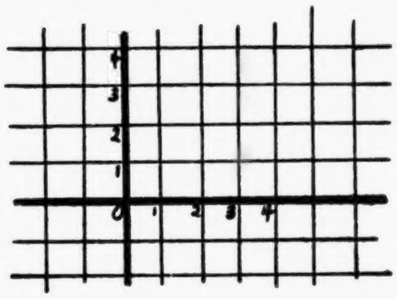
3. Tell children:

   The two numbers necessary to describe position in a plane form an ordered pair of numbers called the coordinates of the point.

   The first number of an ordered pair tells the number on the horizontal axis; the second number of the ordered pair tells the number on the vertical axis.

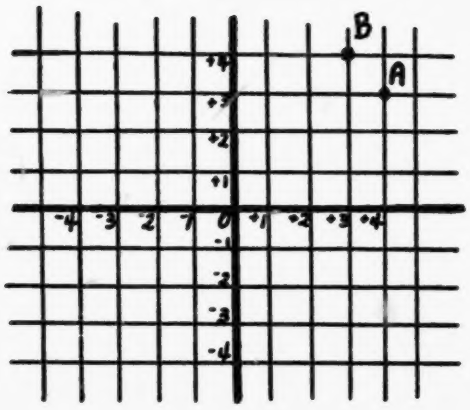
   We write the ordered pair for R as: (2, 3).

4. Discuss use of graph paper to help in locating the position of points in a plane. (The lines are already drawn perpendicular.)



The Importance of Ordered Pairs

Have children examine a graph such as the one below:



Ask children:

What are the coordinates of A? [$(^{+}4, ^{+}3)$]

Which coordinate is the horizontal coordinate; the vertical coordinate?

What are the coordinates of B? $(^{+}3, ^{+}4)$

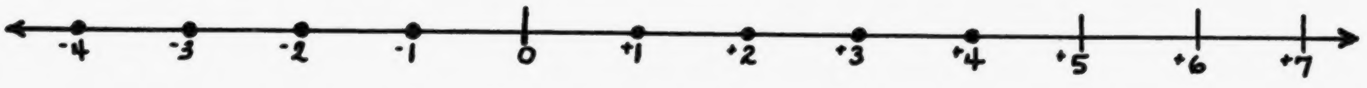
Which coordinate is the horizontal coordinate; the vertical coordinate?

Do the coordinates $(^{+}4, ^{+}3)$ and $(^{+}3, ^{+}4)$ locate the same point? Explain.

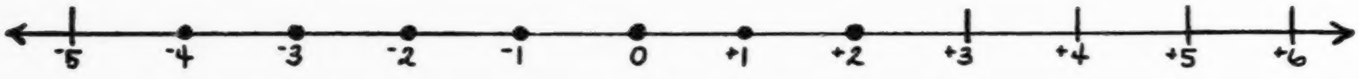
Teacher should emphasize the importance of "ordered pair".

Suggested Practice Exercises

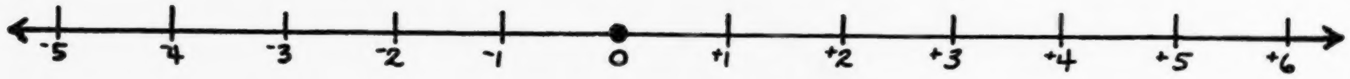
1. List or describe the sets of integers whose graphs are shown below.



[$\{^{-}4, ^{-}3, ^{-}2, ^{-}1, ^{+}1, ^{+}2, ^{+}3, ^{+}4\}$]



$[\{{}^-4, {}^-3, {}^-2, {}^-1, 0, {}^+1, {}^+2\}]$



$[\{0\}]$

2. Draw two number lines showing the integers from ${}^-5$ to ${}^+5$ and graph the sets below; first on a horizontal number line and then on a vertical number line.

   a. $\{{}^-4, {}^-2, {}^+1, {}^+3\}$

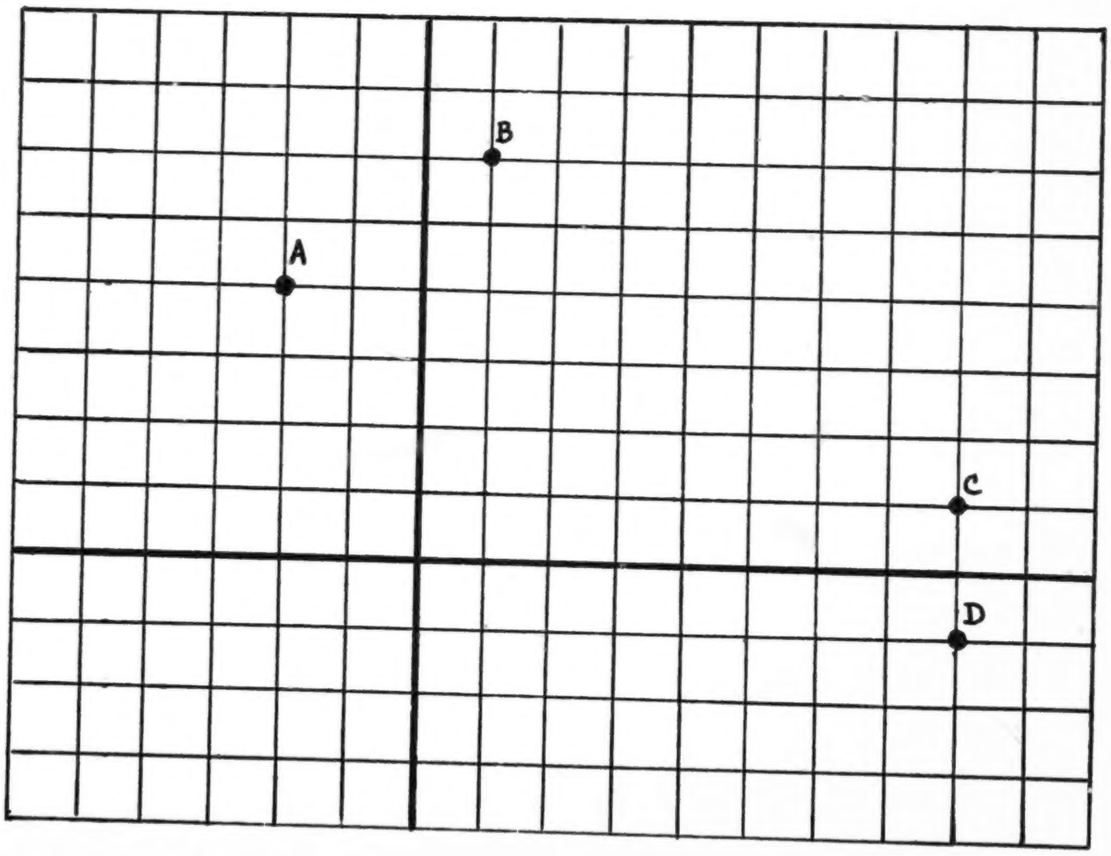
   b. $\{{}^-3, 0, {}^+3, {}^+4, {}^+5\}$

3. Use graph paper. Choose two perpendicular lines for coordinate axes and darken them to show the lines chosen.

   Graph the following ordered pairs and label each point.

   A $({}^+2, {}^+5)$ B $({}^-1, {}^+4)$ C $({}^+3, {}^-2)$

4. Write the coordinates (ordered pairs) for each labeled point on the graph below.



5. Locate four points forming a square on the above set of axes and indicate the coordinates of each point.

6. Do the same (as ex. 5) for other geometric figures.

Graphing the Truth Set For An Open Sentence. (Reference: Student's Discussion Guide - Madison Project - pp 18, 19)

1. Present an open sentence; e.g., □ + △ = 5

Have children find the solution set when the replacement set is {0, 1, 2, 3, 4, 5}.

(Review meaning of replacement set.)

As children respond, teacher may tabulate as follows:

□ + △ = 5

When □ = 0 then △ = 5

When □ = 1 then △ = 4

etc.

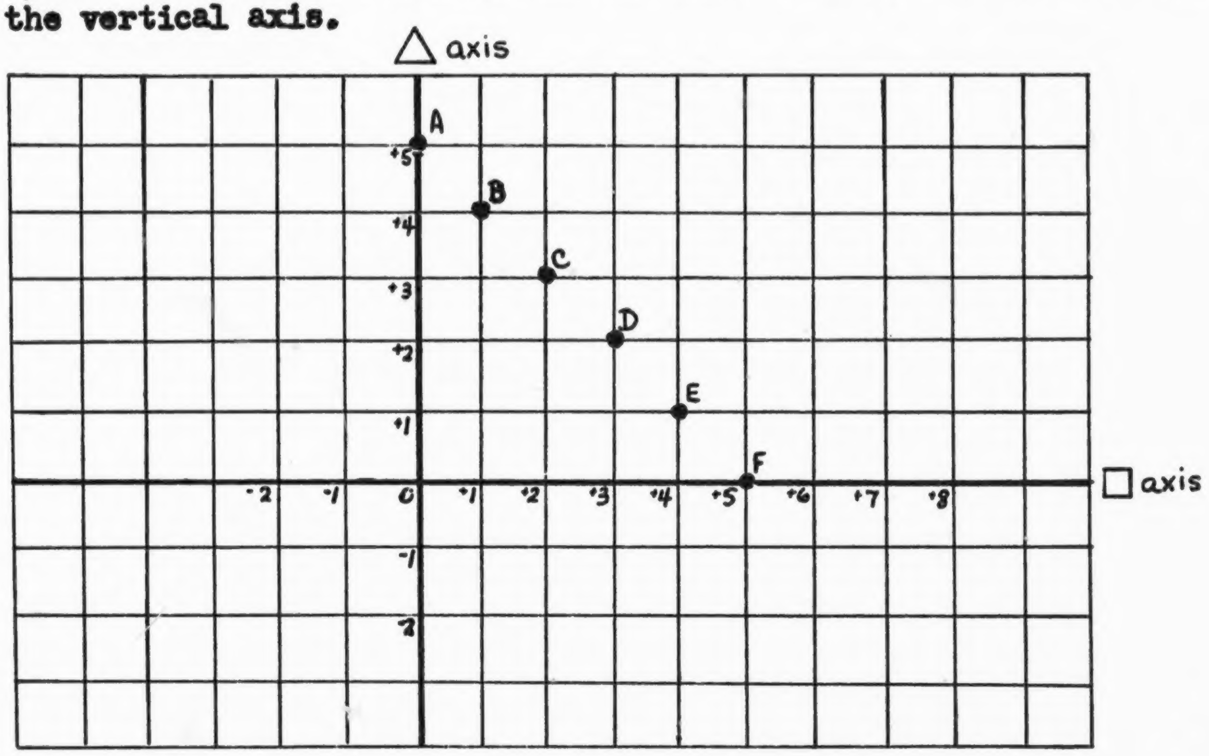
□	△
0	5
1	4
2	3
3	2
4	1
5	0

Ask children:

How many ordered pairs will make the sentence true?

What are the ordered pairs? [0, 5; 1, 4; 2, 3; etc.]

Have children graph each of the ordered pairs in the truth set. They may place the numbers for □ on the horizontal axis, numbers for △ on the vertical axis.



Have children observe that these points lie on a straight line.

*2. Have children explore the same problem when the replacement set includes negative numbers.

For example: if the replacement set is $\{ {}^-3, {}^-2, {}^-1, 0, {}^+1, {}^+2, {}^+3 \}$

3. Have each child prepare a table to represent the truth set for the following open sentence:

$\triangle = \square + 1$

For example:
When $\square = 0, \triangle = 1$
$\square = 1, \triangle = 2$

□	△
0	1
1	2
2	3

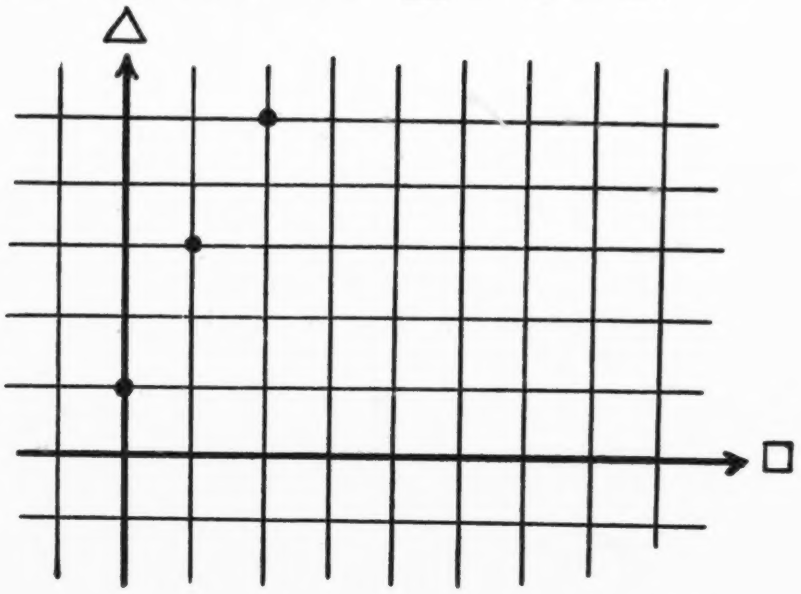
etc.

4. Have children graph the truth set for $\triangle = \square + 1$ when the replacement set is $\{4, 5, 6, 7\}$.

Suggest to children that □ represents the horizontal axis, △ represents the vertical axis.

Observe that the points of the solution set lie on a straight line.

5. Present a graph as follows. Tell children that the graph has been started for the open sentence $\triangle = (2 \times \square) + 1$



Have children mark 3 more points on the graph for the open sentence above.

Ask children to "guess" another point, to explain why they picked it, and then to verify their "guess" by substituting the value for □ and △ in the open sentence.

Suggested Practice Exercises

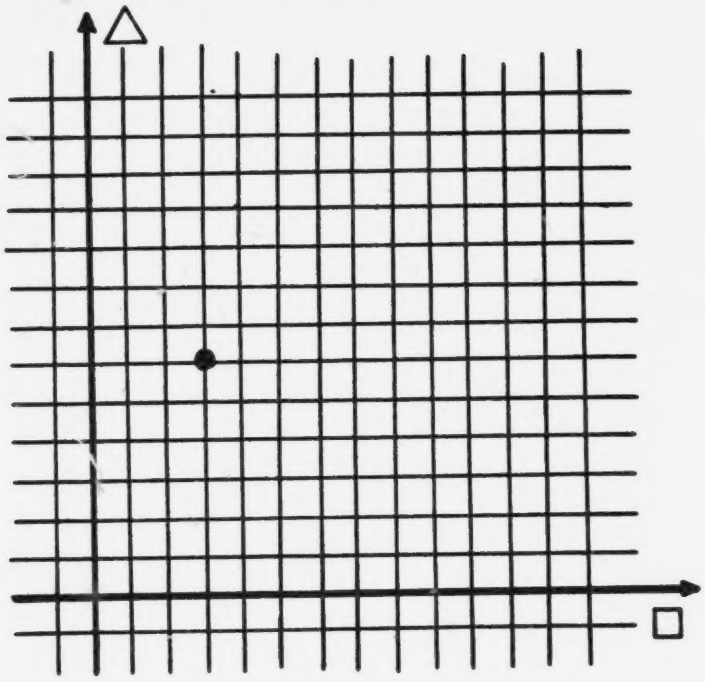
1. Use the table to mark 4 more points on the graph below.
(Reference: Madison Project - Student's Guide - p. 18)

△ = □ + 3

Open Sentence

□	△
3	6
4	7
5	8
6	9

Table for Truth Set



Graph for Truth Set

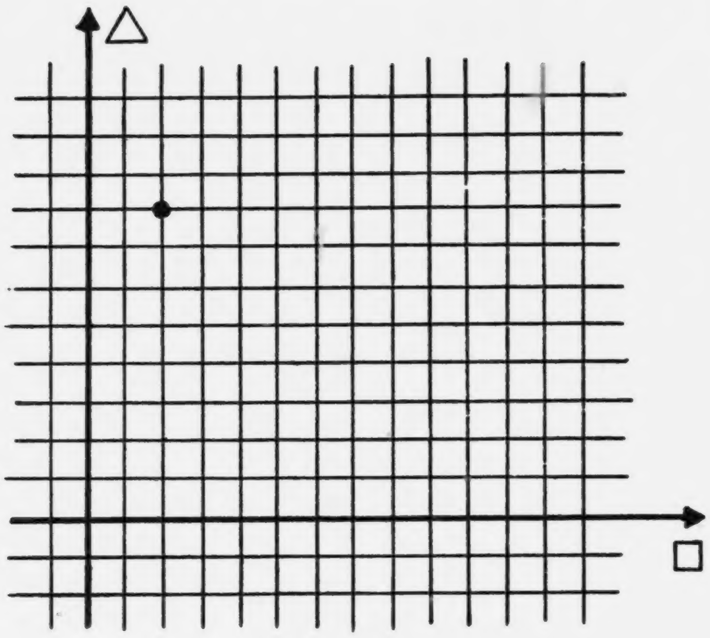
2. Complete the following table to represent the truth set for the open sentence: △ = (4 x □) + 2

□	△
2	4
3	
4	
5	

Children graph the truth set.

Children may connect the points by drawing line segments between them. What do they observe?

3. Write at least 3 true sentences represented by the graph below.



$\triangle 8 = \boxed{2} \times 4$

$\triangle 8 = \boxed{2} + 6$

etc.

SETS; NUMBER; NUMERATION

*UNIT 80 - PROBABILITY (Optional)

NOTE TO TEACHER

Why Probability?

Much that occurs in our lives depends upon chance. Part of our lives is spent considering uncertainties. Things may happen or they may not happen.

Probability concepts and calculations have become increasingly important in our modern age. Medicare, business, weather, insurance, investments all involve consideration of a probability measure. Hence its introduction in its simplest forms into the elementary school curriculum is desirable.

A few illustrations will give us a clue to some simple applications of the mathematics of Probability. If a die (singular of dice) is thrown what are the chances that a 2 will appear? Since there are 6 possible numbers that may appear (a cube has six faces, hence 6 outcomes) and since it is equally likely that any one face may turn up, there is one out of 6 that a 2 will turn up.

If in the dark I reach into a drawer containing 6 gloves, paired and placed in plastic bags, 2 blacks, 2 white, 2 brown, what are my chances of extracting a pair of a given color? The chances are 1 to 3 of finding a pair of brown gloves. ($\frac{1}{3}$ would not be the answer if the 6 gloves were loose in the drawer.)

The Probability of an event may be defined as the ratio of the number of favorable ways that the event can occur to the total number of equally likely outcomes.

To introduce concepts involving probability, experiments should be performed and a count made of the outcomes. For example:

1. Toss a coin many times and observe the number of times a head appears.

2. Toss two coins and observe the number of times
    a. 2 heads
    b. one head and one tail
    c. 2 tails

   come up.

3. Toss a cork from a bottle 5 times.

4. Other experiments involving similar situations.

If children will keep a tally of the number of times a given outcome or set of outcomes (called an Event) occurs and the total number of times the experiment is performed, the ratios involved should prove interesting. For example: In (1) the ratio 1 : 2 (or fraction $\frac{1}{2}$) will show up after many tosses of the coin.

In 50 tosses the likelihood is that heads will turn up about 25 times; in 100 tosses the chances are that heads will turn up about 50 times, etc.

Objectives: To experiment with "What are the Chances".

To introduce beginning concepts of Probability.

TEACHING SUGGESTIONS

Suggested Experimentation

Tossing A Coin

1. Ask children to toss a coin.

When one coin is tossed what is the number of possible ways that the coin can fall? What is the chance of heads turning up out of the 2 possibilities?

[1 chance out of 2 possibilities]

Tell children that another way of saying this is the probability is 1 out of 2 or $\frac{1}{2}$.

2. Have children record results to see how probabilities change when the same action is repeated.

We know that in tossing a coin once the probability is "1 out of 2" that heads will turn up, or that tails will turn up. If we toss the coin a second time, then a third time, etc., what will happen?

Have children keep a tally. For example: The table below shows the results one child may have in 8 tries:

H	T
///	/////

Ask children:

In examining the Tally of Ellen's experiment:

1. How do you know the number of tries? [count]
   How many were there? [8]
   How many times did heads turn up? [3]
   How many times did tails turn up? [5]

   After 8 tries what is the probability that a head will turn up on the 9th try?

3. Have children extend their tallies by experimenting to see what happens after 10 throws, 20 throws, 30 throws, 50 throws.

   Children should experiment as many as 100 times.

4. Have each child compare his tally with the tallies of other children.

Children should observe that in many tosses, a coin may be expected to come down heads as many times as it comes down tails. It does not mean that a coin will come down alternately heads and tails in consecutive throws.

5. Experiment with an irregular object, such as a cork of a bottle or a paper fastener, where there are two possible ways it could fall. Find the Probability of the object falling one way by a tabulation.

6. Have children tabulate for the following experiment:

   If Alan picked a ball from a box, containing blue and red balls, noted the color, put it back and picked again, and did this 30 times, about how many times should he expect a blue ball to be picked? A red ball to be picked?

7. Further Experimentation

Some children may wish to experiment keeping tallies of the probabilities when tossing 2 coins; 3 coins; etc.

For example: In one person's tally, out of 16 throws of 2 coins, heads turned up 4 times, head and tail turned up 9 times, etc.

0 Tails 2 Heads	1 Tail 1 Head	0 Heads 2 Tails
////	///// ////	///

Children should make and tally enough throws to make a prediction.

Have children compare the actual results with the predicted possibilities.

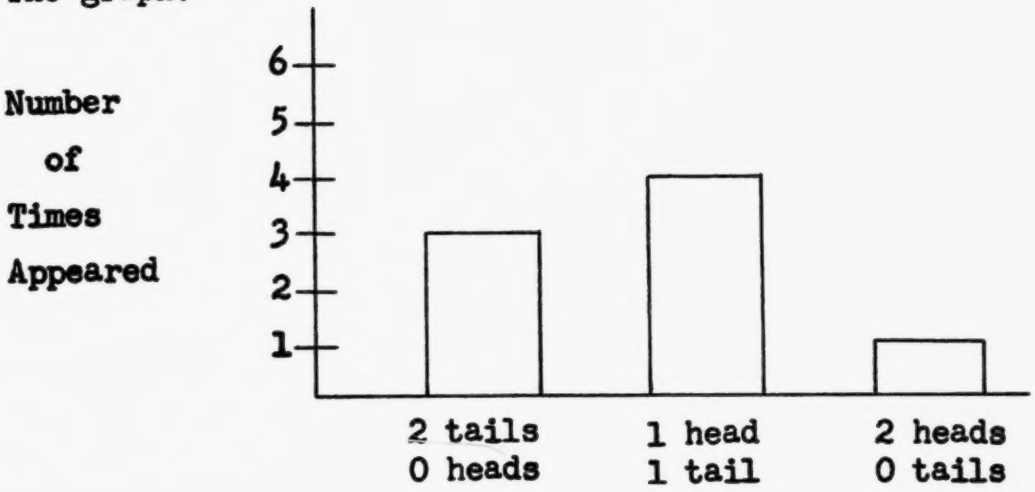
Graphing Probabilities

Have children make a bar graph of the results of their tallies.

a. For 1 child's tally of 8 throws of 2 coins, with the result of:

2 heads,	0 tails	—	3 Times
1 head,	1 tail	—	4 Times
0 heads,	2 tails	—	1 Time

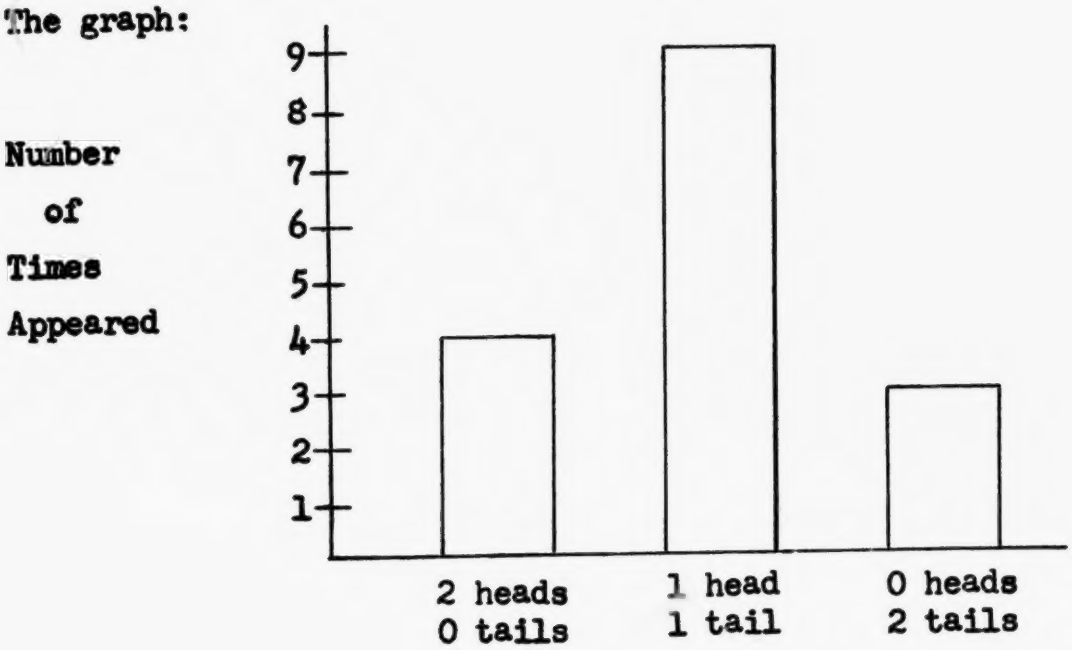
The graph:



b. Have the child graph the result of 16 throws. The result might be as in the tally above:

2 heads,	0 tails	—	4 times
1 head,	1 tail	—	9 times
0 heads,	2 tails	—	3 times

The graph:



c. Children should graph the results of many experiments e.g., 20 throws, 40 throws, etc.

Discuss which possibility is most likely to occur.

Have children compare the various graphs to see the shape of the curve that evolves as the number of tossings increases indefinitely.